SF2943: Application of the GARCH model

Before you start solving this problem, download and open the workspace and read the workspace guide. Both are available from the course homepage.

Background

In this exercise, you will (with the help of Quantlab) model the risk of holding one unit's worth of the OMXS30 index for a few days using a GARCH process and compare it to a naive approach. The OMXS30 index is a weighted mean of the 30 most traded stocks on the Stockholm stock exchange. Let S_t be the value of the index at closing time day t and let $t = 0$ be today. A typical thing to look at when evaluating risk of holding a unit of the index for k days is the quantiles in the left tail of $S_k - S_0$. This gives an estimate of the worst case return (loss of money) in a certain percentage of all possible scenarios. The distribution of the return on the investment will modeled in two ways; a naive approach based on fitting a normal distribution and an approach using a GARCH process.
Start by transforming past index values $(S_{-N}, \dots, S_0)$ into log-returns,

$$X_t = \ln(S_t/S_{t-1}). \tag{1}$$

For the naive approach, assume that the log-returns are i.i.d. $\mathcal{N}(\mu, \sigma)$-distributed random variables. If (μ, σ) is estimated with $(\hat{\mu}, \hat{\sigma})$ (using historical data), one has

$$S_k - S_0 = S_0 \left(e^{X_1 + \cdots + X_k} - 1\right) \overset{d}{\approx} S_0 \left(e^{k\hat{\mu} + \sqrt{k}\hat{\sigma} Z} - 1\right), \tag{2}$$

where Z is a standard normal random variable. By sampling from Z, the empirical quantiles of $-(S_k - S_0)$ can be calculated approximately. Even better, there is an analytical formula for the quantile,

$$F^{-1}_{S_0 - S_k}(p) = S_0 \left(1 - e^{k\hat{\mu} - \sqrt{k}\hat{\sigma}\Phi^{-1}(p)}\right) \tag{3}$$

and for the density of $S_k - S_0$,

$$f_{S_k - S_0}(x) = \left| \frac{1}{\sqrt{2\pi} S_0 \sqrt{k}\hat{\sigma}(1 + x/S_0)} \right| \exp\left(-\frac{\left(\ln(1 + x/S_0) - k\hat{\mu}\right)^2}{2k\hat{\sigma}^2} \right) \tag{4}$$

For the GARCH approach a GARCH process is fitted to the log-returns of $(S_{-N}, \dots, S_0)$ and S_k is retrieved through the following scheme:

1. $\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \beta_j X_{t-j}^2$,
2. $X_t = \sigma_t Z_t, \quad Z_t \sim \mathcal{N}(0, 1)$,
3. $Y_t = \mu + X_t$ (here you may use $\mu = 0$ when solving the exercises),
4. $S_k = S_0 \exp(Y_1 + \cdots + Y_k)$.

From simulations it is now possible to calculate the empirical quantile of

$$-(S_k - S_0) = S_0 \left(1 - \exp(Y_1 + \cdots + Y_k)\right). \tag{5}$$

Exercise a)
Yesterday, you invested one units worth of money in the OMXS30 index and you are planning on selling today. Calculate the 0.05-quantiles your investment with the naive approach, and with the GARCH approach. Your answer should be given in Swedish Krona.

Exercise b)
Calculate or simulate the 0.05-quantiles for a 10-day investment in one units worth of the OMXS30 index, using 250 business days of historical data, if today is

i) 2011-08-18,

ii) 2014-04-24,

iii) 2015-06-18.

Your answes should be given in Swedish Krona.

Exercise c)
In the Quantlab workspace you will find a plot of the log-returns together with the GARCH and the naive volatility of a 1-day investment. Use this plot to interpret your results from b).