

Svar till övning 1, den 23 januari 2015

1a. $(355)_{10}, (3827)_{10}$, b. $(101100011)_2, (111011110011)_2$, c. $(1015)_7, (14105)_7$.

2. Om $m = (abc)_t$ med $a > c$ blir $n = (cba)_t$ och $p = m - n = ([a - c - 1][t - 1][t - a + c])_t$. Det ger $q = ([t - a + c][t - 1][a - c - 1])_t$ och $p + q = (10[t - 2][t - 1])_t$.

3. $(10110011110101)_2 = (10)_2 \cdot 16^3 + (1100)_2 \cdot 16^2 + (1111)_2 \cdot 16 + (0101)_2 = (2CF5)_{16} = (10)_2 \cdot 8^4 + (110)_2 \cdot 8^3 + (011)_2 \cdot 8^2 + (110)_2 \cdot 8 + (101)_2 = (26365)_8$.
 $(364401)_8 = (11110100100000001)_2 = (1E901)_{16}$.

4. $\text{sgd}(14, -49) = 7$, $\text{sgd}(-11, -7) = 1$, $\text{sgd}(1, 1772) = 1$,
 $\text{sgd}(-1632, 0) = 1632$, $\text{sgd}(0, 0) = 0$ (inte definierad med bokens definition av sgd).

5. Euklides algoritm och $\text{sgd}(m, n) \cdot \text{mgm}(m, n) = m \cdot n$.

$$\begin{array}{ll} 2373 &= 1 \cdot 1638 + 735 & 21 &= 63 - 1 \cdot 42 = 63 - 1(168 - 2 \cdot 63) = \\ 1638 &= 2 \cdot 735 + 168 & &= -168 + 3 \cdot 63 = -168 + 3(735 - 4 \cdot 168) = \\ 735 &= 4 \cdot 168 + 63 & &= 3 \cdot 735 - 13 \cdot 168 = \\ 168 &= 2 \cdot 63 + 42 & \text{och} &= 3 \cdot 735 - 13(1638 - 2 \cdot 735) = \\ 63 &= 1 \cdot 42 + 21 & &= -13 \cdot 1638 + 29 \cdot 735 = \\ 42 &= 2 \cdot 21 + 0 & &= -13 \cdot 1638 + 29(2372 - 1638) = \\ & & &= 29 \cdot 2372 - 42 \cdot 1638. \end{array}$$

Så $\text{sgd}(2373, 1638) = 21 = 29 \cdot 2373 - 42 \cdot 1638$, dvs $a = 29$, $b = -42$.

$$\text{mgm}(2373, 1638) = \frac{2373 \cdot 1638}{21} = 185094.$$

6. $\text{sgd}(m, n) = \text{sgd}(m, n - m) = \text{sgd}(m - n, n)$.

$$\text{sgd}(3k + 2, 5k + 3) = \text{sgd}(3k + 2, 2k + 1) = \dots = \text{sgd}(1, k) = 1.$$

7. $d \mid m, n \Rightarrow d \mid am + bn \dots$

$$8. \text{abcabc} = abc \cdot 1001 = abc \cdot 7 \cdot 11 \cdot 13.$$