Multivariable simultaneous stabilization: A modified Riccati approach

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***Abstract*— Simultaneous stabilization problem arises in various systems and control applications. This paper introduces a new approach for addressing this problem in the multivariable scenario, building upon our previous findings in the scalar case. The method utilizes a Riccati-type matrix equation known as the Covariance Extension Equation, which yields all solutions parameterized in terms of a matrix polynomial. The procedure is demonstrated through specific examples.**

I. Introduction

To present and analyze the issues addressed in this paper, it is necessary to establish the following notation.

$\mathbb{C}$: the complex plane

$\mathbb{R}$: the real line

C_- : open left half of the complex plane

$C_+ : [\mathbb{C} - C_-] \cup \{\infty\}$

$R_u : \mathbb{R} \cap C_u$

H : ring of proper rational functions with real co-efficients with poles in C_-

$H^{p\times m}$: set of $p \times m$ matrices whose elements belong to H

J : set of multiplicative units in H

In the realm of control systems engineering, the challenge of achieving stability in multiple systems through a single controller is a topic of significant interest and practical importance [1], [2], [3]. This challenge is encapsulated in the concept of simultaneous stabilization, a pivotal area of study that has garnered considerable attention in both theoretical and applied research [4], [5].

Simultaneous stabilization problem involves the identification of a singular controller capable of stabilizing multiple plants [6], [7]. In our previous paper [8], we addressed the simultaneous stabilization problem for single-input single-output (SISO) systems, which is: Given a family of SISO proper transfer functions denoted as $p_\lambda(s)$ and expressed as:

$$p_\lambda(s) = \frac{\lambda x_1(s) + (1-\lambda)x_0(s)}{\lambda y_1(s) + (1-\lambda)y_0(s)} \tag{1}$$

where λ is a parameter in the interval $[0,1]$, and $x_0(s), x_1(s), y_0(s), y_1(s) \in H$, the objective is to determine a proper compensator $k(s)$ such that the closed-loop systems $p_\lambda(s)(1 + k(s)p_\lambda(s))^{-1}$ are stable for all λ in the interval $[0,1]$.

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As a generalization of the SISO case, the multi-input and multi-output (MIMO) simultaneous stabilization problem solves the following problem: Given a family $P_\lambda(s)$

$$P_\lambda(s) = N_\lambda(s) D_\lambda(s)^{-1} \tag{2}$$

where $\lambda \in [0,1]$ and

$$N_\lambda = \lambda N_1 + (1-\lambda) N_0 \tag{3}$$

$$D_\lambda = \lambda D_1 + (1-\lambda) D_0 \tag{4}$$

with $N_0, N_1, D_0, D_1 \in H^{m\times m}$, find a proper $m \times m$ compensator $K(s)$ which stabilizes all $P_\lambda(s)$.

In [9], [10], [11], BK Ghosh concentrated on addressing the simultaneous partial pole placement problem and introduced an interpolation method to solve this problem, offering a fresh perspective for resolving the simultaneous stabilization problem. In this paper, we employ a more comprehensive interpolation approach based on our prior research on a Riccati-type method for analytic interpolation [21], [20], which is built upon algorithms for the partial stochastic realization problem [14], [15], [16], [17] and on [18]. Specifically, we convert the simultaneous stabilization problem into a matrix analytic interpolation problem, which can be generally formulated as follows: Find a real rational function F of size $\ell \times \ell$, which is analytic within the unit disc $\mathbb{D} = \{z \mid |z| < 1\}$, such that it satisfies the inequality condition

$$F(e^{i\theta}) + F(e^{-i\theta})' > 0, \quad -\pi \le \theta \le \pi, \tag{5}$$

and also fulfills the interpolation conditions

$$\frac{1}{k!} F^{(k)}(z_j) = W_{jk}, \quad j = 0, 1, \cdots, m, \quad k = 0, \cdots n_j - 1, \tag{6}$$

where $'$ denotes transposition, $F^{(k)}(z)$ is the kth derivative of $F(z)$, and $z_0, z_1, \ldots, z_m$ are distinct points in $\mathbb{D}$ and $W_{jk} \in \mathbb{C}^{\ell\times\ell}$ for each (j,k). The complexity of the rational function $F(z)$ is constrained by limiting its McMillan degree to be at most ℓn, where

$$n = \sum_{j=0}^{m} n_j - 1. \tag{7}$$

Typically, this problem has an infinite number of solutions. However, as we will demonstrate in Section III, the freedom to select the $n \times n$ parameter Σ enables us to adjust the solution to specific requirements.

The paper is organized in the following manner: Section II outlines the essential conditions necessary for a group of plants to be simultaneously stabilizable and illustrates how the simultaneous stabilization problem can be converted into an analytical interpolation problem. Section III delves into solving the analytic interpolation problem using the Covariance Extension Equation (CEE). In Section IV, we present simulations to illustrate how the method can be utilized to stabilize multiple plants. Finally, in Section V, we provide concluding remarks and recommendations for future research.

II. The simultaneous stabilization problem

As explained in [9], [10], [11], every SISO system can be written as $x(s)/y(s)$, where $x(s), y(s) \in H$, and an $m \times m$ plant $P(s)$ has the left coprime representation $D_l(s)^{-1}N_l(s)$ and the right coprime representation $N_r(s)D_r(s)^{-1}$, where $N_l(s), D_l(s), N_r(s), D_r(s) \in H^{m\times m}$.

To solve the simultaneous stabilization problem, we firstly consider a simple case: Given two different plants

$$P_i(s) = N_i(s)D_i(s)^{-1}, \quad i = 0, 1 \tag{8}$$

where $N_i(s) \in H^{m\times m}, D_i(s) \in H^{m\times m}$, for $i = 0, 1$, find a proper $m \times m$ compensator $K(s)$ which can stabilize $P_0(s)$ and $P_1(s)$.

Set

$$M(s) = \begin{bmatrix} N_0(s) & N_1(s) \\ D_0(s) & D_1(s) \end{bmatrix} \tag{9}$$

and let $\text{Adj}(M(s))$ be its adjoint matrix. Moreover, suppose $\det(M(s))$ has simple zeros in $\mathbb{C}_+$ at $s_1, \cdots, s_t$ and $\det(M(\infty)) \neq 0$.

Proposition 1: The pair of distinct plants P_0, P_1 can be simultaneously stabilized by a proper compensator if and only if there exists $\Delta_i(s) \in H^{m\times m}$, $\det \Delta_i(s) \in J, i = 0, 1$, such that if $s_1, s_2, \cdots, s_t$ are the zeros of $\det(M)$ in $\mathbb{C}_+$, then

$$\begin{bmatrix} \Delta_0(s) & \Delta_1(s) \end{bmatrix} \text{Adj}(M(s)) = \mathbf{0} \tag{10}$$

at $s_1, s_2, \cdots, s_t$.

Proof: The proof of this proposition is based on the work of BK Ghosh [11]. Let us represent the compensator as

$$K(s) = D_c(s)^{-1}N_c(s) \tag{11}$$

where $N_c(s) \in H^{m\times m}, D_c(s) \in H^{m\times m}, N_c(s), D_c(s)$ are coprime. Then the compensator stabilize $P_0(s), P_1(s)$ if and only if

$$N_c(s)N_i(s) + D_c(s)D_i(s) = \Delta_i(s) \tag{12}$$

for some $\Delta_i(s) \in H^{m\times m}$, $\det \Delta_i(s) \in J, i = 0, 1$ respectively. We may write this in matrix form as

$$\begin{bmatrix} N_c & D_c \end{bmatrix} \begin{bmatrix} N_0(s) & N_1(s) \\ D_0(s) & D_1(s) \end{bmatrix} = \begin{bmatrix} \Delta_0(s) \ \Delta_1(s) \end{bmatrix} \tag{13}$$

and then

$$\begin{bmatrix} N_c & D_c \end{bmatrix} = \begin{bmatrix} \Delta_0(s) \ \Delta_1(s) \end{bmatrix} \frac{\text{Adj}(M(s))}{\det(M(s))} \tag{14}$$

In order that $N_c(s) \in H^{m\times m}, D_c(s) \in H^{m\times m}$, it is necessary and sufficient that

$$[\Delta_0(s) \ \Delta_1(s)]\text{Adj}(M(s)) = \mathbf{0} \tag{15}$$

at $s_1, s_2, \cdots, s_t$, where $s_1, s_2, \cdots, s_t$ are the simple zeros of $\det(M)$ in $\mathbb{C}_+$. ∎

Assume generically that $\text{Adj}(M(s_i))$ are of rank 1, If $\text{Adj}(M(s_i))$ are spanned by a column vector $\mathbf{r_i} = [\mathbf{r_{i1}}, \mathbf{r_{i2}}]'$, then

$$\Delta_0(s_i)\mathbf{r_{i1}}' + \Delta_1(s_i)\mathbf{r_{i2}}' = \mathbf{0} \tag{16}$$

for $i = 1, \cdots, t$. From equation (16), we can derive $(\Delta_0(s_i))^{-1}\Delta_1(s_i)$.

Now we consider the more general case, suppose

$$P_\lambda(s) = N_\lambda(s)D_\lambda(s)^{-1} \tag{17}$$

where $\lambda \in [0, 1]$ and

$$N_\lambda = \lambda N_1 + (1 - \lambda)N_0 \tag{18}$$

$$D_\lambda = \lambda D_1 + (1 - \lambda)D_0 \tag{19}$$

Proposition 2: If $(\Delta_0)^{-1}\Delta_1$ is diagonalizable at $s \in \mathbb{C}_+$, then the family of plants $P_\lambda(s)$ for $\lambda \in [0, 1]$ can be simultaneously stabilized by a proper compensator if and only if there exist $\Delta_i(s) \in H^{m\times m}$, with $\det \Delta_i(s) \in J$ for $i = 0, 1$, satisfying the condition of Proposition 1, along with the additional condition that the eigenvalues of $\Delta_0(s)^{-1}\Delta_1(s)$ do not intersect the nonpositive real axis, including infinity, at any point in $\mathbb{C}_+$.

Proof: Suppose $K(s)$ is the required compensator. To stabilize the plants $P_0(s), P_1(s)$ simultaneously, a necessary and sufficient condition is given by Proposition 1. Additionally, $K(s)$ simultaneously stabilizes every other plants $P_\lambda(s)$ if and only if there exist $\Delta_\lambda(s) \in H^{m\times m}$, $\det \Delta_\lambda(s) \in J$ such that

$$N_c(s)N_\lambda(s) + D_c(s)D_\lambda(s) = \Delta_\lambda(s) \tag{20}$$

By calculation,

$$\lambda\Delta_1(s) + (1 - \lambda)\Delta_0(s) = \Delta_\lambda(s) \tag{21}$$

Since $\Delta_0(s) \in H^{m\times m}$ and $\Delta_1(s) \in H^{m\times m}$, we have $\Delta_\lambda(s) \in H^{m\times m}, \lambda \in [0, 1]$. To have $\det \Delta_\lambda(s) \in J, \lambda \in [0, 1]$, we need

$$\det(\lambda\Delta_2(s) + (1 - \lambda)\Delta_1(s)) \neq 0, \lambda \in [0, 1] \tag{22}$$

at $s_i \in \mathbb{C}_+$, which means the eigenvalues of $\Delta_0(s)^{-1}\Delta_1(s)$ do not intersect the nonpositive real axis for $s_i \in \mathbb{C}_+$. ∎

Next, we reformulate the MIMO simultaneous stabilization problem as multivariable analytic interpolation problem. Let

$$F_1(s) = (\Delta_0(s)^{-1}\Delta_1(s))^{1/2} \tag{23}$$

be the square-root of $\Delta_0(s)^{-1}\Delta_1(s)$. Then the eigenvalues of $F_1(s)$ have positive real parts at $s \in \mathbb{C}_+$. Using the Möbius transformation $z = (1 - s)(1 + s)^{-1}$, which maps $\mathbb{C}_+$ into the interior of the unit disc, we set

$$F(z) := F_1((1 - z)(1 + z)^{-1}) \tag{24}$$

Then the problem is simplified to finding a Carathéodory function $F(z)$ such that the interpolation constraints (16) is satisfied. This is the matrix case analytic interpolation problem. After obtaining the solution for $F(z)$, we may apply the following transformations to obtain the compensator $K(s)$. Denote

$$\frac{Adj M(s)}{\det(M(s)} = \begin{bmatrix} m_{11}(s) & m_{12}(s) \\ m_{21}(s) & m_{22}(s) \end{bmatrix} \quad (25)$$

Then

$$F_1(s) = F((1-s)(1+s)^{-1}) \quad (26)$$

$$K(s) = D_c^{-1} N_c = (m_{12} + F_1^2 m_{22})^{-1}(m_{11} + F_1^2 m_{21}) \quad (27)$$

III. The analytic interpolation problem

In this section, we demonstrate how to address the analytic interpolation problem (6) using the Covariance Extension Equation [19], [20], [21]. We can assume, without loss of generality, that $z_0 = 0$ and $W_0 = \frac{1}{2}I$. Then, $F(z)$ can be realized as

$$F(z) = \tfrac{1}{2}I + zH(I - zF)^{-1}G, \quad (28)$$

where $H \in \mathbb{R}^{\ell \times \ell n}$, $F \in \mathbb{R}^{\ell n \times \ell n}$, $G \in \mathbb{R}^{\ell n \times \ell}$, (H, F) is an observable pair, and the matrix F has all its eigenvalues in $\mathbb{D}$.

Defining $\Phi_+(z) := F(z^{-1})$ we have

$$\Phi_+(z) = \tfrac{1}{2}I + H(zI - F)^{-1}G, \quad (29)$$

which has all its poles in the unit disc $\mathbb{D}$. In view of (5)

$$\Phi_+(e^{i\theta}) + \Phi_+(e^{-i\theta})' > 0, \quad -\pi \le \theta \le \pi, \quad (30)$$

and thus $\Phi_+(z)$ is positive real [12, Chapter 6]. The problem is then reduced to finding a rational positive real function $\Phi_+(z)$ of degree at most ℓn that meets the interpolation constraints (6). Through a coordinate transformation $(H, F, G) \to (HT^{-1}, TFT^{-1}, TG)$ we can select (H, F) in the observer canonical form

$$H = \operatorname{diag}(h_{t_1}, h_{t_2}, \dots, h_{t_\ell}) \in \mathbb{R}^{\ell \times n\ell} \quad (31)$$

with $h_\nu := (1, 0, \dots, 0) \in \mathbb{R}^\nu$, and

$$F = J - AH \in \mathbb{R}^{n\ell \times n\ell} \quad (32)$$

where $J := \operatorname{diag}(J_{t_1}, J_{t_2}, \dots, J_{t_\ell})$ with J_ν the $\nu \times \nu$ shift matrix

$$J_\nu = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \quad (33)$$

and $A \in \mathbb{R}^{n\ell \times \ell}$. The numbers $t_1, t_2, \dots, t_\ell$ are the *observability indices* of $\Phi_+(z)$, and

$$t_1 + t_2 + \dots + t_\ell = n\ell. \quad (34)$$

Next define $\Pi(z) := \operatorname{diag}(\pi_{t_1}(z), \pi_{t_2}(z), \dots, \pi_{t_\ell}(z))$, where $\pi_\nu(z) = (z^{\nu-1}, \dots, z, 1)$, and the $\ell \times \ell$ matrix polynomial

$$A(z) = D(z) + \Pi(z)A, \quad (35)$$

where

$$D(z) := \operatorname{diag}(z^{t_1}, z^{t_2}, \dots, z^{t_\ell}). \quad (36)$$

From Lemma 1 in [21],

$$H(zI - F)^{-1} = A(z)^{-1}\Pi(z), \quad (37)$$

and consequently

$$\Phi_+(z) = \tfrac{1}{2}A(z)^{-1}B(z), \quad (38)$$

where

$$B(z) = D(z) + \Pi(z)B$$

with

$$B = A + 2G. \quad (39)$$

Moreover let $V(z)$ be the minumum-phase spectral factor of

$$V(z)V(z^{-1})' = \Phi(z) := \Phi_+(z) + \Phi_+(z^{-1})'. \quad (40)$$

We know [12, Chapter 6] that $V(z)$ has a realization of the form

$$V(z) = H(zI - F)^{-1}K + R, \quad (41)$$

which, by (37), can be written

$$V(z) = A(z)^{-1}\Sigma(z)R, \quad (42)$$

where

$$\Sigma(z) = D(z) + \Pi(z)\Sigma \quad (43)$$

with

$$\Sigma = A + KR^{-1}. \quad (44)$$

From stochastic realization theory [12, Chapter 6] we have

$$K = (G - FPH')(R')^{-1} \quad (45)$$

$$RR' = I - HPH' \quad (46)$$

where P is the unique minimum solution of the algebraic Riccati equation

$$P = FPF' + (G - FPH')(I - HPH')^{-1}(G - FPH')'. \quad (47)$$

Now, from (32), (45) and (46) we have

$$\begin{aligned} G &= JPH' - AHPH' + KR^{-1}(I - HPH') \\ &= \Gamma PH' + KR^{-1}, \end{aligned}$$

where, in view of (44),

$$\Gamma = J - \Sigma H. \quad (48)$$

Hence, by (44),

$$G = \Gamma PH' + \Sigma - A. \quad (49)$$

Since $F = \Gamma + KR^{-1}H$ and $G - \Gamma PH' = KR^{-1}$, (47) can be written

$$P = \Gamma(P - PH'HP)\Gamma' + GG'. \quad (50)$$

The article [20] demonstrates that G can be expressed as $u + U(\Sigma + \Gamma PH')$, with u and U being fully determined by the interpolation data (6). The analytic interpolation problem involves determining the values of (A, B) based on the given interpolation data (6) and a specific matrix polynomial $\Sigma(z)$.

In [20] we conclude that the conditions for the existence of solutions to this problem only depend on the interpolation data. If the solution exists, it is also shown in [20] that the *Covariance Extension Equation (CEE)*

$$P = \Gamma(P - PH'HP)\Gamma' + G(P)G(P)' \tag{51a}$$

with

$$G(P) = u + U(\Sigma + \Gamma PH'), \tag{51b}$$

has a unique symmeric solution $P \geq 0$ with the property that $HPH' < 1$. Additionally, for every Σ, there exists a unique solution to the analytic interpolation problem, which is expressed as follows:

$$A = (I - U)(\Gamma PH' + \Sigma) - u \tag{52a}$$

$$B = (I + U)(\Gamma PH' + \Sigma) + u \tag{52b}$$

$$R = (I - HPH')^{\frac{1}{2}}, \tag{52c}$$

The equation (51) can be solved using a homotopy continuation approach, as described in [20]. Distinct selections of matrix polynomial $\Sigma(z)$ yield diverse viable solutions.

IV. Computational Examples

A. Example 1

First we consider a simple MIMO simultaneous stabilization problem with

$$N_0 = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \quad D_0 = \begin{bmatrix} \frac{s-2}{s+6} & 1 \\ 3 & \frac{s-2.7}{s+10} \end{bmatrix}$$
$$N_1 = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \quad D_1 = \begin{bmatrix} \frac{s-3.2}{s+2.2} & 1 \\ 3 & \frac{s-7.7}{s+1} \end{bmatrix} \tag{53}$$

There exist poles that are not stable when the parameter λ changes within the range of values from 0 to 1. In order to enhance the visibility of the poles, we utilize the transformation defined by equation (54),

$$z = \frac{1+s}{1-s} \tag{54}$$

This transformation effectively maps left half plane to the interior of the unit circle, while also converting the right half plane to the exterior of the unit circle. A stable system is characterized by having all poles located within the unit circle. Fig. 1 displays the whole set of poles of P_λ as λ ranges from 0 to 1 in increments of 0.1.

Based on the information provided in Fig. 1, it is evident that some systems lack stability. Applying the technique described in this paper, we first observe that $\det(M(s))$ has two zeros at $s_0 = 12.24$ and $s_1 = 1.494$ in $\mathbb{C}_+$. In order to achieve system stability, it is necessary to satisfy the interpolation criteria (16), which result in the following equation:

$$(\Delta_0(s_0))^{-1}\Delta_1(s_0) = M_0, \ (\Delta_0(s_1))^{-1}\Delta_1(s_1) = M_1 \tag{55}$$

and through the use of the Möbius transformation $z = (1 - s)(1 + s)^{-1}$, the open right half plane is mapped into the interior of the unit disk. Here, the eigenvalues of M_0 and

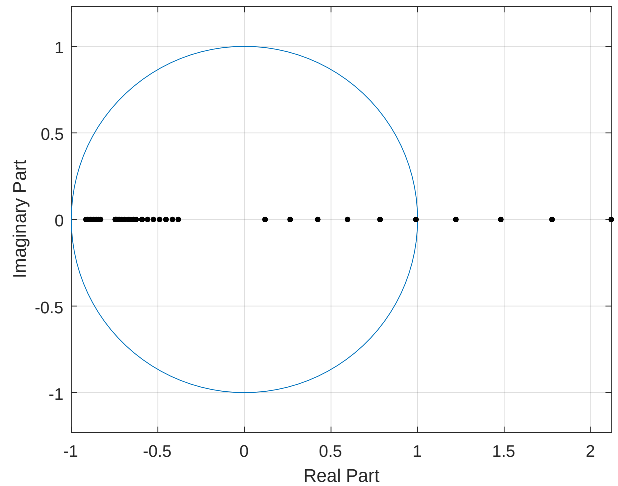


Fig. 1. The poles of P_λ before stabilization

M_1 have positive real parts, the problem is then reduced to the analytic interpolation problem with $n_0 = n_1 = 1$. The interpolation constraints are

$$\begin{aligned} (\Delta_0^{-1}\Delta_1)(\frac{1-s_0}{1+s_0}) &= M_0 \\ (\Delta_0^{-1}\Delta_1)(\frac{1-s_1}{1+s_1}) &= M_1 \end{aligned} \tag{56}$$

By the theory in [20], there exists solutions. Here we choose $\Sigma = [0.3\ 0; 0\ 0.5]$ and get

$$(\Delta_0(s))^{-1}\Delta_1(s) = \begin{bmatrix} 1 & \frac{0.4571s^2+38.42s+572.4}{s^2+35.22s+279.8} \\ -1 & \frac{0.1414s^2+38.5s+869.2}{s^2+35.22s+279.8} \end{bmatrix} \tag{57}$$

After stabilization, the locations of the poles of $P_\lambda, \lambda \in [0, 1]$ are depicted in Figure 2. As all the poles are situated within the open unit disc, it indicates that all feedback systems are stable.

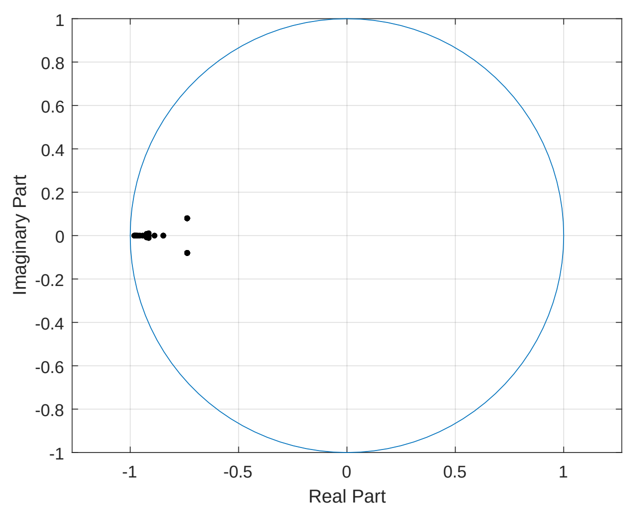
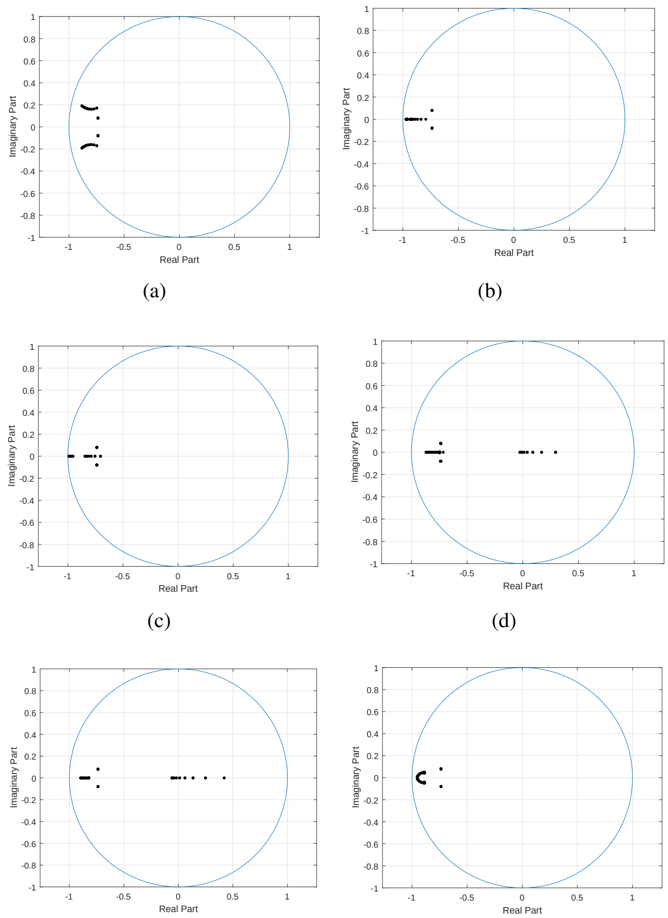


Fig. 2. The poles of P_λ after stabilization

To demonstrate that different choices of Σ produce different feasible solutions, we take Σ to be

$$a=\begin{bmatrix}-0.1 & -0.9\\ 0.4 & -0.6\end{bmatrix}\qquad b=\begin{bmatrix}0.4 & 0.1\\ 0.5 & 0.4\end{bmatrix}$$
$$c=\begin{bmatrix}0.2 & 0.35\\ 0.6 & 0.4\end{bmatrix}\qquad d=\begin{bmatrix}-0.8 & 0.1\\ 0.6 & -0.2\end{bmatrix}\tag{58}$$
$$e=\begin{bmatrix}-0.65 & 0.22\\ 0.8 & -0.2\end{bmatrix}\qquad f=\begin{bmatrix}0.8 & -0.33\\ 0.9 & 0.7\end{bmatrix}.$$

respectively. Fig. 3 shows the corresponding results, indicating that the solution changes with different Σ.



(a) (b) (c) (d) (e) (f)

Fig. 3. The poles of the stabilized system with diferent Σ

B. Example 2

Next we consider a more complex system which includes complex unstable zeros. Suppose

$$N_0=\frac{1}{z^2+2z+10}\begin{bmatrix}3(z^2-2z+1) & 5(z^2+2z+10)\\ 4(z^2+2z+10) & 2(z-4)(z-2)\end{bmatrix}$$

$$D_0=\frac{1}{z^2+2z+10}\begin{bmatrix}2(z-2)(z-3) & -(z^2+2z+10)\\ 3(z^2+2z+10) & (z-2)^2\end{bmatrix}$$

and

$$N_1=\frac{1}{z^2+2z+15}\begin{bmatrix}(z-1)(z+1) & 6(z^2+2z+15)\\ 7(z^2+2z+15) & (z-8)(z-1)\end{bmatrix}$$

$$D_1=\frac{1}{z^2+2z+15}\begin{bmatrix}(z-6)(z+2) & 2(z^2+2z+15)\\ -(z^2+2z+15) & 3(z-5)(z-9)\end{bmatrix}$$

There are numerous poles that are unstable as λ changes within the range of 0 to 1. In Fig. 4 we show the poles of P_λ as λ varies from 0 to 1 at intervals of 0.1. Unlike the previous example, there are many complex poles in this example.

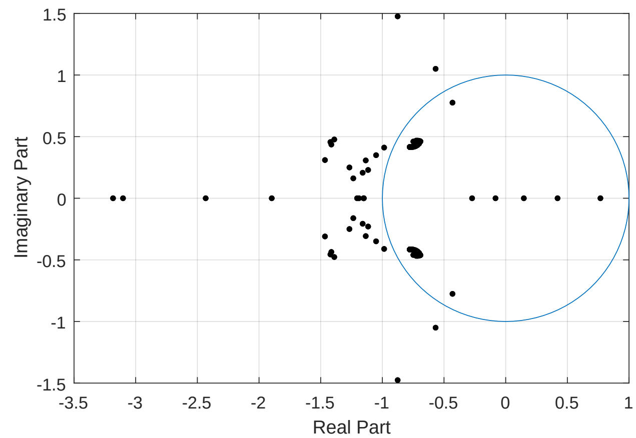


Fig. 4. The poles of P_λ before stabilization

From Fig. 4, we can see there are some systems that are not stable. Using the method in this paper, we first observe that $\det(M(s))$ has three zeros at $s_0 = 0.2322$, $s_1 = 0.9862 - 3.5291i$, $s_2 = 0.9862 + 3.5291i$ in $\mathbb{C}_+$. To make the systems stable, we therefore need the interpolation conditions (16), which yield

$$\begin{aligned}(\Delta_0(s_0))^{-1}\Delta_1(s_0) &= M_0,\\ (\Delta_0(s_1))^{-1}\Delta_1(s_1) = M_1,&(\Delta_0(s_2))^{-1}\Delta_1(s_2) = M_2\end{aligned}\tag{59}$$

using the Möbius transformation $z=(1-s)(1+s)^{-1}$ and since M_0, M_1, M_2 have negative eigenvalues, so we take the square-root, the problem is then reduced to the analytic interpolation problem with the interpolation conditions

$$\begin{aligned}(\Delta_0^{-1}\Delta_1)^{1/2}\Big(\frac{1-s_0}{1+s_0}\Big) &= (M_0)^{1/2}\\ (\Delta_0^{-1}\Delta_1)^{1/2}\Big(\frac{1-s_1}{1+s_1}\Big) &= (M_1)^{1/2}\\ (\Delta_0^{-1}\Delta_1)^{1/2}\Big(\frac{1-s_2}{1+s_2}\Big) &= (M_2)^{1/2}\end{aligned}\tag{60}$$

Here we choose

$$\Sigma=\begin{bmatrix}0.4 & 0.2\\ 0.3 & -0.5\\ 0.8 & -0.1\\ 0.6 & -0.2\end{bmatrix}.\tag{61}$$

Then we get the result

$$(\Delta_0(s)^{-1}\Delta_1(s))^{1/2}=\begin{bmatrix}K_{11} & K_{12}\\ K_{21} & K_{22}\end{bmatrix}\tag{62}$$

$$K_{11}=\frac{s^4+0.923s^3+0.6752s^2+0.1567s+0.02193}{s^4+0.3846s^3+0.6211s^2+0.1852s+0.0186}$$

$$K_{12} = \frac{-0.325s^4 + 1.821s^3 + 0.3929s^2 - 0.0064s - 0.0111}{s^4 + 0.3846s^3 + 0.6211s^2 + 0.1852s + 0.0186}$$

$$K_{21} = \frac{-0.5581s^4 + 0.9026s^3 - 0.2138s^2 - 0.01s - 0.03345}{s^4 + 0.3846s^3 + 0.6211s^2 + 0.1852s + 0.0186}$$

$$K_{22} = \frac{0.2714s^4 + 3.214s^3 + 1.429s^2 + 0.1429s + 0.017}{s^4 + 0.3846s^3 + 0.6211s^2 + 0.1852s + 0.0186}$$

Fig. 5 displays the poles of $P_\lambda, \lambda \in [0, 1]$ after stabilization. All feedback systems are stable because all of the poles are in the open unit disc.

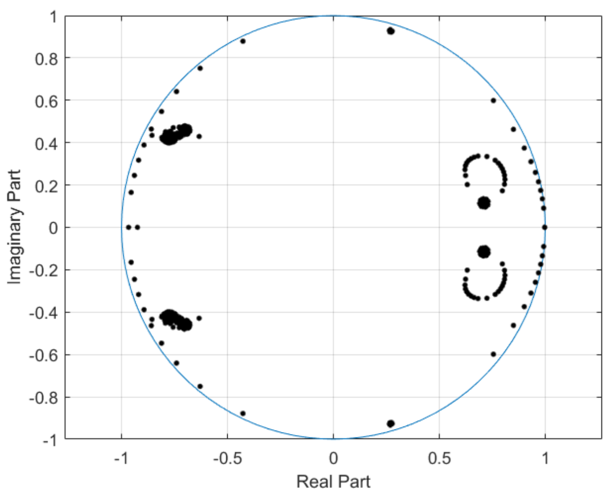


Fig. 5. The poles of P_λ after stabilization

V. CONCLUSION

In this study, we focus on the MIMO simultaneous stabilization issue and reframe it as an analytic interpolation problem. We resolve this problem by employing a Riccati-type algebraic matrix equation known as the Covariance Extension Equation. Furthermore, we present various solutions by selecting different matrix polynomials. In future research, we intend to incorporate derivative constraints.

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