An Optimization Approach to Joint Cell, Channel and Power Allocation in Multicell Networks

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***Abstract*—In multicell wireless networks the resource allocation task includes the selection of the serving cell and the allocation of channels and transmission power. While all of these tasks have been studied in the past, all three jointly are seldom addressed. In this paper we formulate the joint cell, channel and power allocation problem as an optimization task whose purpose is to maximize either the minimum user throughput or the multicell sum throughput or a weighted combination of these two objectives. The max-min problem and a simplified max-throughput problem are both NP-hard and we therefore propose heuristic solution approaches. In particular, we develop a successive allocation update algorithm that iteratively improves the cell, channel and power allocations while having to solve one of these tasks at a time. We compare the performance of this allocation update procedure with that of the optimization based techniques. We present numerical results that give new and valuable insights into the trade off between fair and sum throughput optimal joint resource allocation strategies both for the downlink and the uplink.**

I. Introduction

In multicell wireless networks mobile stations (MS) need to be assigned to a serving base station such that MSs enjoy continuous service coverage. This task is referred to as the (serving) cell or link selection and it has been studied ever since cellular telecommunication systems started to gain popularity [1]- [3]. Link selection can be optimized according to different objective functions, such as overall system throughput [2], individual quality of service (QoS) targets [3] or other suitable utility functions [4].

Once an MS is assigned to a cell, radio resources – most importantly frequency/time channels and transmission power – need to be allocated. Due to their relevance and complexity, channel assignment and power allocation have a vast literature, including classical papers from the late eighties (for a survey see [1]) to the most recent research results [5]- [20]. The authors of [5] propose a hybrid of centralized and distributed algorithms for subcarrier (i.e., channel) assignment to maximize the overall throughput. This scheme has been extended to include power allocation by [15]. Paper [6] studies three adaptive schemes for subcarrier (channel) allocation by means of cross-layer techniques for the purpose of throughput enhancement. Recently, it has been widely recognized that joint allocation of various radio resources has a clear potential over techniques that deal with a single resource, see for example [7], [8], [9] and [11]. However, cell selection is out of the scope of these papers.

Along another line, recent advances in power control suggest that allocating power taking into account instantaneous channel variations and targeting throughput maximization (rather than a predefined signal-to-noise-and-interference, SINR) target has a great potential for data traffic [10], [17]. However, such *opportunistic* power control strategies need to deal with fairness issues, as it has been pointed out by [10], [17] and [19]. These papers do not deal with either channel or cell selection (link assignment).

Therefore, in this paper we examine joint cell (link), channel and power assignment for the purpose of gaining insight into the gains when these three tasks are dealt with jointly. To this end, we consider two extreme schemes in terms of throughput optimization and fairness. The max-min scheme has the objective of maximizing the throughput of the minimum-throughput (the least happy) user. In contrast, the maximum throughput scheme neglects fairness. As we shall see, it is possible to define an objective function that balances between these extreme cases. Since the joint task turns out to be NP-hard, we use heuristic algorithms to obtain numerical results.

We focus on centralized algorithms because of two reasons. First, centralized algorithms provide an insight into the potential gains of addressing the resource allocation tasks jointly and often serve as a starting point to distributed algorithm development [12] [28]. Secondly, recent technology advancements indicate that centralized architectures and algorithms across multiple access points, including base band processing and radio resource management functions may be an attractive technical solution

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in future wireless networks [31]. For cell selection and channel allocation a centralized entity may cover multiple cells, as proposed in [5] in the form of the "Radio Network Controller (RNC) algorithm", while fast power control can be advantageously implemented by means of distributed approaches. For this reason, we also develop a power control algorithm (termed Power Optimization within Cells) that relies on own cell information and can be readily combined with our proposed cell and channel allocation algorithms.

We study these algorithms in orthogonal frequency division multiple access (OFDMA) context by means of a dynamic system level simulator called Rudimentary Network Simulator (RUNE) [21], [22]. The results give new and valuable insights into the trade off between max-min fair and maximum throughput allocation of link, channel and power resources in multicell networks.

The rest of the paper is structured as follows. Section II describes our system model. Next, in Section III we formulate the max-min fair and maximum throughput resource allocation problems. Section IV proposes solution approaches for link and channel assignment as well as for power allocation. In this section we also describe an update of the channel allocation, which in combination with the power allocation algorithms allows to successively improve the resource allocation. Section V presents numerical results and Section VI highlights our conclusions.

II. System Model

We consider a multicell wireless network consisting of a set $\mathcal{B}$ of base stations (BS), such that each BS maintains the coverage area of its associated cell. In the coverage area of the multicell network, there is the set of mobile stations (MS), denoted by $\mathcal{M}$. To allow for convenient handling of both the downlink and uplink, we denote the set of information sources by $\mathcal{S}$ and the set of destinations by $\mathcal{D}$. We say that there is a communication *link* between a MS and its serving BS. Furthermore, we assume that the radio resources that are used in each cell (for example subcarriers, time slots or codes) are orthogonal *channels* such that there is no intracell interference. We denote the set of channels $\mathcal{C}$. In general, a link may comprise multiple channels. The assumption on negligable intracell interference is valid for virtually all major modern telecommunication standards, including orthogonal frequency division multiple access (OFDMA) or orthogonal code division multiple access (OCDMA) schemes and is often used in the literature of multi-cell models, see for example [23]. Finally, we allow for a complete reuse of all channels in each cell, that is we consider a Reuse-1 system. The sets used in the mathematical formulation are summarized in Table I.

Table I: Definition of the sets of the system model

$\mathcal{B}$	Set of base stations
$\mathcal{M}$	Set of mobile users
$\mathcal{M}_b$	Set of mobile users in cell $b \in \mathcal{B}$
$\mathcal{S}$	Set of sources
$\mathcal{D}$	Set of destinations
$\mathcal{C}$	Set of channels per base station

For ease of presentation, in this paper we assume that the basic radio resource is the transmission bandwidth W that is a known given constant. This bandwidth is allocated in terms of frequency channels for the communication links and it is reused in each cell. The most important variables and constants of this (rather general) system are summarized by Table II.

Table II: Definition of the variables and constants used in our system model

x_{ijk}	$\begin{cases} 1, & \text{if } i \in \mathcal{S} \text{ and } j \in \mathcal{D} \text{ communicate on channel } k \in \mathcal{C} \\ 0, & \text{otherwise} \end{cases}$
y_{ij}	$\begin{cases} 1, & \text{if } x_{ijk} = 1 \text{ on any channel } k \in \mathcal{C} \\ 0, & \text{otherwise} \end{cases}$
p_{ik}	the power that source i uses on channel k
η_{ijk}	the throughput between i and j on channel k
g_{ij}	path gain on link i to j, without channel variations
g_{ijk}	path gain on link i to j on channel k
σ_j^2	thermal noise at receiver $j \in \mathcal{D}$
$P_i^{\max}$	the maximum transmit power of sender $i \in \mathcal{S}$
W	transmission frequency bandwidth

The signal-to-interference-and-noise-ratio (SINR) on channel k between source i and destination j is given by

$$\gamma_{ijk} = \frac{g_{ijk} p_{ik}}{\sigma_j^2 + \sum_{n \in \mathcal{S} \setminus \{i\}} g_{njk} p_{nk}}, \; i \in \mathcal{S}, j \in \mathcal{D}, k \in \mathcal{C}; \quad (1)$$

while the throughput on channel k between i and j is given by

$$\eta_{ijk} = W \log_2(1 + \gamma_{ijk}), \quad i \in \mathcal{S}, j \in \mathcal{D}, k \in \mathcal{C}. \quad (2)$$

III. Problem Formulation

A. The Max-Min Problem

Given the model setup, we now introduce a joint cell, channel and power allocation optimization problem. For ease of presentation, we first focus on the max-min problem for the downlink and later discuss how this problem can be generalized to the uplink and to include the maximum throughput problem (in both directions). This max-min problem for the downlink is stated as follows:

$$\underset{\eta, p_{ik}, x_{ijk}, y_{ij}}{\text{maximize}} \quad \eta \tag{3a}$$

$$\text{subject to} \quad y_{ij}\eta \le \sum_{k\in\mathcal{C}} x_{ijk}\eta_{ijk}, \quad i \in \mathcal{S}, j \in \mathcal{D}, \tag{3b}$$

$$\sum_{i\in\mathcal{B}} y_{ij} = 1, \quad j \in \mathcal{M}, \tag{3c}$$

$$\sum_{j\in\mathcal{M}} x_{ijk} \le 1, \quad i \in \mathcal{B}, k \in \mathcal{C}, \tag{3d}$$

$$x_{ijk} \le y_{ij}, \quad i \in \mathcal{S}, j \in \mathcal{D}, k \in \mathcal{C}, \tag{3e}$$

$$0 \le y_{ij} \le 1, \quad i \in \mathcal{S}, j \in \mathcal{D}, \tag{3f}$$

$$x_{ijk} \in \{0, 1\}, \quad i \in \mathcal{S}, j \in \mathcal{D}, k \in \mathcal{C}, \tag{3g}$$

$$\sum_{k\in\mathcal{C}} p_{ik} \le P_i^{\max}, \quad i \in \mathcal{S}, \tag{3h}$$

$$p_{ik} \ge 0, \quad i \in \mathcal{S}, k \in \mathcal{C}. \tag{3i}$$

In this formulation, η is an auxiliary variable that is defined by constraint (3b). Constraint (3b) ensures that for all active links η is less than or equal to the sum of the throughput on the active channels on that link. Maximizing η in the objective function (3a) ensures that η is equal to the minimum sum throughput on the active links, and that this minimum is maximized. Constraint (3c) ensures that all MSs are connected to exactly one BS, while constraint (3d) ensures that each BS at most has one MS per channel. Inequality (3e) define the connection between x and y, while the constraints (3f) and (3g) define those variables. Finally constraints (3h) and (3i) ensure that the total power of each sender is at most that senders maximum and that each transmission power is nonnegative. The corresponding uplink problem is given by interchanging i and j in constraints (3c) and (3d).

In [20], the complexity of the joint cell, channel and power allocation optimization problem (3) has been studied and shown to be NP-hard and not approximable for a general problem instance, unless P is equal to NP. In other words, there exists no problem solvable in polynomial time that approximates (3). Therefore, we will consider heuristic methods in the following section.

B. A Generalization of Problem (3)

In this paper we choose to study the price of fairness in terms of maximizing the minimum throughput among all users and maximizing the total user throughput. More precisely, we study the optimization problem

$$\underset{\eta, p_{ik}, x_{ijk}, y_{ij}}{\text{maximize}} \quad (1-\alpha)\eta + \frac{\alpha}{|\mathcal{M}|} \sum_{i\in\mathcal{S}, j\in\mathcal{D}, k\in\mathcal{C}} x_{ijk}\eta_{ijk} \tag{4a}$$

$$\text{subject to} \quad \text{constraints (3b) to (3i)}, \tag{4b}$$

with various choices of the parameter $\alpha \in [0, 1]$. Note that the additional $|\mathcal{M}|$ is merely introduced to give η a more suitable value in the objective function, in comparision with the total throughput. With $\alpha = 0$, as in model (3), the objective becomes to maximize the minimum user throughput, i.e., maximize the user worst off, while with $\alpha = 1$ the objective becomes to maximize the total system throughput.

IV. Solution Approach

Due to the complexity of the problem (3) and (4), we resort to heuristic algorithms that are based on the decomposition of the problems to the separate tasks of link, channel and power allocation. An initial feasible allocation in terms of link, channel and power is going to be referred to as an initial point. Given an initial point, an allocation update approach then aims to improve the current point, by updating the channel allocation and then updating the power allocation. The allocation update is repeated until no further improvement is obtained. We refer to the obtained allocation update solution as the final point.

Furthermore, to avoid modeling the details of scheduling algorithms, we assume that the overall number of MSs does not exceed the (number of BSs) $\times$ (number of channels per BS). In other words, we assume that all MSs currently in $\mathcal{M}$ can be allocated at least one channel.

A. Link Allocation

In the link allocation it is decided which base stations each mobile user is to be communicating with, i.e., regular cell allocation. We focus on link allocation algorithms which ensures that the number of MS at any BS does not exceed the total number of channels, i.e., $|\mathcal{M}_b| \le |\mathcal{C}|$ for each $b \in \mathcal{B}$. Given this link allocation condition it will be possible to assign at least one channel to each MS in the Channel allocation. Two different link allocation algorithms are presented below. Both are considering the path gain g_{ij}, $i \in \mathcal{S}$, $j \in \mathcal{D}$. This path gain will also be referred to as $\tilde{g}_{ij}$, $i \in \mathcal{B}$, $j \in \mathcal{M}$.

1) Link allocation by Greedy assignment (LaG): The Link allocation by Greedy assignment (LaG), assigns mobile users to a cell according to the largest available path gain (breaking ties arbitrarily). This link allocation is a reference algorithm, Algorithm LaG. In GLA, the desired BS for MS j is denoted b_j. Eventually, the MS with the highest path gain gets assigned to its desired base station. Finally, the variables are updated in order to remove that

Algorithm 1 Link allocation by Greedy (LaG)

Consider the path gain $\tilde{g}_{ij}$, $i \in \mathcal{B}$, $j \in \mathcal{M}$.
Let $y_{ij} \leftarrow 0$, $\quad i \in \mathcal{B}, j \in \mathcal{M}$.
for $\Delta = 1$ **to** $|\mathcal{M}|$ **do**
$(b_m, m) \leftarrow \arg\max_{i\in\mathcal{B}, j\in\mathcal{M}} \tilde{g}_{ij}$, breaking ties arbitrarily.
Let $y_{b_m m} \leftarrow 1$ and $\tilde{g}_{im} \leftarrow -1$, $i \in \mathcal{B}$,
{which removes mobile m}.
if $\sum_{j\in\mathcal{M}} y_{b_m j} = |\mathcal{C}|$ **then**
  $\tilde{g}_{b_m j} \leftarrow -1$, $j \in \mathcal{M}$, {which removes base station b_m}.
end if
end for

MS and if the BS has no more channels available the BS is removed from the set of available BSs.

2) Link allocation by Optimization (LaO): The Link allocation by Optimization (LaO) minimizes the gain that cause disturbance in the system by maximizing the total gain over the active links. This gives the following optimization problem

$$\underset{y_{ij}}{\text{maximize}} \quad \sum_{i\in\mathcal{B},j\in\mathcal{M}} \tilde{g}_{ij}y_{ij} \tag{5a}$$

$$\text{subject to} \quad \sum_{j\in\mathcal{M}} y_{ij} \le |\mathcal{C}|, \quad i\in\mathcal{B}, \tag{5b}$$

$$\sum_{i\in\mathcal{B}} y_{ij} = 1, \quad j\in\mathcal{M}, \tag{5c}$$

$$0 \le y_{ij} \le 1, \quad i\in\mathcal{B}, j\in\mathcal{M}, \tag{5d}$$

where constraint (5b) ensures that no cell contains more MSs than channels, while constraints (5c) and (5d) defines our general cell setting, see (3c) and (3f).

The optimization problem (5) is a linear programming problem known to give integer solutions, see, e.g., [24]. The Link allocation by Optimization (LaO) algorithm solves this optimization problem, see Algorithm LaO.

Algorithm 2 Link allocation by Optimization (LaO)

Consider the path gain $\tilde{g}_{ij}$, $i\in\mathcal{B}$, $j\in\mathcal{M}$.
Solve optimization problem (5).

B. Channel Allocation

Given a feasible link allocation, y_{ij}, $i\in\mathcal{S}$, $j\in\mathcal{D}$, the channel allocation assigns the channels to use on the different links. As an initial step, we assign the number of channels that each MS is allowed to allocate, according to Algorithm NC.

Algorithm 3 Number of Channels (NC)

Consider y_{bm}, $b\in\mathcal{B}$, $m\in\mathcal{M}$.
Let $N_m^{\mathcal{C}}$, $m\in\mathcal{M}$.
for $b=1$ **to** $|\mathcal{B}|$ **do**
if $|\mathcal{M}_b|\ge 1$ **then**
  Let $n \leftarrow |\mathcal{C}|/|\mathcal{M}_b|$ rounded to the nearest integer towards minus infinity. Let $N_m^{\mathcal{C}} \leftarrow n$, $m\in\mathcal{M}_b$.
  While $|\mathcal{C}| < \sum_{m\in\mathcal{M}_b} N_m^{\mathcal{C}}$, let $N_m^{\mathcal{C}} \leftarrow N_m^{\mathcal{C}}+1$ for one arbitrary mobile $m\in\mathcal{M}_b$.
end if, end for

Given y and $N^{\mathcal{C}}$, three different channel allocation algorithms are presented below. They are all considering the path gain g_{ijk}, $i\in\mathcal{S}$, $j\in\mathcal{D}$, $k\in\mathcal{C}$. This path gain will also be referred to as $\tilde{g}_{ijk}$, $i\in\mathcal{B}$, $j\in\mathcal{M}$, $k\in\mathcal{C}$.

1) Channel allocation by Greedy assignment (CaG): The Channel allocation by Greedy assignment (CaG), assigns channels to existing links according to the largest available path gain. This greedy channel allocation is a reference algorithm, Algorithm CaG.

Algorithm 4 Channel allocation by Greedy (CaG)

Consider path gain $\tilde{g}_{bmk}$, $b\in\mathcal{B}$, $m\in\mathcal{M}$, $k\in\mathcal{C}$.
Let $x_{ijk}\leftarrow 0$, $i\in\mathcal{B}, j\in\mathcal{M}$, $k\in\mathcal{C}$.
for $b=1$ **to** $|\mathcal{B}|$ **do**
if $|\mathcal{M}_b|\ge 1$ **then**
  for $c=1$ **to** $|\mathcal{C}|$ **do**
  Let $m\leftarrow \arg\max_{j\in\mathcal{M}_b}\tilde{g}_{bjc}$. Update $x_{bmc}\leftarrow 1$.
  if $\sum_{k\in\mathcal{C}} x_{bmk} = N_m^{\mathcal{C}}$ **then**
   $\tilde{g}_{bmk}\leftarrow -1$, $k\in\mathcal{C}$, {which removes mobile m}.
  end if, end for
end if, end for

GCA assigns channels to the MS within each cell, given a feasible link assignment. Eventually, in each BS the MS with the highest path gain of the considered channel gets assigned. Finally, the variables are updated in order to remove that channel and if the MS has no more channels to receive, then it is removed from the set of available MSs.

2) Channel allocation by Integer Programming (CaIP) and Channel allocation by Relaxed Optimization (CaRO): Let us, as in (4), introduce a parameter $\beta\in[0,1]$ that serves as a weight between the total gain and the minimum gain in the objective function. The channel assignment is now obtained by solving the following optimization problem

$$\underset{\eta,x_{ijk}}{\text{maximize}} \quad (1-\beta)\eta + \frac{\beta}{|\mathcal{M}|}\sum_{i\in\mathcal{S},j\in\mathcal{D},k\in\mathcal{C}} f_{ijk}x_{ijk} \tag{6a}$$

$$\text{subject to} \quad y_{ij}\eta \le \frac{1}{N_j^{\mathcal{C}}}\sum_{k\in\mathcal{C}} f_{ijk}x_{ijk}, \quad i\in\mathcal{S}, j\in\mathcal{D}, \tag{6b}$$

$$\sum_{b\in\mathcal{B},k\in\mathcal{C}} x_{bmk} = N_m^{\mathcal{C}}, \quad m\in\mathcal{M}, \tag{6c}$$

$$\text{constraints (3c) to (3f)}, \tag{6d}$$

$$x_{ijk}\in\{0,1\}, \quad i\in\mathcal{S}, j\in\mathcal{D}, k\in\mathcal{C}, \tag{6e}$$

$$\eta \ge f_{\min}, \tag{6f}$$

where $f_{\min} = \min_{i\in\mathcal{S},j\in\mathcal{D},k\in\mathcal{C}}\{f_{ijk}\}$. As in (3) and (4), η serve as the auxiliary variable, here defined by constraint (6b). This constraint ensures that η is less than or equal to the minimum path gain over all active links in the system. Constraint (6c) ensures that the number of channels allocated are given by Algorithm NC. Finally, constraints (6d) and (6e) define our general cell and channel setting.

The two channel allocation optimization approaches are now given by solving (6). See Algorithm CaIP and Algorithm CaRO.

C. Power Allocation

In line with recent advances in power control [10], [17], here we consider a power control scheme that allocates transmission power taking into account small scale fading.

Algorithm 5 Channel allocation by Integer Programming (CaIP)

Given y_{ij}, $i \in \mathcal{B}$, $j \in \mathcal{M}$. Let $\beta \leftarrow 1$ and $f_{ijk} \leftarrow g_{ijk}\, 1e+11$, $i \in \mathcal{S}$, $j \in \mathcal{D}$, $k \in \mathcal{C}$. Solve optimization problem (6).

Algorithm 6 Channel allocation by Relaxed Optimization (CaRO)

Given y_{ij}, $i \in \mathcal{B}$, $j \in \mathcal{M}$. Let $\beta \leftarrow 1$ and $f_{ijk} \leftarrow g_{ijk}\, 1e+11$, $i \in \mathcal{S}$, $j \in \mathcal{D}$, $k \in \mathcal{C}$. Solve a relaxed version of the optimization problem (6), where the constraint (6e) has been replaced by $x_{ijk} \in [0,1]$, $i \in \mathcal{B}$, $j \in \mathcal{M}$, $k \in \mathcal{C}$. Let x^* denote the optimal solution.
Let $x_{ijk} \leftarrow 1$, in decreasing order of x^*_{ijk}, $i \in \mathcal{B}$, $j \in \mathcal{M}$, $k \in \mathcal{C}$, as long as it is feasible (in terms of $N^{\mathcal{C}}$).

To this end, we introduce some additional notation in Table III.

Table III: Definition of the sets and variables used for power control

$\mathcal{L} = \{(i,j) : y_{ij} = 1, i \in \mathcal{S}, j \in \mathcal{D}\}$,	Set of active links
$\mathcal{C}_{ij} = \{k \in \mathcal{C} : x_{ijk} = 1\}$, $(i,j) \in \mathcal{L}$,	Set of active channels
$\mathcal{C}_i = \bigcup_{j \in \mathcal{D}:(i,j)\in\mathcal{L}} \mathcal{C}_{ij}$, $i \in \mathcal{S}$,	Set of active channels by each sender
$\eta_{ij} = \sum_{k \in \mathcal{C}_{ij}} \eta_{ijk}$, $(i,j) \in \mathcal{L}$.	Link throughput

Given a feasible cell and channel assignment, it remains to allocate the transmission powers, i.e., to solve optimization problem (4) with y_{ij}, $i \in \mathcal{S}$, $j \in \mathcal{D}$ and x_{ijk}, $i \in \mathcal{S}$, $j \in \mathcal{D}$, $k \in \mathcal{C}$ as known constants. We formulate this transmission power allocation optimization problem as:

$$\underset{\eta, \eta_{ij}, p_{ik}}{\text{maximize}} \quad (1-\alpha)\eta + \frac{\alpha}{|\mathcal{M}|} \sum_{(i,j)\in\mathcal{L}} \eta_{ij} \tag{7a}$$

$$\text{subject to} \quad \eta \le \eta_{ij}, \quad (i,j) \in \mathcal{L}, \tag{7b}$$

$$\sum_{k \in \mathcal{C}_i} p_{ik} \le P_i^{\max}, \quad i \in \mathcal{S}, \tag{7c}$$

$$p_{ik} \ge 0, \quad i \in \mathcal{S}, k \in \mathcal{C}_i. \tag{7d}$$

Constraint (7b) ensures that η is less than or equal to all active links' total throughput, while constraints (7c) and (7d) define our general power setting, see (3h) and (3i).

This continuous optimization problem has no binary variables as the optimization problem (4). Despite this, the continuous optimization problem (7) remains hard to solve in general, see, e.g., [16] and [20]. Hence, to solve the task of power allocation we resort to heuristic algorithms.

1) Power allocation Optimization (PaO): The Power allocation Optimization (PaO) heuristic applies a nonlinear optimization solver on the optimization problem (7), see Algorithm PaO.

2) Power Optimization within Cells (POC): The Power Optimization within Cells (POC) solves a convex optimization problem within each cell given a fix interference. When the interference term in the denominator of the

Algorithm 7 Power allocation Optimization (PaO)

Solve optimization problem (7) with a nonlinear optimization solver.

SINR (1) is fix, then it is well known, see, e.g., [16], that (7) turns out to be a convex optimization problem. A sufficiently small change of transmission powers within a cell should not significantly affect the other cells, in terms of interference.

This motivates a decentralized heuristic that considers the interferences and solves a convex transmission power optimization problem in each cell. Note that the interferences are updated on a central level after each iteration. Before stating the algorithm, let us define the interference as

$$I_{jk} = \sigma_j^2 + \sum_{n \in \mathcal{S} \setminus \{i : y_{ij}=1\}} g_{njk} p_{nk}, \quad j \in \mathcal{D}, k \in \mathcal{C}; \tag{8}$$

and let the non-negative parameter Δ denote the largest transmission power deviation allowed, during one iteration. Now, let the lower and upper bounds of the transmission powers be given by

$$L_{ik} = \max\{0,\, p_{ik} - \Delta\}, i \in \mathcal{S}, k \in \mathcal{C}, \tag{9a}$$

$$U_{ik} = \min\{P_i^{\max},\, p_{ik} + \Delta\}, i \in \mathcal{S}, k \in \mathcal{C}. \tag{9b}$$

Given (8) and (9), the following optimization problem is solved within in each cell.

$$\underset{\eta, \eta_{ij}, p_{ik}}{\text{maximize}} \quad (1-\alpha)\eta + \frac{\alpha}{|\mathcal{M}|} \sum_{(i,j)\in\mathcal{L}} \eta_{ij} \tag{10a}$$

$$\text{subject to} \quad \text{constraint (7b) and constraint (7c)}, \tag{10b}$$

$$L_{ik} \le p_{ik} \le U_{ik}, \quad i \in \mathcal{S}, k \in \mathcal{C}_i, \tag{10c}$$

Note that it is only the transmission powers within the cell that are variables. The Power Optimization within Cells is given in Algorithm POC.

3) Power allocation by Derivative approximation (PaD): We once again consider fix interferences as in POC, but here we update by approximating derivatives instead of solving optimization problems. Also here we let Δ denote the largest possible transmission power deviation. Let

$$R_{ik}(\rho) = x_{ijk} \frac{\rho}{P_i^{\max}} W \log_2(1 + \rho \tilde{g}_{ijk}/I_{jk}), \quad i \in \mathcal{B},\, k \in \mathcal{C}, \tag{11}$$

where j denotes the MS of BS i on channel k, i.e., $j \in \mathcal{M}$ such that $x_{ijk} = 1$. The Power allocation by Derivative approximation is given in Algorithm PaD.

4) Power allocation by Greedy (PaG): The Power allocation by Greedy (PaG) assigns powers to the sources $i \in \mathcal{S}$ given a link allocation and a channel allocation, see Algorithm PaG.

Algorithm 8 Power Optimization within Cells (POC)

Given feasible Δ and p_{ik}, $i \in \mathcal{S}$, $k \in \mathcal{C}$. Let I, L and U be given by (8) and (9). Let $\Gamma \leftarrow 0$. Choose accuracy.
while $\Gamma = 0$ **do**
$\tilde{p} \leftarrow p$.
for b = 1 **to** $|\mathcal{B}|$
if $|\mathcal{M}_b| > 0$ **then**
  $p^b \leftarrow$ solve (10) in cell $b \in \mathcal{B}$, using $\tilde{p}$ as starting point.
end if
end for
$p \leftarrow$ combine p^b, $b \in \mathcal{B}$.
if obj(p)−obj($\tilde{p}$) $\leq$ tolerance **then**
  $\Delta \leftarrow \Delta/2$. $p \leftarrow \tilde{p}$.
  if $\Delta <$ accuracy **then**
    $\Gamma \leftarrow 1$.
  end if
end if
Update I, L and U from (8) and (9).
end while

Algorithm 9 Power allocation by Derivative approximation (PaD)

Choose $\Gamma \in (0, 1/2)$, $\gamma \in (0, \Gamma)$, Δ^{initial} and accuracy.
Given a feasible p_{ik}, $i \in \mathcal{S}$, $k \in \mathcal{C}$. Let I be given by (8) and $\Delta \leftarrow \Delta^{\text{initial}}$.
while $\Gamma \geq 0$ **do**
Let $\tilde{p} \leftarrow p$, $R^+_{ik} \leftarrow R_{ik}(p_{ik} + \Delta)$, $R_{ik} \leftarrow R_{ik}(p_{ik})$ and $R^-_{ik} \leftarrow R_{ik}(p_{ik} - \Delta)$, $i \in \mathcal{S}$, $k \in \mathcal{C}$.
Let $r_{ik} \leftarrow R^+_{ik} - 2R_{ik} + R^-_{ik}$, $i \in \mathcal{S}$, $k \in \mathcal{C}$.
for b = 1 **to** $|\mathcal{B}|$
if $|\mathcal{M}_b| > 0$ **then**
  $\Xi_b \leftarrow$ sort r_{ik}, $i \in \mathcal{S}$, $k \in \mathcal{C}$, of cell b in decreasing order. Let $n \leftarrow \max\{1, \Gamma/|\mathcal{M}_b|$ rounded to the nearest integer towards minus infinity$\}$.
  for s = 1 **to** the number of sources in cell b
  Let $\tilde{n}_i \leftarrow \min\{n$, the number of elements fulfulling $p_{ik} - \Delta \geq 0\}$, $i \in \mathcal{S}$.
  Let $p_{ik} \leftarrow p_{ik} - \Delta$ for the last $\tilde{n}_i$ of elements $k \in \Xi_b$, $i \in \mathcal{S}$.Let $p_{ik} \leftarrow p_{ik} + \Delta$ for the first $\tilde{n}$ of elements $k \in \Xi_b$, $i \in \mathcal{S}$.
  end for
end if, end for
if obj(p)−obj($\tilde{p}$) $\leq$ tolerance **then**
  $p \leftarrow \tilde{p}$.
  if $\Delta >$ accuracy **then**
    $\Delta \leftarrow \Delta/2$.
  else
    $\Gamma \leftarrow \Gamma - \gamma$. $\Delta \leftarrow \Delta^{\text{initial}}$.
  end if
end if
Update I, L and U from (8) and (9).
end while

Algorithm 10 Power allocation by Greedy (PaG)

If in Downlink:
Let $p_{ik} \leftarrow P^{\max}_i/|\mathcal{C}_i|$, $i \in \mathcal{B}$. Where $|\mathcal{C}_i|$ is the number of channels that BS i uses.

If in Uplink:
Let $p_i \leftarrow \sigma^2_j \text{SNR}^{\text{target}}/g_{ij}$, $i \in \mathcal{M}$, and BS j of MS i.
$p_{ik} \leftarrow p_i/|\mathcal{C}_i|$ if $k \in \mathcal{C}_i$, and $p_{ik} \leftarrow 0$ otherwise, $i \in \mathcal{M}$.
if $\sum_{k\in\mathcal{C}} p_{ik} > P^{\max}_i$ **then**
  $p_{ik} \leftarrow \frac{P^{\max}_i}{\sum_{k\in\mathcal{C}} p_{ik}} p_{ik}$, $i \in \mathcal{M}$, $k \in \mathcal{C}$.
end if

D. Update the Channel allocation in Downlink (UCaD)

Here we aim to update an already obtained initial point, i.e., a feasible link, channel and power allocation. The solution after this update is referred to as the final point.

Given links and transmission powers in downlink an update of the channel allocation might improve the current solution. Note that in uplink the channel allocation is almost given when the links and transmission powers are fix, part from unused channels. Here we propose an update in downlink that aims to solve a mixed integer linear programming problem (4), which in general is an NP-complete problem, e.g., see [27]. If $\alpha = 1$, and each MS has been assigned its number of channels, then this problem can be solved to global optimality by using efficient polynomial weighted matching algorithms, see, e.g., [25] and [26]. The two sets of nodes within the cell weighted matching is $\mathcal{C}$ and $\mathcal{M}_b$, where each MS is duplicated to its number of pre-assigned channels. Whereas the arc-weight between $k \in \mathcal{C}$ and $j \in \mathcal{M}_b$ is given by the throughput in (2). One matching is solved within each cell $b \in \mathcal{B}$. These weighted matching solutions then form the solution of (4) with $\alpha = 1$, given the additional constraints of the number of channels that each MS is to be assigned, see, e.g., (6c).

E. Allocation Update

After allocating cells, channels and transmission powers a feasible allocation of optimization problem (4) has been obtained. Here we propose a heuristic that tries to improve the current solution by first reallocating the channels and the powers alternately, until no improvement (in terms of objective value) is obtained.

V. Numerical Results

A. Simulation Environment

Our simulation environment has been MATLAB, where the RUdimentary Network Emulator (RUNE) was used to simulate realistic cellular systems. A detailed description of RUNE is available in [21]. Two different optimization solvers have been used. To solve linear programming problems and mixed integer linear programming problems CPLEX 10.2 [29] has been used. While the nonlinear programming problems have been solved using SNOPT [30].

Algorithm 11 Update the Channel allocation in Downlink (UCaD)

Given feasible y_{ij} and p_{ik}, $i \in \mathcal{B}$, $j \in \mathcal{M}$, $k \in \mathcal{C}$.

If $\alpha = 1$, let $\beta \leftarrow 1$.
Solve a modified relaxed version of the optimization problem (6), where constraint (6c) and (6f) have been removed and where constraint (6e) has been replaced by constraint $x_{ijk} \in [0, 1]$, $i \in \mathcal{B}$, $j \in \mathcal{M}$, $k \in \mathcal{C}$.

If $\alpha < 1$, we choose an accuracy on η.
Let $\beta \leftarrow 0$ and solve a relaxed version of the optimization problem (6), where constraint (6e) has been replaced by $x_{ijk} \in [0, 1]$, $i \in \mathcal{B}$, $j \in \mathcal{M}$, $k \in \mathcal{C}$. Let $\bar{\eta}$ denote η of the optimal solution.
if $\bar{\eta} = 0$ **then**
  Fix $\eta \leftarrow 0$, $\beta \leftarrow 1$ and solve optimization problem (6) given η.
else
  $\beta \leftarrow \alpha$, and perform bisection on $\eta \in [0, \bar{\eta}]$ by solving a modified version of the binary optimization problem (6): where $N_j^{\mathcal{C}} = 1$, $j \in \mathcal{D}$, in constraint (6b) and where constraint (6c) has been removed. Repeat the bisection, until a feasible solution is obtained. Let $\tilde{\eta}$ denote the η variable of this feasible solution. Let $\Delta \leftarrow \tilde{\eta}$.
  while $\Delta \geq$ accuracy **do**
  $\Delta \leftarrow \Delta/2$.
  $\eta^+ \leftarrow \tilde{\eta} + \Delta$. Solve (6) with fix $\eta = \eta^+$.
  $\eta^- \leftarrow \tilde{\eta} - \Delta$. Solve (6) with fix $\eta = \eta^-$.
  $\eta \leftarrow$ best solution (in terms of objective value) of η^+, η and η^-.
  end while
end if

Algorithm 12 Allocation Update

Given feasible y_{ij}, x_{ijk} and $p_{ik} \leftarrow$, $i \in \mathcal{S}$, $j \in \mathcal{D}$, $k \in \mathcal{C}$.
while the objective value improves **do**
$\tilde{x} \leftarrow$ given y and p, solve UCaD algorithm.
If the objective value was improved, then let $x \leftarrow \tilde{x}$.
$(p, \eta) \leftarrow$ given y and x, solve the power allocation by either PaO, POC, PaD or PaG.
end while

The computations were run under 64-bit Linux on a single Intel Xeon 3 GHz processor core with hyperthreading disabled and with 32 GB of memory.

We consider a geographical location consisting of a number of hexagons. In the center of each hexagon a BS with the same fix bandwidth is located. In total we consider 3, 7 or 19 BSs. The fix bandwidth is devided into a number of channels $|\mathcal{C}|$. Within this study we consider $|\mathcal{C}| = 20$. The MSs are randomly distributed within the hexagons, all using the uniform distribution. Given this data, the program RUNE consider path loss and fading effects to generate the channel gain between each BS and MS in the network. The main parameters of this system are described in Table IV while Table V details some further parameters relevant for generating the path gain matrix.

Table IV: The main parameters in the system

Parameter	Value
Cell Radius	500 m
Number of Sectors per Site	1
Number of Clusters per System	1
Maximum Power of Mobile	24 dB
Maximum Power of Base Station	43 dB
Carrier Frequency	2 GHz
Chunk Bandwidth	0.2 MHz

Table V: Path gain specific parameters

Parameter	Value
Gain at 1 meter Distance	- 28 dBm
Noise	-103 dBm
Distance Dependant Path Gain Coefficient	3.5
Standard Deviation for the log-normal Fading	6 dB
Log-normal Correlation Downlink	0.5
Correlation Distance	110 m
Fast Fading	Rayleigh

In all simulations, we consider the following six initial power starting points: $p_{ik} = 0$, $i \in \mathcal{S}, k \in \mathcal{C}$, $p_{ik} = P_i^{\max}/|\mathcal{C}|$, $i \in \mathcal{S}, k \in \mathcal{C}$, and $p_{ik} = P_i^{\max}$, $i \in \mathcal{S}, k \in \mathcal{C}$, together with three randomly generated startingpoints. The power allocation algorithms PaO, POC and PaD use these starting points and only return the best solution (in terms of optimal value) as their solution. Also, some of the input parameters of the power allocation algorithms POC and PaD are listed in Table VI.

Table VI: Parameter values in the simulations

Downlink	Parameter and Value
POC	Δ =8 and accuracy = 2
PaD	Γ =0 .25, $\gamma = 0.1$, $\Delta^{\text{initial}} = 10$ and accuracy = 2
UCaD	accuracy = 1e-2
Uplink	**Parameter and Value**
POC	Δ =8 and accuracy $= 2\, P_m^{\max}/P_b^{\max}$, $m \in \mathcal{M}$, $b \in \mathcal{B}$
PaD	Γ =0 .25, $\gamma = 0.1$, $\Delta^{\text{initial}} = 10$ and accuracy $= 2\, P_m^{\max}/P_b^{\max}$, $m \in \mathcal{M}$, $b \in \mathcal{B}$
PaG	$\text{SNR}^{\text{target}} = 13$ [dB]

B. Numerical Results in Downlink

In this section we present some simulation results in downlink. Mainly we consider $|\mathcal{B}| = 7$ and study the system setting of having either 50% of MS in the system (i.e., $|\mathcal{M}| = 70$) or 90% of MS in the system (i.e., $|\mathcal{M}| = 126$). When investigating the performace of the CaIP algorithm, a system setting consisting of $|\mathcal{B}| = 3$ is considered. Also a larger system with $|\mathcal{B}| = 19$ is investigated briefly.

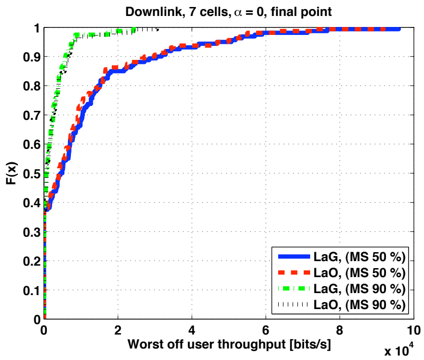

Figure 1: Comparing the LaG and LaO link allocation algorithms in terms of the worst off user throughput (final point), $\alpha = 0$, with $|\mathcal{M}| = 70$ and $|\mathcal{M}| = 126$.

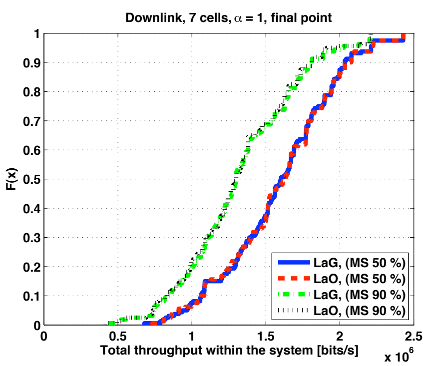

Figure 2: Comparing the LaG and LaO link allocation algorithms in terms of the total throughput (final point), $\alpha = 1$, with $|\mathcal{M}| = 70$ and $|\mathcal{M}| = 126$.

Figures 1-2 compare the performance of link allocation by greedy (LaG) and optimization methods (LaO) for the cases $\alpha = 0$ and $\alpha = 1$ respectively. We notice that for both the 50% and 90% load cases, the greedy assignment of links (LaG) yields similar performance as maximizing the total gain over the active links (LaO), both in terms of the worst off user throughput and the total throughput. In other words, selecting the best cell (in terms of largest path gain) for each user de facto maximizes the total gain over the active links as well. On the other hand, LaG executes significantly faster, especially in the max-min case ($\alpha = 0$), but depending on the channel and power allocation algorithms, also in the throughput maximization case ($\alpha = 1$).

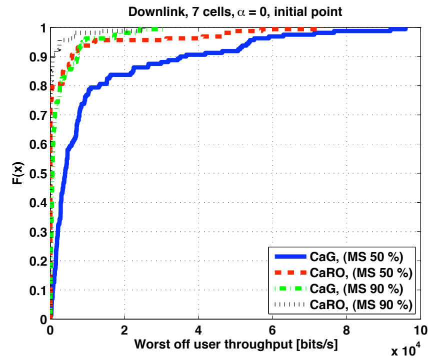

Figure 3: Comparing the CaG and CaRO channel allocation algorithms in terms of the worst off user throughput (initial point), $\alpha = 0$, with $|\mathcal{M}| = 70$ and $|\mathcal{M}| = 126$.

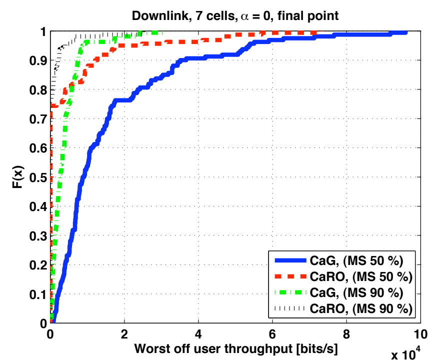

Figure 4: Comparing the CaG and CaRO channel allocation algorithms in terms of the worst off user throughput (final point), $\alpha = 0$, with $|\mathcal{M}| = 70$ and $|\mathcal{M}| = 126$.

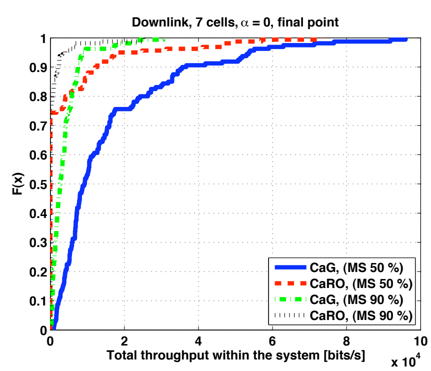

Figure 5: Comparing the CaG and CaRO channel allocation algorithms in terms of total user throughput, when the objective is to maximize the worst off user throughput (final point), $\alpha = 0$, with $|\mathcal{M}| = 70$ and $|\mathcal{M}| = 126$.

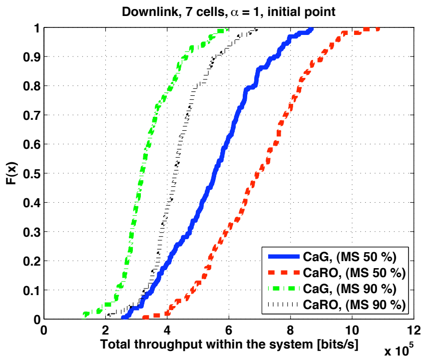

Figure 6: Comparing the CaG and CaRO channel allocation algorithms in terms of the total user throughput (initial point), $\alpha = 1$, with $|\mathcal{M}| = 70$ and $|\mathcal{M}| = 126$.

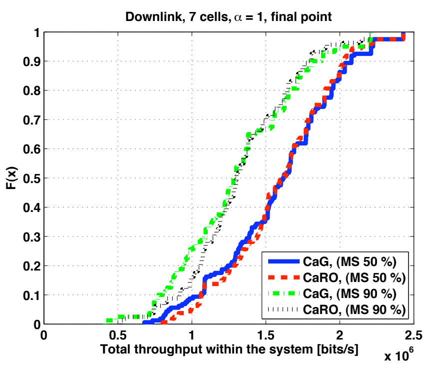

Figure 7: Comparing the CaG and CaRO channel allocation algorithms in terms of the total user throughput (final point), $\alpha = 1$, with $|\mathcal{M}| = 70$ and $|\mathcal{M}| = 126$.

Next, we are interested in the performance of the greedy (CaG) and the relaxed optimal channel assignment (CaRO) approaches. For the $\alpha = 0$ case, (Figures 3-4), CaG results in a higher throughput for the worst off user (both at the initial and final points of allocation updates). The reason for this is that both CaIP and CaRO are more suitable for throughput maximization (due to $\beta \leftarrow 1$) and also that CaG achieves a high total throughput allowing for an acceptable performance even for the worst off user. In Figure 5 one can see that the CaG gives a higher total user throughput for the worst off user case ($\alpha = 0$). This is further highlighted in Figures 6-7, that show the CaG channel assignment performs very well; in fact CaG with allocation updates yields similar total user throughput to that of CaRO. On the other hand, updating the allocations comes at the price of increased run time (Tables VII-XIV), but all in all our results clearly show that greedy channel allocations by CaG (possibly improved by allocation updates) provide a viable solution to the channel allocation problem.

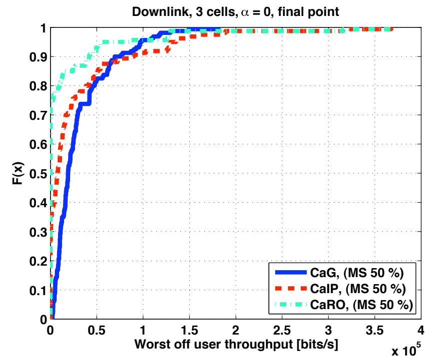

Figure 8: Comparing the CaIP and CaRO channel allocation algorithms in terms of worst off user throughput (final point), $\alpha = 0$, $|\mathcal{B}| = 3$ with $|\mathcal{M}| = 30$.

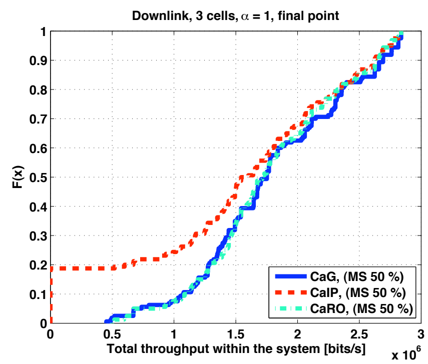

Figure 9: Comparing the CaIP and CaRO channel allocation algorithms in terms of total user throughput (final point), $\alpha = 1$, $|\mathcal{B}| = 3$ with $|\mathcal{M}| = 30$.

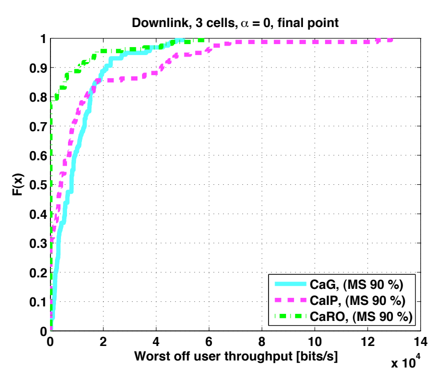

Figure 10: Comparing the CaIP and CaRO channel allocation algorithms in terms of worst off user throughput (final point), $\alpha = 0$, $|\mathcal{B}| = 3$ with $|\mathcal{M}| = 56$.

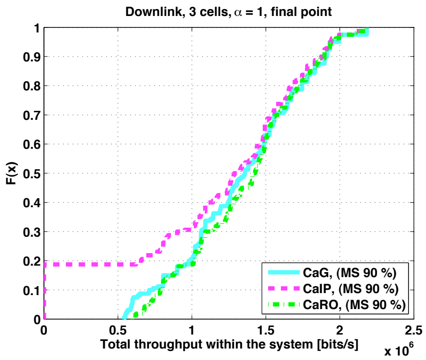

Figure 11: Comparing the CaIP and CaRO channel allocation algorithms in terms of total user throughput (final point), $\alpha = 1$, $|\mathcal{B}| = 3$ with $|\mathcal{M}| = 56$.

Figures 8-11 examine the performance of the relaxed optimization based channel allocation (CaRO) with that based on integer programming (CaIP). (We recall that the motivation for developing CaRO is to reduce the computational complexity of CaIP.) For the $\alpha = 0$ case, although CaIP outperformes CaRO, the worst off user throughput difference is not large (Figure 8 and Figure 10.) For $\alpha = 1$, CaRO performs better than CaIP in terms of total user throughput (Figure 9 and Figure 11). In this case, CaIP sometimes cannot solve the channel assignment problem yielding zero throughput, which clarifies the poor performance at the low percentiles.

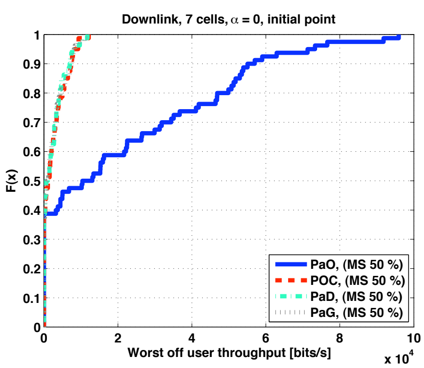

Figure 12: Comparing the PaO, POC, PaD and PaG power allocation algorithms in terms of the worst off user throughput (initial point), $\alpha = 0$, with $|\mathcal{M}| = 70$.

We now examine the performance of the power allocation algorithms under 50% and 90% load. (Figures 12-15 and Figures 16-20 respectively.) For $\alpha = 0$ (Figures 12-13 and 16-18), the power allocation optimization PaO (using SNOPT) yields superior performance for both the initial and final points. For $\alpha = 1$, although PaO gives the best total throughput initially (Figure 14), successively updating the allocations of the cell-wise optimization method (PoC) eventually gives better results. In fact, allocation updates combined with power allocation by derivative

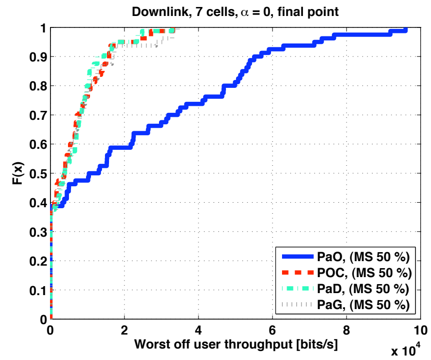

Figure 13: Comparing the PaO, POC, PaD and PaG power allocation algorithms in terms of the worst off user throughput (final point), $\alpha = 0$, with $|\mathcal{M}| = 70$.

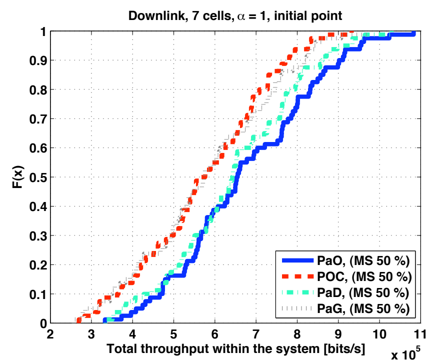

Figure 14: Comparing the PaO, POC, PaD and PaG power allocation algorithms in terms of total user throughput (initial point), $\alpha = 1$, with $|\mathcal{M}| = 70$.

approximation (PaD) and by greedy allocation (PaG) all perform better than PaO (Figure 15). Similar results are shown for high load by Figures 19-20.

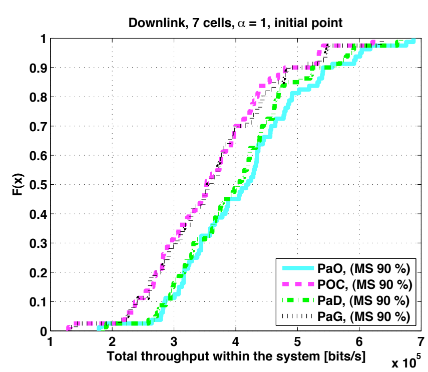

Figure 19: Comparing the PaO, POC, PaD and PaG power allocation algorithms in terms of total user throughput (initial point), $\alpha = 1$, with $|\mathcal{M}| = 126$.

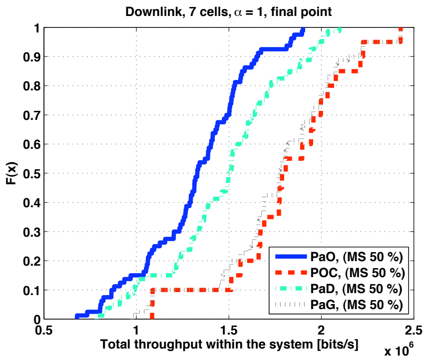

Figure 15: Comparing the PaO, POC, PaD and PaG power allocation algorithms in terms of total user throughput (final point), $\alpha = 1$, with $|\mathcal{M}| = 70$.

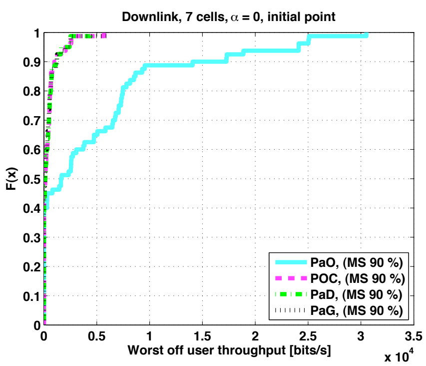

Figure 16: Comparing the PaO, POC, PaD and PaG power allocation algorithms in terms of the worst off user throughput (initial point), $\alpha = 0$, with $|\mathcal{M}| = 126$.

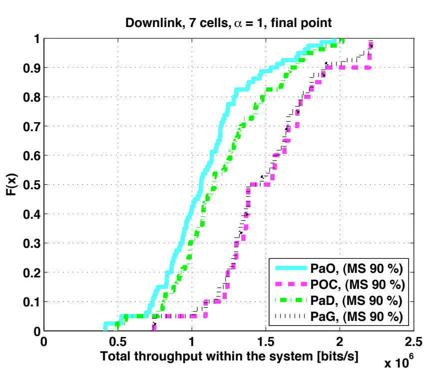

Figure 20: Comparing the PaO, POC, PaD and PaG power allocation algorithms in terms of total user throughput (final point), $\alpha = 1$, with $|\mathcal{M}| = 126$.

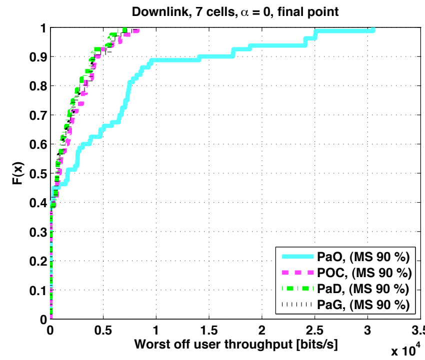

Figure 17: Comparing the PaO, POC, PaD and PaG power allocation algorithms in terms of the worst off user throughput (final point), $\alpha = 0$, with $|\mathcal{M}| = 126$.

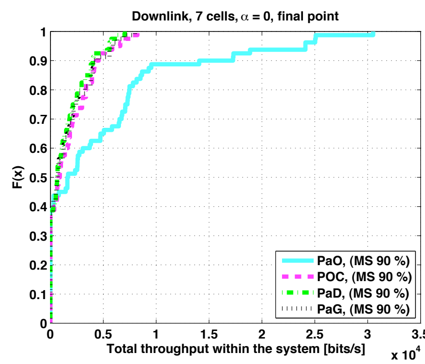

Figure 18: Comparing the PaO, POC, PaD and PaG power allocation algorithms in terms of total user throughput, when the objective is to maximize the worst off user throughput (final point), $\alpha = 0$, with $|\mathcal{M}| = 126$.

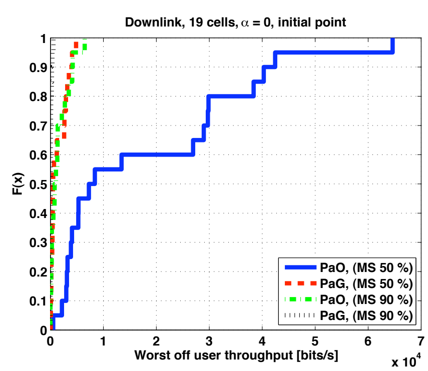

Figure 21: Comparing the PaO and PaG power allocation algorithms in terms of the worst off user throughput (initial point), $\alpha = 0$, $|\mathcal{B}| = 19$ with $|\mathcal{M}| = 190$ and $|\mathcal{M}| = 342$.

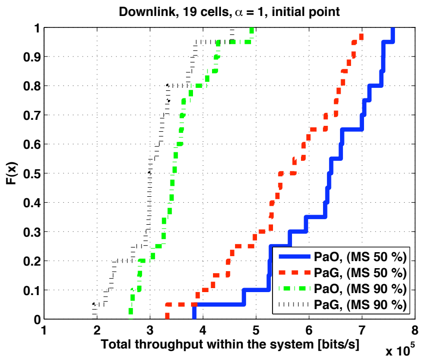

Figure 22: Comparing the PaO and PaG power allocation algorithms in terms of total user throughput (initial point), $\alpha = 1$, $|\mathcal{B}| = 19$ with $|\mathcal{M}| = 190$ and $|\mathcal{M}| = 342$.

Figures 21-22 show results for a larger system ($\mathcal{B} = 19$) that are similar to Figures 12-13 respectively (using LaG and CaG link and channel allocations). These figures indicate that also for a larger system, PaO yields highest total user throughput for the initial point.

C. Numerical Running Times in Downlink

In this section we present running times from the simulations with $|\mathcal{B}| = 7$ in downlink of section V-B. First we consider the setting of $|\mathcal{M}| = 70$ and present running times for four different configurations: $\alpha = 0$, (initial point); $\alpha = 0$, (final point); $\alpha = 1$, (initial point); $\alpha = 1$, (final point). Thereafter we present running times for the same four configurations, but with the setting $|\mathcal{M}| = 126$.

	PaO	**POC**	**PAD**	**PaG**	
LaG	2.6e+02	20	1.3	0.02	**CaG**
LaO	55	15	4.6	3.4	**CaRO**
LaG	30	25	4.8	3.6	**CaRO**
LaO	2.1e+02	23	1.5	0.031	**CaG**

Table VII: Average time [s] spent to find an initial point by solving a Testproblem ($|\mathcal{B}| = 7$, $|\mathcal{M}| = 70$ and $|\mathcal{C}| = 20$) with the different heuristic approaches, given $\alpha = 0$.

	PaO	**POC**	**PAD**	**PaG**	
LaG	2.7e+02	3e+02	1.9e+02	1.9e+02	**CaG**
LaO	57	70	56	45	**CaRO**
LaG	32	1.2e+02	63	52	**CaRO**
LaO	2.1e+02	3.1e+02	1.9e+02	1.7e+02	**CaG**

Table VIII: Average time [s] spent to find an final point, given an initial point, by solving a Testproblem ($|\mathcal{B}| = 7$, $|\mathcal{M}| = 70$ and $|\mathcal{C}| = 20$) with the different heuristic approaches, given $\alpha = 0$.

	PaO	**POC**	**PAD**	**PaG**	
LaG	11	3.7	10	0.018	**CaG**
LaO	15	8	18	3.8	**CaRO**
LaG	13	6.9	18	4.2	**CaRO**
LaO	12	3.7	11	0.044	**CaG**

Table IX: Average time [s] spent to find an initial point by solving a Testproblem ($|\mathcal{B}| = 7$, $|\mathcal{M}| = 70$ and $|\mathcal{C}| = 20$) with the different heuristic approaches, given $\alpha = 1$.

	PaO	**POC**	**PAD**	**PaG**	
LaG	29	13	31	4.3	**CaG**
LaO	36	13	39	4.9	**CaRO**
LaG	28	12	40	5.2	**CaRO**
LaO	31	14	37	5.1	**CaG**

Table X: Average time [s] spent to find an final point, given an initial point, by solving a Testproblem ($|\mathcal{B}| = 7$, $|\mathcal{M}| = 70$ and $|\mathcal{C}| = 20$) with the different heuristic approaches, given $\alpha = 1$.

	PaO	**POC**	**PAD**	**PaG**	
LaG	2.2e+02	25	2.5	0.031	**CaG**
LaO	76	29	22	19	**CaRO**
LaG	46	33	13	13	**CaRO**
LaO	2.3e+02	30	2.7	0.078	**CaG**

Table XI: Average time [s] spent to find an initial point by solving a Testproblem ($|\mathcal{B}| = 7$, $|\mathcal{M}| = 126$ and $|\mathcal{C}| = 20$) with the different heuristic approaches, given $\alpha = 0$.

	PaO	**POC**	**PAD**	**PaG**	
LaG	2.6e+02	7.6e+02	6e+02	6.1e+02	**CaG**
LaO	75	2.9e+02	1.6e+02	1.4e+02	**CaRO**
LaG	53	1.7e+02	1.2e+02	1.5e+02	**CaRO**
LaO	2.6e+02	7.5e+02	5.3e+02	5.8e+02	**CaG**

Table XII: Average time [s] spent to find an final point, given an initial point, by solving a Testproblem ($|\mathcal{B}| = 7$, $|\mathcal{M}| = 126$ and $|\mathcal{C}| = 20$) with the different heuristic approaches, given $\alpha = 0$.

	PaO	**POC**	**PAD**	**PaG**	
LaG	15	4	12	0.03	**CaG**
LaO	41	20	30	15	**CaRO**
LaG	57	18	28	11	**CaRO**
LaO	15	4.1	12	0.066	**CaG**

Table XIII: Average time [s] spent to find an initial point by solving a Testproblem ($|\mathcal{B}| = 7$, $|\mathcal{M}| = 126$ and $|\mathcal{C}| = 20$) with the different heuristic approaches, given $\alpha = 1$.

	PaO	POC	PAD	PaG	
LaG	55	24	52	14	**CaG**
LaO	68	24	49	13	**CaRO**
LaG	57	25	49	14	**CaRO**
LaO	56	25	51	14	**CaG**

Table XIV: Average time [s] spent to find an final point, given an initial point, by solving a Testproblem ($|\mathcal{B}| = 7$, $|\mathcal{M}| = 126$ and $|\mathcal{C}| = 20$) with the different heuristic approaches, given $\alpha = 1$.

D. Numerical Results for the Uplink

In this section we present some simulation results in uplink. Mainly we consider $|\mathcal{B}| = 7$ and study the system setting of having either 50% of MS in the system (i.e., $|\mathcal{M}| = 70$) or 90% of MS in the system (i.e., $|\mathcal{M}| = 126$). Also a larger system with $|\mathcal{B}| = 19$ is investigated briefly.

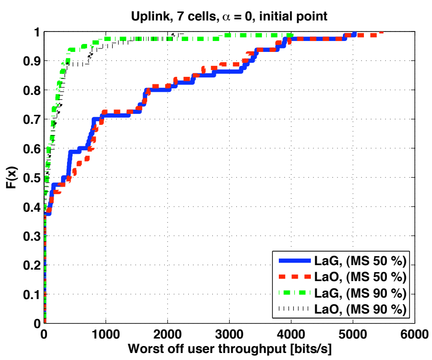

Figure 23: Comparing the LaG and LaO link allocation algorithms in terms of the worst off user throughput (initial point), $\alpha = 0$, with $|\mathcal{M}| = 70$ and $|\mathcal{M}| = 126$.

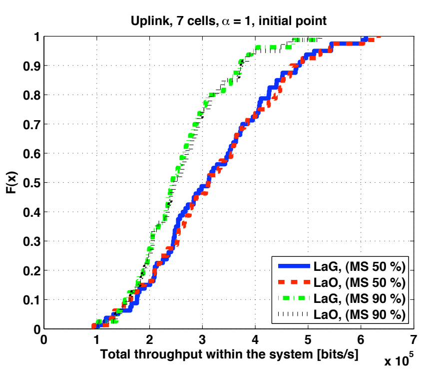

Figure 24: Comparing the LaG and LaO link allocation algorithms in terms of the total throughput (initial point), $\alpha = 1$, with $|\mathcal{M}| = 70$ and $|\mathcal{M}| = 126$.

Figures 23-24 compare the performance of the LaG and LaO link allocation algorithms for different load values. Similarly to the downlink (Figure 1-2), these results indicate that greedy link allocation performs similarly to link allocation by optimization.

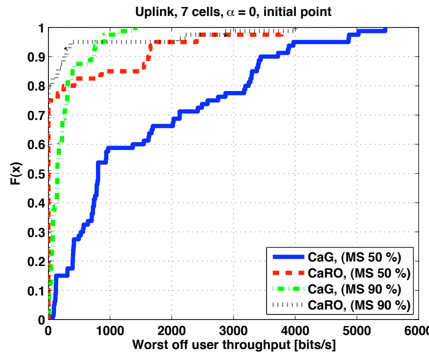

Figure 25: Comparing the CaG and CaRO channel allocation algorithms in terms of the worst off user throughput (initial point), $\alpha = 0$, with $|\mathcal{M}| = 70$ and $|\mathcal{M}| = 126$.

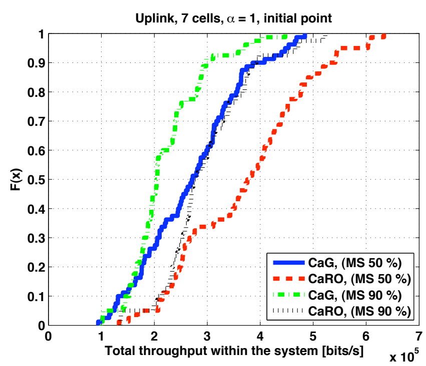

Figure 26: Comparing the CaG and CaRO channel allocation algorithms in terms of the total user throughput (initial point), $\alpha = 1$, with $|\mathcal{M}| = 70$ and $|\mathcal{M}| = 126$.

Figures 25 and 26 compare the performance of the CaG and CaRO channel allocations and give a similar insight to that of Figures 3 and 6. For $\alpha = 0$, CaG provides higher throughput to the worst off user than CaRO, while CaRO is superior in terms of the total throughput for the $\alpha = 1$ case.

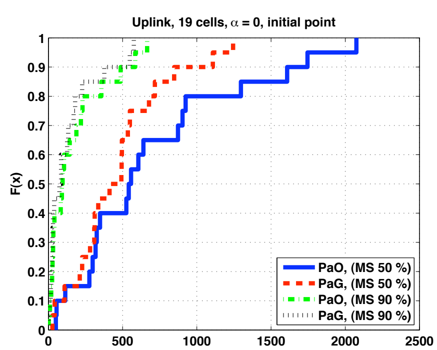

Figure 27: Comparing the PaO and PaG power allocation algorithms in terms of the worst off user throughput (initial point), $\alpha = 0$, $|\mathcal{B}| = 19$ with $|\mathcal{M}| = 190$ and $|\mathcal{M}| = 342$.

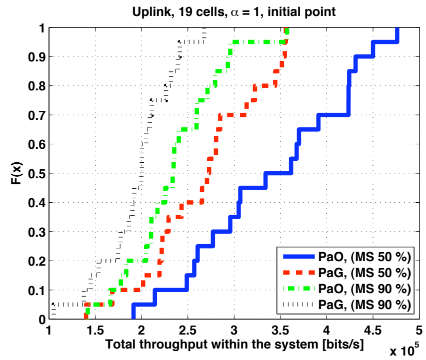

Figure 28: Comparing the PaO and PaG power allocation algorithms in terms of total user throughput (initial point), $\alpha = 1$, $|\mathcal{B}| = 19$ with $|\mathcal{M}| = 190$ and $|\mathcal{M}| = 342$.

Finally, we compare the results obtained by the PaO and PaG power allocation algorithms for the uplink (Figure 27-28). Similarly to the downlink case (Figures 21-22), these figures indicate that PaO yields highest worst off user throughput as well as the total user throughput.

VI. Conclusions

In this work, we considered the joint serving cell (link), channel and power allocation problem in multicell cellular systems that use orthogonal channel assignments within cells. The complexity of this rather general problem motivated to decompose it to separate link, channel and power assignment problems. To each of these tasks, we proposed a low complexity reference algorithm and a higher complexity optimization technique based algorithm as well as a heuristic allocation update (AU) procedure that iteratively reallocates the link channel and power resources.

We implemented the algorithms that solve the decomposed allocation tasks and iteratively update the overall resource allocations in a realistic system simulator called RUNE and generated numerical results both for the downlink and the uplink. One of the main results is that low complexity heuristics based on a greedy resource allocation of the link, channel and power resources (LaG, CaG and PaG respectively) perform surprisingly well when combined with an iterative update procedure in comparison with the optimization based approaches. The update procedure (AU) is attractive because due to its low complexity, its run time performance is more favorable than that of the optimization based techniques. The proposed update procedure shows good performance in various load conditions for both maximizing the minimum user throughput and maximizing the overall total throughput.

To develop distributed algorithms based on the insights of this paper is left for future work.

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