

Project II - SF2975 Financial Derivatives

Henrik Hult, Fredrik Viklund, Alexander Aurell

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Project II: Calibrating the Hull-White Model

Preliminaries

The objective is to calibrate a one-factor Hull-White model to market data, obtained via QuantLab. The following is available from the QuantLab workspace that can be found on the course webpage.

1. A workspace with bond selection where you can select government bonds with a variety of maturities and coupons to be included in the calibration.
2. A selection of interpolation method used to obtain the yield curve.
3. An output yield curve on a time grid with daily resolution, from today at time 0 until the longest maturity among the selected bonds. The output yield curve can be exported to, e.g, MATLAB.

Hull-White Model

Recall that the one-factor *Hull-White model* for the continuously compounded yearly short rate is given as follows:

$$dr(t) = (\theta(t) - ar(t))dt + \sigma dW(t), \quad r(0) = r_0.$$

The time refers to years with the longest maturity being $T^* < \infty$. Let $a = 5\%$ and $\sigma = 1\%$, say, and $\theta(t)$ is a deterministic function.

Assignments

1. The first objective is to compute the function $\theta(t)$ numerically, as a piecewise constant function over each day, from the yield curve implied by market data. This can be done in the following steps.

- (a) Transform the yield curve to a (piecewise constant) forward rate curve $f^*(0, T)$, $0 \leq T \leq T^*$.
 - (b) Express $\theta(t)$ in terms of the forward rate curve. See Chapter 24 of Björk.
 - (c) Illustrate the piecewise constant function $\theta(t)$ graphically.
2. Derive formulas for pricing caplets and swaptions in the Hull-White model. The formulas can be expressed using the formula for zero-bond put options as described on p5 of the paper “Calibration Methods of Hull-White Model” that can be found on the course webpage. Hint: In the case of swaptions, use the so-called Jamshidian trick, see the article and/or Wikipedia. See also Chapter 26 of Björk.
3. Optional: Use the formulas you derived together with your calibrated model to numerically compute prices of either caplets or swaptions (or both).