Midterm REPORT
Pendubot

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2006-04-07

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Chapter 1

Introduction

1.1 Description of the problem

The system that will be controlled in this project is the Pendubot.The control system should have the following features:

- Swing up the outer link of the Pendubot to upright position:
  - when the inner link is pointing downwards : this will be called the ”Down-up” control
  - when the inner link is pointing upwards : this will be called the ”Up-up” control
- Stabilize the links in these configurations, while moving the inner link slowly from one peripheral position to the other.
- Implement a software safety net, such that when the system enters an unsafe state or gets disturbed, both links should be controlled to their hanging down position. This should also be the procedure at a system shut down.

This project will be run in several steps :

- Derivation of the physical model of the system
- Linearization of the non-linear model
- Choice of a control strategy for the up-up and down-up positions
- Choice of a control strategy for the peripheral movement
- Choice of a control strategy for the swing-up movements
- Implementation and validation

1.2 Description of the Pendubot

The Pendubot is a two-link robot. It consists of a motor which is connected to an arm (arm 1), to this arm another arm (arm 2) is connected. By applying a voltage to the motor the motor will give out a torque acting on arm 1. The torque on arm 1 will also via some non-linear relation affect arm 2.

The system can as shown in 1.1 be described as a 2^{nd}order mechanical system. As shown later in this report one can using some control theory achieve impressive performance on this system.

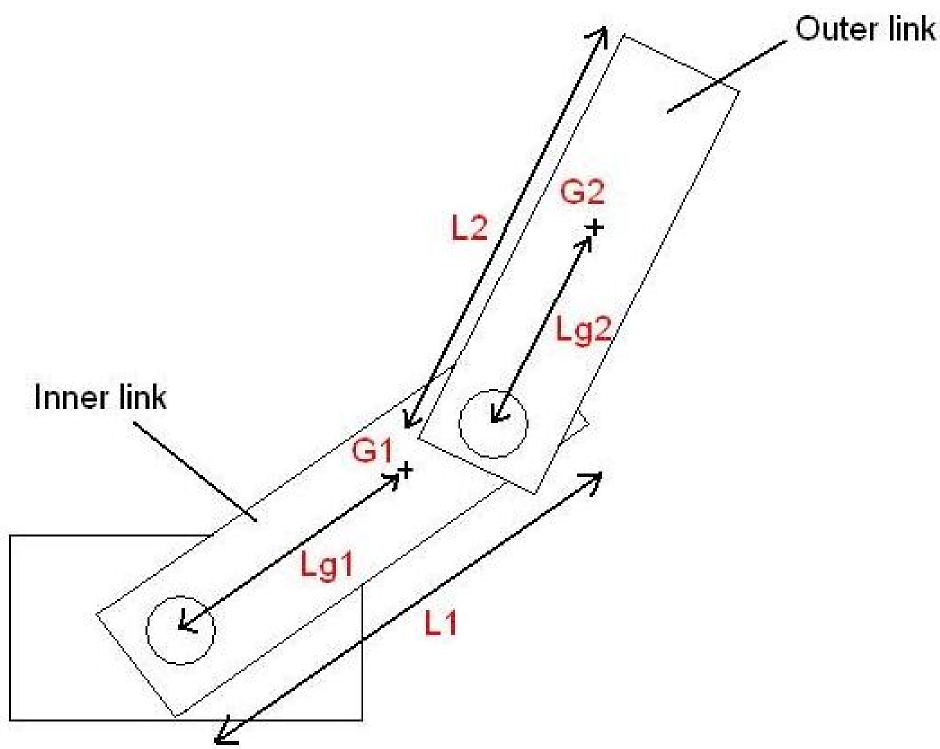

Figure 1.1: Scheme of the Pendubot

Chapter 2

Modelization of the Pendubot

2.1 Modelization's strategy

In mechanics two approaches are possible:

- The first one, Lagrange's approach, is based on a energetic description of the system.
- The second one, Newton's approach, is based on the fundamental principle of dynamics.

In this case, Lagrange's approach seems easier as only the kinetic and potential energies of the system are necessary to describe it. This theory assumes that one system can entirely be described with a convenient set of parameters. Here, the knowledge of the angles of the inner and the upper link is enough to describe completely the system in every positions.

The derivation of the physical model will thus be completed in steps:

1. Derivation of all the physical parameters of the Pendubot
2. Derivation of the nonlinear equations using Lagrange's theory
3. Linearization of the nonlinear model

2.2 Derivation of the parameters

2.2.1 Inner link

The following parameters are needed for the description of the system:

Total length of the arm	$l_1 = 0,21\,m$
Distance from the axis of rotation to the center of gravity	$l_g1 = 0,116\,m$
Width of the link	$w = 0,005\,m$
Weight of the link	$M_{inner} = 138,75\,g$
Total weight of the arm	$M_1 = 482,5\,g$
Weight of the wheel	$M_{Wheel} = 51,9\,g$
Weight of the sensor	$M_{Sensor} = 291,85\,g$
Radius of the wheel	$R_{Wheel} = 0,015\,m$
Distance from the axis of rotation to the wheel	$l_{Wheel} = 0,08\,m$

The moment of inertia of the inner link can now be calculated. It can be computed by adding the moments of every parts of the arm:

$$J_1 = J_{Inner} + J_{Wheel} + J_{Sensor} \tag{2.1}$$

Where:

- $J_{Inner} = \frac{1}{12} M_{inner} \left(l_1^2 + w^2\right) = 0,51.10^{-3}\,kg.m^2$
- $J_{Wheel} = \frac{1}{2} M_{Wheel} R_{Wheel}^2 + M_{Wheel} l_{Wheel}^2 = 0,338.10^{-3}\,kg.m^2$
- $J_{Sensor} = M_{Sensor} l_{Sensor}^2 = 7,19.10^{-3}\,kg.m^2$

The total moment of inertia of the inner link is thus:

$$\boxed{J_1 = 8,03.10^{-3}\,kg.m^2}$$

2.2.2 Outer link

The following parameters are needed for the description of the system:

Total length of the arm	$l_2 = 0,233\,m$
Distance from the axis of rotation to the center of gravity	$l_g2 = 0,134\,m$
Width of the link	$w = 0,005\,m$
Weight of the link	$M_{Outer} = 117\,g$
Total weight of the arm	$M_2 = 220,8\,g$
Weight of the wheels	$M_{Wheel} = 51,9\,g$
Radius of the wheels	$R_{Wheel} = 0,015\,m$
Distance from the axis of rotation to the 1^{st} wheel	$l_{Wheel1} = 0,117\,m$
Distance from the axis of rotation to the 2^{nd} wheel	$l_{Wheel2} = 0,221\,m$

The moment of inertia of the outer link can now be calculated. It can be computed by adding the moments of every parts of the arm:

$$J_2 = J_{Outer} + J_{Wheel1} + J_{Wheel2} \tag{2.2}$$

Where:

- $J_{Outer} = \frac{1}{12} M_{Outer} \left(l_2^2 + w^2 \right) = 0,529.10^{-3}\ kg.m^2$
- $J_{Wheel1} = \frac{1}{2} M_{Wheel} R_{Wheel}^2 + M_{Wheel} l_{Wheel1}^2 = 0,716.10^{-3}\ kg.m^2$
- $J_{Wheel2} = \frac{1}{2} M_{Wheel} R_{Wheel}^2 + M_{Wheel} l_{Wheel2}^2 = 0,567.10^{-3}\ kg.m^2$

The total moment of inertia of the outer link is thus:

$$\boxed{J_2 = 1,812.10^{-3}\ kg.m^2}$$

2.3 Derivation of the Nonlinear Model

2.3.1 Dynamic equations

According to Lagrange's theory, a system can be totally described by using a convenient set of parameters q_i . Then, the studied system will be ruled by the following set of Lagrange's equations:

$$L = E_k - E_p \tag{2.3}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \sum Non\ conservative\ torques \tag{2.4}$$

Where L is called the Lagrangian of the system. In this case, a convenient set of parameters is $\{\theta_1; \theta_2\}$. Indeed, no more parameter is needed to know exactly the position of the Pendubot at any time.
It is now necessary to calculate the energy of both arms.

Kinetic energy of the inner link

The inner link only has a movement of rotation around one axis. Its moment of inertia can thus be simply computed as follows:

$$E_{k1} = \frac{1}{2} J_1 \dot{\theta}_1^{\,2} \tag{2.5}$$

Where $\dot{\theta}_1$ is the angular velocity of the inner link.

Kinetic energy of the outer link

The movement of the outer link is composed by one movement of rotation and one movement of translation. Thus, in order to calculate its total kinetic energy the energy from both movements must be added. Koenig's relation thus has to be applied as follows :

$$E_{k2} = \frac{1}{2} J_2 \dot{\theta_2}^2 + \frac{1}{2} M_2 {V_{G2}}^2 \tag{2.6}$$

Where $\dot{\theta}_2$ is the angular velocity of the link and V_{G2} is the velocity of its center of gravity. It can be calculated using the classical laws of kinematics and the geometry of the system :

$$\vec{V_{G2}} = \frac{d\vec{OG}_2}{dt} \tag{2.7}$$

with $\vec{OG}_2 = (l_1 cos(\theta_1) + l_{G2} cos(\theta_2))\, \vec{x} + (l_1 sin(\theta_1) + l_{G2} sin(\theta_2))\, \vec{y}$

This leads to the following result:

$$\vec{V_{G2}} = \left(-\dot{\theta}_1 l_1 sin(\theta_1) - \dot{\theta}_2 l_{G2} sin(\theta_2)\right) \vec{x} + \left(\dot{\theta}_1 l_1 cos(\theta_1) + \dot{\theta}_2 l_{G2} sin(\theta_2)\right) \vec{y} \tag{2.8}$$

Finally, the kinetic energy of the outer link is :

$$E_{K2} = \frac{1}{2} J_2 \dot{\theta}_2^2 + \frac{1}{2} M_2 \left(\dot{\theta}_1^2 l_1^2 + \dot{\theta}_2^2 l_{G2}^2 + +2 l_1 l_2 \cos(\theta_1 - \theta_2) \dot{\theta}_1 \dot{\theta}_2\right) \tag{2.9}$$

Potential energy

The potential energy of the systems only comes from the action of the gravity. It can be easily calculated using the geometry of the system:

$$E_{P1} = M_1 l_{G1} g sin\theta_1 \tag{2.10}$$

$$E_{P2} = M_2 g \left(l_1 \sin\theta_1 + l_{G2} \sin\theta_2\right) \tag{2.11}$$

Where $g = 9,81\, m.s^{-2}$ is the constant of gravity.

Derivation of the equations

Now that all the energies are known it is possible to calculate the Lagrangian of the system as follows:

$$\begin{aligned} L &= E_k - E_p \\ &= \dot{\theta}_1^2 \left[\frac{1}{2} J_1 + \frac{1}{2} M_2 l_1^2\right] + \dot{\theta}_2^2 \left[\frac{1}{2} J_2 + \frac{1}{2} m_2 l_{G2}^2\right] + M_2 l_1 l_{G2} \cos\left(\theta_1 - \theta_2\right) \dot{\theta}_1 \dot{\theta}_2 \\ &\quad - \left(M_1 l_{G1} + M_2 l_1\right) g \sin\theta_1 - M_2 g l_{G2} \sin\theta_2 \end{aligned} \tag{2.12}$$

Then, all the necessary calculations are made in several steps :

- Partial derivative with respect to the angular velocities :

$$\frac{\partial L}{\partial \dot{\theta}_1} = \dot{\theta}_1 \left[J_1 + M_2 l_1^2 \right] + M_2 l_1 l_{G2} \cos\left(\theta_1 - \theta_2\right) \dot{\theta}_2 = A\left(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2\right) \tag{2.13}$$

$$\frac{\partial L}{\partial \dot{\theta}_2} = \dot{\theta}_2 \left[J_2 + l_{G2}^2 M_2 \right] + M_2 l_1 l_{G2} \cos\left(\theta_1 - \theta_2\right) \dot{\theta}_1 = B\left(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2\right) \tag{2.14}$$

- Partial derivative of the previous equations with respect to the time :

$$\begin{aligned} \frac{d}{dt}\left(A\left(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2\right)\right) =& \frac{\partial A}{\partial \theta_1}\dot{\theta}_1 + \frac{\partial A}{\partial \theta_2}\dot{\theta}_2 + \frac{\partial A}{\partial \dot{\theta}_1}\ddot{\theta}_1 + \frac{\partial A}{\partial \dot{\theta}_2}\ddot{\theta}_2 \\ =& - M_2 l_1 l_{G2} \sin\left(\theta_1 - \theta_2\right) \dot{\theta}_1 \dot{\theta}_2 + M_2 l_1 l_{G2} \dot{\theta}_2^2 \sin\left(\theta_1 - \theta_2\right) \\ &+ \ddot{\theta}_1 \left(J_1 + M_2 l_1^2\right) + \ddot{\theta}_2 M_2 l_1 l_{G2} \cos\left(\theta_1 - \theta_2\right) \end{aligned} \tag{2.15}$$

$$\begin{aligned} \frac{d}{dt}\left(B\left(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2\right)\right) =& \frac{\partial B}{\partial \theta_1}\dot{\theta}_1 + \frac{\partial B}{\partial \theta_2}\dot{\theta}_2 + \frac{\partial B}{\partial \dot{\theta}_1}\ddot{\theta}_1 + \frac{\partial B}{\partial \dot{\theta}_2}\ddot{\theta}_2 \\ =& - \dot{\theta}_1^2 M_2 l_1 l_{G2} \sin\left(\theta_1 - \theta_2\right) + \dot{\theta}_1 \dot{\theta}_2 M_2 l_1 l_{G2} \sin\left(\theta_1 - \theta_2\right) \\ &+ \ddot{\theta}_1 M_2 l_1 l_{G2} \cos\left(\theta_1 - \theta_2\right) + \ddot{\theta}_2 \left(J_2 + M_2 l_{G2}^2\right) \end{aligned} \tag{2.16}$$

- Partial derivative with respect to the angles:

$$\frac{\partial L}{\partial \theta_1} = -M_2 l_1 l_{G2} \sin\left(\theta_1 - \theta_2\right) \dot{\theta}_1 \dot{\theta}_2 - \left[l_{G1} M_1 + M_2 l_1\right] g \cos\theta_1 \tag{2.17}$$

$$\frac{\partial L}{\partial \theta_2} = -M_2 l_1 l_{G2} \sin\left(\theta_1 - \theta_2\right) \dot{\theta}_1 \dot{\theta}_2 - M_2 l_{G2} g \cos\theta_2 \tag{2.18}$$

The final relations of the model can then be derived:

$$Torque = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} \tag{2.19}$$

$$0 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} \tag{2.20}$$

‘ Which gives :

$$\begin{aligned} Torque =& \ddot{\theta}_1 \left(J_1 + M_2 l_1^2\right) + \ddot{\theta}_2 M_2 l_1 l_{G2} \cos(\theta_1 - \theta_2) + M_2 l_1 l_{G2} \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) \\ &+ \left(M_1 l_{G1} + M_2 l_1\right) g \cos\theta_1 \end{aligned} \tag{2.21}$$

$$\begin{aligned} 0 =& \ddot{\theta}_1 M_2 l_1 l_{G2} \cos(\theta_1 - \theta_2) + \ddot{\theta}_2 \left(J_2 + M_2 l_{G2}^2\right) - M_2 l_1 l_{G2} \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) \\ &+ M_2 g l_{G2} \cos\theta_2 \end{aligned} \tag{2.22}$$

2.3.2 State space equations

The previous equations are the general relations that describe the model. To simplify them, new constants will be introduced as follows :

$$\begin{cases} A_1 = J_1 + l_{G1}^2 M_1 + m_2 l_1^2 \\ A_2 = M_2 l_1 l_{G2} \\ A_3 = M_2 l_1 l_{G2} \\ A_4 = (M_1 l_{G1} + M_2 l_1)\, g \end{cases} \qquad \begin{cases} B_1 = M_2 l_1 l_{G2} \\ B_2 = J_2 + M_2 l_{G2}^2 \\ B_3 = M_2 l_1 l_{G2} \\ B_4 = M_2 g l_{G2} \end{cases}$$

The input torque will be noted τ .

The two variables $\ddot{\theta}_1$ and $\ddot{\theta}_2$ must now be decoupled. Thanks to the second equation, $\ddot{\theta}_2$ can be expressed as a function of all the other variables:

$$\ddot{\theta}_2 = \frac{1}{B_2}\left[-B_1\ddot{\theta}_1\cos(\theta_1-\theta_2) + B_3\dot{\theta}_1^2\sin(\theta_1-\theta_2) - B_4\cos\theta_2\right] \tag{2.23}$$

Then, this relation can be plugged into the first equation :

$$\begin{aligned}\tau =& \theta_1 A_1 + \frac{A_2}{B_2}\cos(\theta_1-\theta_2)\left[-B_1\ddot{\theta}_1\cos(\theta_1-\theta_2) + B_3\dot{\theta}_1^2\sin(\theta_1-\theta_2) - B_4\cos\theta_2\right] \\ &+ A_3\sin(\theta_1-\theta_2)\dot{\theta}_2^2 + A_4\cos\theta_1\end{aligned} \tag{2.24}$$

A first relation involving $\ddot{\theta}_1$ but not $\ddot{\theta}_2$ is thus obtained :

$$\boxed{\begin{aligned}\ddot{\theta}_1 =& \frac{1}{\frac{B_1A_2}{B_2}\cos^2(\theta_1-\theta_2) - A_1}\left[\frac{A_2B_3}{2B_2}\sin\left(2(\theta_1-\theta_2)\right)\dot{\theta}_1^2 + A_3\sin(\theta_1-\theta_2)\dot{\theta}_2^2\right. \\ &\left. - A_2\frac{B_4}{B_2}\cos\theta_2\cos(\theta_1-\theta_2) + A_4\cos\theta_1 - \tau\right]\end{aligned}} \tag{2.25}$$

In a similar way, $\ddot{\theta}_1$ can be expressed as a function of all the other variables and plugged into the second equation :

$$\begin{aligned}0 =& \frac{B_1}{A_1}\cos(\theta_1-\theta_2)\left[-\ddot{\theta}_2A_2\cos(\theta_1-\theta_2) - A_3\dot{\theta}_2^2\sin(\theta_1-\theta_2) - A_4\cos\theta_1 + \tau\right] \\ &+ B_2\ddot{\theta}_2 - B_3\dot{\theta}_1^2\sin(\theta_1-\theta_2) + B_4\cos\theta_2\end{aligned} \tag{2.26}$$

A second relation involving $\ddot{\theta}_2$ but not $\ddot{\theta}_1$ is thus obtained :

$$\boxed{\begin{aligned}\ddot{\theta}_2 =& \frac{1}{\frac{B_1 A_2}{A_1}\cos^2(\theta_1-\theta_2) - B_2}\Big[-B_3\dot{\theta}_1^2\sin(\theta_1-\theta_2) - B_1\frac{A_3}{2A_1}\sin\left(2(\theta_1-\theta_2)\right)\dot{\theta}_2^2 \\ &- B_1\frac{A_4}{A_1}\cos\theta_1\cos(\theta_1-\theta_2) + B_4\cos\theta_2 + \frac{B_1}{A_1}\cos(\theta_1-\theta_2)\tau\end{aligned}} \tag{2.27}$$

Thus, the system is now in a state-space form. If $X = \left[\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2\right]^T$ denotes the state vector containing the four state variables, the following relation is obtained:

$$\dot{X} = F\left(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2\right) + G\left(\tau\right) \tag{2.28}$$

Where

$$F\left(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2\right) = \begin{bmatrix} 0 \quad 0 \quad 1 \quad 0 \\ 0 \quad 0 \quad 0 \quad 1 \\ f_1\left(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2\right) \\ f_2\left(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2\right) \end{bmatrix} \tag{2.29}$$

$$G\left(\tau\right) = \begin{bmatrix} 0 \\ 0 \\ \tau \\ 0 \end{bmatrix} \tag{2.30}$$

$f_1\left(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2\right)$ denotes equation (2.25) and $f_2\left(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2\right)$ denotes equation (2.27).

2.3.3 Validation of the model

Validation of the equilibrium points

It is first necessary to verify that the equilibrium points are correctly placed. The Pendubot has three sets of equilibrium points:

- $[\theta_1 = -\frac{\pi}{2}; \theta_2 = -\frac{\pi}{2}]$: this is the only stable equilibrium
- $[\theta_1 = -\frac{\pi}{2}; \theta_2 = \frac{\pi}{2}]$: this equilibrium is unstable
- $[\theta_1 = \frac{\pi}{2}; \theta_2 = \frac{\pi}{2}]$: this equilibrium is unstable

The behavior of the system in those three positions corresponds to what was expected.

Comparison with the real system

Thanks to the Data Acquisition function of Labview, it is possible to observe the behavior of the system.

Schemes will be add in the final report

This result can be compared to the graph obtained with the Simulink model of the system :

Schemes will be add in the final report

2.4 Linearization

2.4.1 Theoretical approach

The previous model is not directly exploitable. It is necessary to linearize it in order to control it in an easier way. If $\left[\theta_{1eq}, \theta_{2eq}, \dot{\theta}_{1eq}, \dot{\theta}_{2eq}\right]^T$ denotes the values of the state variables in an equilibrium point, the linearization around this point will be derived by using Taylor's expansion:

$$\dot{X} = \dot{X}_{eq} + Jac_F.\Delta X + Jac_G.\Delta U \tag{2.31}$$

Further, the following notations will be used:

$$A = Jac_F \tag{2.32}$$

$$B = Jac_G \tag{2.33}$$

A and B can be calculated as follows:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\partial f_1}{\partial \theta_1} & \frac{\partial f_1}{\partial \theta_2} & \frac{\partial f_1}{\partial \dot{\theta}_1} & \frac{\partial f_1}{\partial \dot{\theta}_2} \\ \frac{\partial f_2}{\partial \theta_1} & \frac{\partial f_2}{\partial \theta_2} & \frac{\partial f_2}{\partial \dot{\theta}_1} & \frac{\partial f_2}{\partial \dot{\theta}_2} \end{bmatrix}_{\left[\theta_1,\theta_2,\dot{\theta}_1,\dot{\theta}_2\right]^T=\left[\theta_{1eq},\theta_{2eq},\dot{\theta}_{1eq},\dot{\theta}_{2eq}\right]^T} \tag{2.34}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{\partial \tau}{\partial \tau} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \tag{2.35}$$

Finally, the general relation that will be used in the project is:

$$\boxed{\Delta\dot{X} = A\Delta X + B\Delta U} \tag{2.36}$$

2.4.2 Calculations

Regarding the complexity of the equations, all the calculations of the partial derivatives are performed with Maple.
The matrices A and B must be computed for each studied equilibrium point. Those equilibrium points will be defined in the next chapter, when the controllers will be chosen.

Chapter 3

Control design

The control of the Pendubot is divided into two main problems, swing-up and balancing. These two problems are solved with different approaches.

3.1 Balancing control

To stabilize the Pendubot in position state-feedback with LQR-control is used[2] . LQR is used since it gives an optimal stabilizing controller. The LQ-parameters are derived using the linearized systems evaluated around the different equilibria. This gives a feedback torque as

$$\tau = \tau_0 - K(x - x_0) \tag{3.1}$$

were K is the 1×4 vector containing the state feedback gains. K is calculated by minimizing eq. 3.2. This is done by solving the Algebraic Riccati Equation eq. 3.3, then K is given by eq. 3.4.

$$min \int \Delta x^T Q \Delta x + \Delta \tau^T R \Delta \tau \tag{3.2}$$

$$A^T P + PA + Q = PBR^{-1} B^T P \tag{3.3}$$

$$K = R^{-1} B^T P \tag{3.4}$$

3.1.1 Tuning the balancing controllers

As mentioned in section 3.1 LQR is optimal, however it is important to remember that it is optimal with respect to the weightings Q and R. Hence the controller can be tuned by changing the values of these parameters. In this application it was found that the below values of Q and R gave a good result.

$$\mathbf{Q} = \begin{pmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{R} = 50$$

3.1.2 Robustness of stabilizing controllers

Since the controllers are calculated based on the linearized system they need to be robust so that they function on the true system. The controller derived with LQR are robust and function in a neighborhood around the linearization point. Since this neighborhood only covers a part of a revolution several controllers are needed. Testing has shown that one revolution needs to be divided into 6 sectors with one balancing controller in each sector.

3.2 Swing-up control

The swing-up control design **??**will be divided in two parts:

- A partial linearization of the model
- The design of the PD controller

3.2.1 Partial linearization

The equation noted (2.21) and (2.22) will be used here:

$$\tau = \ddot{\theta}_1 A_1 + \ddot{\theta}_2 A_2 \cos(\theta_1 - \theta_2) + A_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) + A_4 \cos(\theta_1) \tag{3.5}$$

$$0 = \ddot{\theta}_1 B_1 \cos(\theta_1 - \theta_2) + \ddot{\theta}_2 B_2 - A_2 \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) + B_4 \cos\theta_2 \tag{3.6}$$

From the equation (3.6)the relations of $\ddot{\theta}_1$ and $\ddot{\theta}_2$ can be obtained as showed in the previous chapter:

$$\ddot{\theta}_1 = \frac{\dot{\theta_1}^2 A_2 \sin(\theta_1 - \theta_2) - B_4 \cos(\theta_2) - \ddot{\theta}_2 B_2}{A_2 \cos(\theta_1 - \theta_2)} \tag{3.7}$$

$$\ddot{\theta}_2 = \frac{1}{B_2} \left[\theta_1^2 A_2 \sin(\theta_1 - \theta_2) - B_4 \cos(\theta_1 - \theta_2) - \ddot{\theta}_1 A_2 \cos(\theta_1 - \theta_2)) \right] \tag{3.8}$$

A new equation of the torque can be derivated by replacing $\ddot{\theta}_2$ in the equation (3.5)

$$\tau = \ddot{\theta}_1 A_1 + A_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) + A_4 \cos(\theta_1) + fracA_2 B_2 \cos(\theta_1 - \theta_2) \left[\theta_1^2 A_2 \sin(\theta_1 - \theta_2) \right. \\ \left. - B_4 \cos(\theta_1 - \theta_2) - \ddot{\theta}_1 A_2 \cos(\theta_1 - \theta_2)\right) \tag{3.9}$$

which can be rewritten:

$$\tau = \ddot{\theta}_1 \left[A_1 - \frac{A_2^2}{B_2} \cos^2(\theta_1 - \theta_2)\right] + \dot{\theta}_2^2 A_2 \sin(\theta_1 - \theta_2) + \dot{\theta}_1^2 \frac{A_2^2}{B_2} \cos(\theta_1 - \theta_2) \sin(\theta_1 - \theta_2) \\ + A_4 \cos(\theta_1) - B_4 \frac{A_2}{B_2} \cos(\theta_2) \cos(\theta_1 - \theta_2) \tag{3.10}$$

$$\alpha\left(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2\right) = \dot{\theta}_2^2 A_2 \sin(\theta_1 - \theta_2) + \dot{\theta}_1^2 \frac{A_2^2}{B_2} \cos(\theta_1 - \theta_2) \sin(\theta_1 - \theta_2) + A_4 \cos(\theta_1) \\ - B_4 \frac{A_2}{B_2} \cos(\theta_2) \cos(\theta_1 - \theta_2) \tag{3.11}$$

$$\beta(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2) = A_1 - \frac{A_2^2}{B_2} \cos^2(\theta_1 - \theta_2) \tag{3.12}$$

The feedback will be design in order to nullify the terms α and β:

After the application of the feedback linearization a new set of state equation can be computed:

$$\ddot{\theta}_1 = v_1 \tag{3.13}$$

$$\ddot{\theta}_2 = \frac{1}{B_2} \left[\theta_1^2 A_2 \sin(\theta_1 - \theta_2) - B_4 \cos(\theta_1 - \theta_2) - v_1 A_2 \cos(\theta_1 - \theta_2))\right] \tag{3.14}$$

The system is now linear with respect to θ_1. Furthermore, as it is a second order system a PD controller must now be implemented. The global corrected system will be looped as follows:

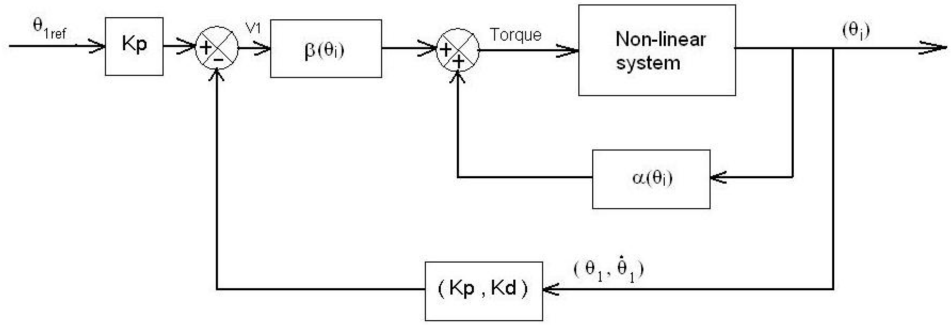

Figure 3.1: Swing-up control

3.3 Safety net

The safety net is not yet designed nor implemented. The current idea is to first monitor if the states or the calculated output are unsafe. If they are the controller will let the system free-fall until both arms are pointing downwards and θ_1 and θ_2 are in a sufficiently small sector around $-\pi/2$. What remains is to decide when the system is unsafe. Currently that is said to be when the calculated torque is too high and when θ_2 is not sufficiently close to $\pi/2$.

3.4 Switched system

All of the above parts of the controller must be put together in a switched system. A pilot version of this switch system has been implemented revealing new problems. Apart from the pure implementation problems there are a few theoretical. The first one of these is how to do the hand-over between different sector controllers, currently a large step occurs when this is done. How to set the references in the different sectors to follow a global reference is another.

Chapter 4

Implementation

4.1 The Pendubot

The Pendubot has got two encoders that gives the angle of the inner and the outer link, θ_1, θ_2. The encoders have got a resolution of 1250 points per revolution, which means that it is possible to detect an angular deviation of $5 \cdot 10^{-3}$ rad on each link. To apply a torque to the inner link the Pendubot uses a 90 volts DC motor. Since the power to the Pendubot is taken from a wall socket the Pendubot is equipped with an AC-DC converter. This converter provides a control unit with DC power. The control unit also receives control signals from the users control program, amplifies them by a factor 10 and send them out to the DC motor.

4.2 DAQ

To control the Pendubot Labview is used. Labview uses a Data Aquisition Card, or a DAQ card, to collect and send data from and to the Pendubot. The DAQ card that is used in this project is a National Instruments PCI 6221, which collects the angular measurements from the two encoders in the Pendubot and sends out a calculated voltage to the DC motor.

4.3 Samplingrate

In one sampling interval Labview collects data from the two encoders, does all the stabilizing calculations and sends out a voltage to the motor in the Pendubot. Therefor one important parameter to set is the sampling frequency. With a to low sampling rate the controllers will not be able to stabilize the system. A high sampling rate requires a fast program that is able to do all the calculations within the sampling time. A good sapling frequency to stabilize the system seems to be 200 Hz. This result is derived by practical testing.

4.4 Problems

When using 200 Hz sampling frequency the system has 5 ms to collect data, make all the computations, send out data and react to the new output signal. It is essential to give the system time to react to the new output signal before collecting the new angles. The stabilizing controllers have been implemented in Labview and tested on the real system one by one. They work perfect on the real system with 200 Hz sampling rate when used one at a time. However, when the entire system is implemented with data processing, all the stabilizing controllers and switching criterions, the program uses almost the entire 5 ms time space to complete the calculations an sending out data. This means that the system does not have any time to respond to the new output signal before new data is collected. The result is an overcompensating system that starts to shake. To get the entire system to work a new faster control program has to be developed.

When stabilizing the Pendubot a bias can be detected. For example, if the system is stabilized in the up-up position the inner link does not have the angular output $\theta_1 = \frac{\pi}{2}$ rad like it is supposed to have. Instead $\theta_1 = \frac{\pi}{2} + 5 \cdot 10^{-2}$ rad, which corresponds to a deviation of about 3° in the counter clockwise direction. This problem is probably due to an amplification error in the control unit in the Pendubot. By measuring the voltage output from the control unit to the motor a difference in amplification between the positive and the negative output has been detected. The positive output voltage has a somewhat higher amplification than the negative output. Since a positive output will result in revolutions in the counter clockwise direction it seems reasonable that this is the source to the bias.

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