Mathematics, KTH
B.Ek

SF2736 Discrete mathematics
autumn 2015

Homework assignment set number 2, autumn 2015

Solutions (handwritten, in English or in Swedish) must be submitted not later than **Monday 21 December**, either in the black box on the wall to the left immediately inside the front door Lindstedtsvägen 25 (marked "To Bengt Ek") or by post (arriving not later than that Monday) to:

Bengt Ek
Matematik
KTH
100 44 Stockholm.

Write **your name and your e-mail address** on your solutions.

It is allowed to discuss the problems with classmates, but not to obtain help from others (also not from fora on the internet). Everyone must write his/her own solutions, so copying is not allowed (and would be considered as cheating).

To give full points, the solutions must be clear and well explained.

The points given for the solutions will be added to the points on the exam, as explained on the course's web page (the total number of points on homework will be rounded to the nearest integer; $x + 0.5$ ($x \in \mathbb{N}$) is rounded to $x + 1$).

1. (0.4p) Show that for all $m, n \in \mathbb{Z}_+$

$$\sum_{k=1}^{m} S(m,k) \cdot (n)_k = n^m.$$

($S(m,k)$ is, as usual, the Stirling number (of the second kind). $(n)_k = n \cdot (n-1) \cdot \ldots \cdot (n-k+1)$.)

2. (0.4p) Let, for all $n \in \mathbb{Z}_+$, $\sigma(n)$ be the sum of all positive divisors of n and $\mu(n)$ be the Möbius function. What is for $n \in \mathbb{Z}_+$ (sum over all positive divisors of n)

$$\sum_{d|n} \mu(d)\sigma(\tfrac{n}{d})?$$

3. (0.4p) In how many essentially different ways can the edges of a cube be coloured with four red, four blue and four yellow edges? ("Essentially different" means that none of them can be rotated to look exactly like any other.)

4. (0.4p) Show that, for any $n \in \mathbb{Z}_+$, the number of partitions of n such that there are at most k parts of size k (for $k \in \mathbb{Z}_+$) is equal to the number of partitions of n with no part of size k^2+k (for $k \in \mathbb{Z}_+$). I.e., in Biggs' notation:

$$p(n \mid \text{for } k \in \mathbb{Z}_+\text{: at most } k \text{ parts of size } k) =$$
$$= p(n \mid \text{for } k \in \mathbb{Z}_+\text{: no part of size } k^2 + k).$$

5. (0.4p) Given a (finite) set A of people, each of them with a list of books from a set B (also finite) he or she wants to borrow.
For $k \in \mathbb{Z}_+$, find a (reasonably simple) necessary and sufficient condition on the lists of the persons in A for it to be possible for each of them to borrow k books from her or his list (each book can only be borrowed by one person).