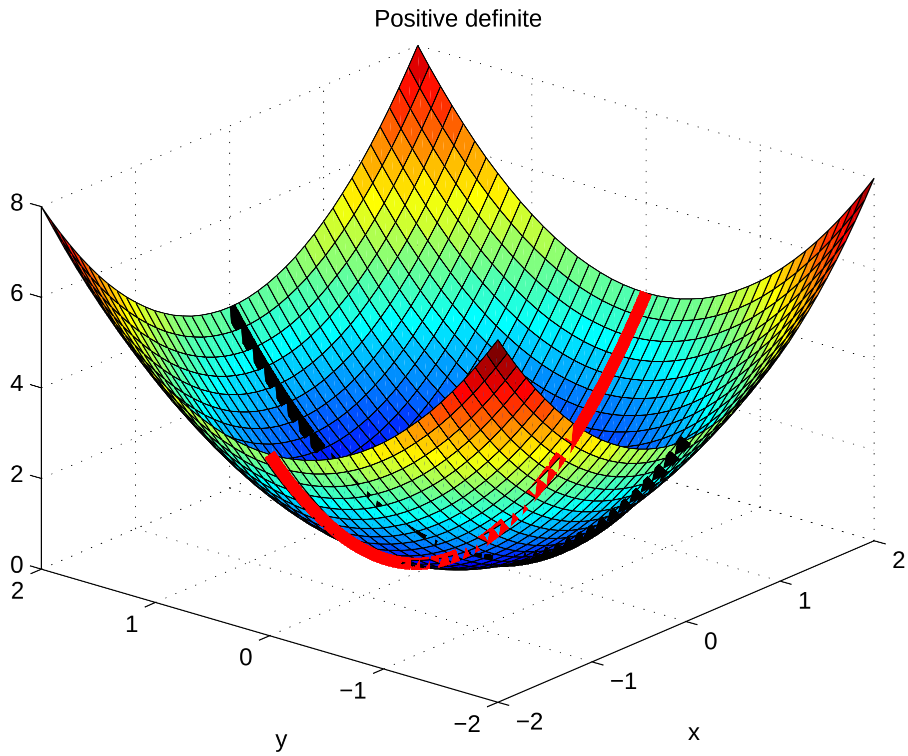

Figure 1: Quadratic function with linear equality constraints. Positive definite.

0.1 Quadratic forms

0.1.1 Positive definite

$$f = \frac{1}{2}x^T H x + c^T x + c_0$$

where

$$H = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad c = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad c_0 = 0$$

The constraints are given by

- (Red) $A_1 x = b$, $A_1 = [0, 1]$ and $b = 0$.
- (Black) $A_2 x = b$, $A_2 = [1, 0]$ and $b = 0$.

The optimization problem

$$\begin{aligned} \min \ & f(x) \\ \text{s.t. } & A_i x = b \end{aligned}$$

has a unique optimal solution for $i = 1, 2$.

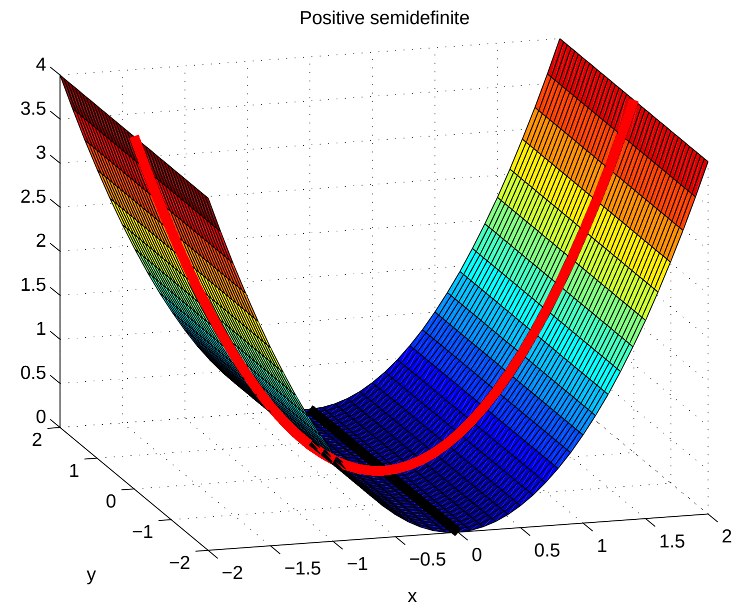

Figure 2: Quadratic function with linear equality constraints. Positive semidefinite, Case 1

0.1.2 Positive semidefinite, Case 1

$$f = \frac{1}{2}x^T Hx + c^T x + c_0$$

where

$$H = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, \quad c = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad c_0 = 0$$

The constraints are given by

- (Red) $Ax = b$, $A = [0, 1]$ and $b = 0$.
- (Black) $Ax = b$, $A = [1, 0]$ and $b = 0$.

The optimization problem

$$\begin{aligned} \min \; & f(x) \\ \text{s.t.} \; & A_1 x = b \end{aligned}$$

has a unique optimal solution, and

$$\begin{aligned} \min \; & f(x) \\ \text{s.t.} \; & A_2 x = b \end{aligned}$$

has infinitely many optimal solutions.

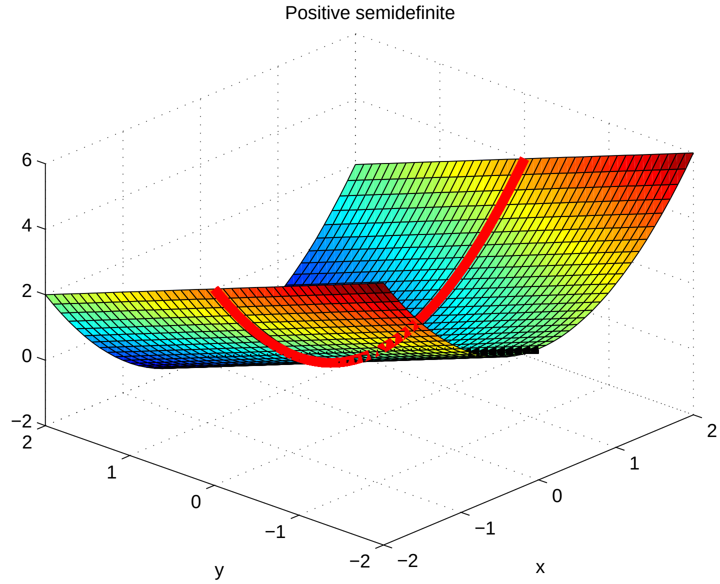

Figure 3: Quadratic function with linear equality constraints. Positive semidefinite, Case 2

0.1.3 Positive semidefinite, Case 2

$$f = \frac{1}{2}x^T H x + c^T x + c_0$$

where

$$H = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, \quad c = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \quad c_0 = 0$$

The constraints are given by

- (Red) $A_1 x = b$, $A_1 = [0, 1]$ and $b = 0$.
- (Black) $A_2 x = b$, $A_2 = [1, 0]$ and $b = 0$.

The optimization problem

$$\begin{aligned} &\min \ f(x) \\ &\text{s.t. } A_1 x = b \end{aligned}$$

has a unique optimal solution, and

$$\begin{aligned} &\min \ f(x) \\ &\text{s.t. } A_2 x = b \end{aligned}$$

is unbounded from below.

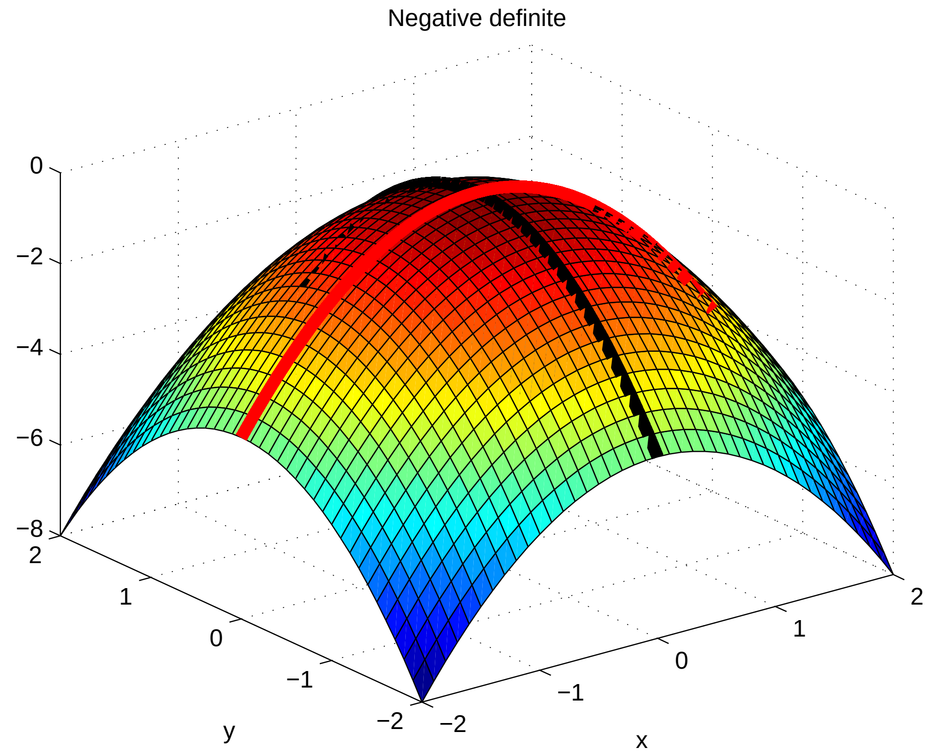

Figure 4: Quadratic function with linear equality constraints. Negative definite

0.1.4 Negative definite

$$f = \frac{1}{2}x^T H x + c^T x + c_0$$

where

$$H = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}, \quad c = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad c_0 = 0$$

The constraints are given by

- (Red) $A_1 x = b$, $A_1 = [0, 1]$ and $b = 0$.
- (Black) $A_2 x = b$, $A_2 = [1, 0]$ and $b = 0$.

The optimization problem

$$\begin{aligned} \min \; & f(x) \\ \text{s.t.} \; & A_i x = b \end{aligned}$$

is unbounded from below for $i = 1, 2$.

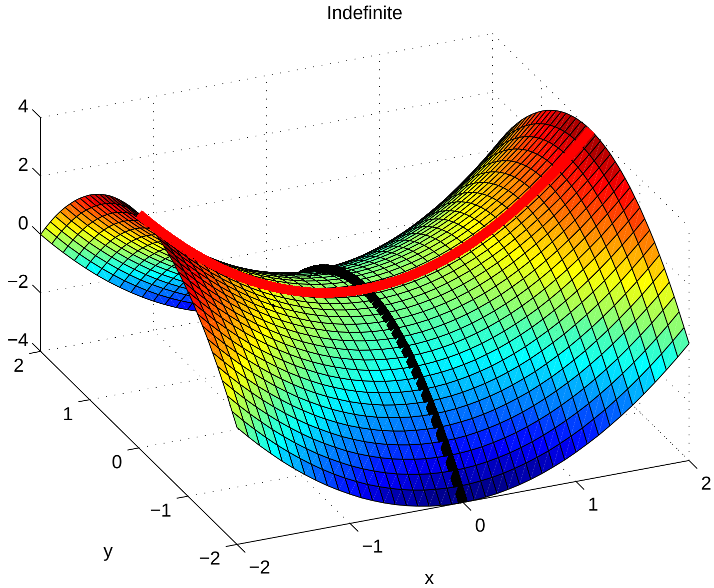

Figure 5: Quadratic function with linear equality constraints. Indefinite

0.1.5 Indefinite

$$f = \frac{1}{2}x^T Hx + c^T x + c_0$$

where

$$H = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}, \quad c = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad c_0 = 0$$

The constraints are given by

- (Red) $A_1 x = b$, $A_1 = [0, 1]$ and $b = 0$.
- (Black) $A_2 x = b$, $A_2 = [1, 0]$ and $b = 0$.

The optimization problem

$$\begin{aligned} \min \ & f(x) \\ \text{s.t. } & A_1 x = b \end{aligned}$$

has a unique optimal solution, and

$$\begin{aligned} \min \ & f(x) \\ \text{s.t. } & A_2 x = b \end{aligned}$$

is unbounded from below.