Project Part 3, Analysis of financial time series

Before you start solving this problem, download and open the workspace and read the workspace guide. Both are available from Alexander Aurell's homepage `https://people.kth.se/~aaurell/Teaching/SF2943_VT18/garch.html`.

Background

Your task is to model (with the help of Finance Lab) the risk of holding one unit's worth of the OMXS30 index for a few days using a GARCH process and compare it to a naive approach. The OMXS30 index is a weighted mean of the 30 most traded stocks on the Stockholm stock exchange. Let S_t be the value of the index at closing time day t and let $t = 0$ be today. A typical thing to look at when evaluating risk of holding a unit of the index for $k > 0$ days is the quantiles in the left tail of $S_k - S_0$. This gives an estimate of the worst case return (loss of money) in a certain percentage of all possible scenarios.
The distribution of the return on the investment should be modeled in two ways: A naive approach based on fitting a normal distribution and using a GARCH process. First, transform past index values $(S_{-N}, \dots, S_0)$ into log-returns

$$X_t = \log(S_t/S_{t-1}).$$

For the naive approach, assume the the log-returns are i.i.d. $\mathcal{N}(\mu, \sigma^2)$ -distributed random variables. One can than estimate (μ, σ) by $(\hat{\mu}, \hat{\sigma})$ using historical data and model according to

$$S_k - S_0 = S_0 \left(e^{X_1+\dots+X_k} - 1\right) \overset{d}{\approx} S_0 \left(e^{k\hat{\mu}+\sqrt{k}\hat{\sigma}Z} - 1\right), \tag{1}$$

where Z is a standard normal random variable. Quantiles of the expression in (1) can be obtained from the theoretical quantiles of a normal distribution.
For the second approach, a GARCH(p, q) process should be fitted to the log-returns of $(S_{-N}, \dots, S_0)$ and S_k is retrieved through the following scheme:

$$\begin{aligned}
X_t &= \sigma_t Z_t, \quad \{Z_t\} \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0,1), \\
\sigma_t^2 &= \alpha_0 + \sum_{i=1}^{p} \alpha_i X_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \\
Y_t &= \mu + X_t \\
S_k &= S_0 e^{Y_1 + \dots Y_k}.
\end{aligned}$$

From simulations it is now possible to calculate the empirical quantile of

$$S_k - S_0 = S_0(e^{Y_1+\dots Y_k} - 1).$$

Problems

Note: For the short time horizons of this exercise, you can simply assume that $\mu = 0$.

a1) Calculate the 0.05-quantiles for the value S_1 "tomorrow" of the OMXS30-index, if "today" ($t = 0$) is

i) January 11th, 2018

ii) February 9th, 2018

For each "today", use both the naive and the GARCH approach described above and compare the resulting quantiles. You should start with a GARCH(1,1) model for the second approach, but feel free to see how different orders of the GARCH-model affect the outcome. The GARCH-model should not be fitted to periods longer than 4-5 years. The naive volatility estimator should be based on the last n observations, for n between 50 and 100.

a2) The same as in a1), but now for $k = 5$, i.e. you look five days ahead. For the naive approach you can calculate again, for the GARCH approach base your answer on simulations. Note that the GARCH-volatility will also change from day to day!

b) In the Finance Lab workspace you will also find a plot of the log-returns together with the GARCH-implied and the naive volatility σ_t. Use this plot to interpret your results from a).