Problem Set 4

Sum of Squares Seminar

October 3, 2017, Due at the end of the seminar

Problem 1: Sparsest Cut (50 points)

In this problem, we will run our algorithms for the sparsest cut problem on the following graphs (to be added)

(a) 25 points: Run the Leighton-Rao linear programming relaxation and the Goemans-Linial semidefinite programming relaxation (analyzed by Arora-Rao-Vazirani) and give the values obtained for each graph

(b) 25 points: Run the rounding algorithm for the linear programming relaxation and give the corresponding cuts.

Problem 2: Minimizing a Homogeneous Polynomial over the Sphere (50 points)

Consider the following problem. Minimize the value of a homogeneous degree k polynomial h subject to the constraint that $\sum_{i=1}^{n} x_i^2 = 1$.

(a) 10 points: What are the constraints for the primal and dual semidefinite programs (giving the pseudo-expectation values and Positivstellensatz proof, respectively)? Write these constraints as linear constraints on the entries of the appropriate matrices and matrix PSDness constraints (i.e. the form you would put them in for implementing the semidefinite program).

(b) 15 points: Show that in the special case where $d = k$ is even, it is sufficient to consider indices of degree exactly $\frac{d}{2}$. In other words, show the following:

   (a) If we have pseudo-expectation values $\tilde{E}$ for all degree d polynomials and the corresponding submatrix of the moment matrix is PSD, then these can be extended to valid pseudo-expectation values $\tilde{E}$ for all polynomials of degree $\leq d$.

(b) If there is a Positivstellensatz proof that $h \geq c$ with the the constraint that $\sum_{i=1}^{n} x_i^2 = 1$ then there is a Positivstellensatz proof that $h \geq c(\sum_{i=1}^{n} x_i^2)^{\frac{d}{2}}$ with no constraints.

What happens if d, k are even and $d > k$?

(c) 25 points: Run both the primal and dual program on the polynomial $h = x_4^4 + x_1^2 x_2^2 + x_1^2 x_3^2 + x_2^2 x_3^2 - 4x_1 x_2 x_3 x_4$. Letting c be the value you obtain, give both the pseudo-expectation values $\tilde{E}$ such that $\tilde{E}[h] = c$ (giving the values for degree 4 polynomials is sufficient) and the Positivstellensatz proof that $h \geq c(\sum_{i=1}^{n} x_i^2)^2$