Application of GARCH: risk modelling

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OMXS30

OMXS30 is a weighted mean of the 30 most traded stocks the Stockholm stock exchange.

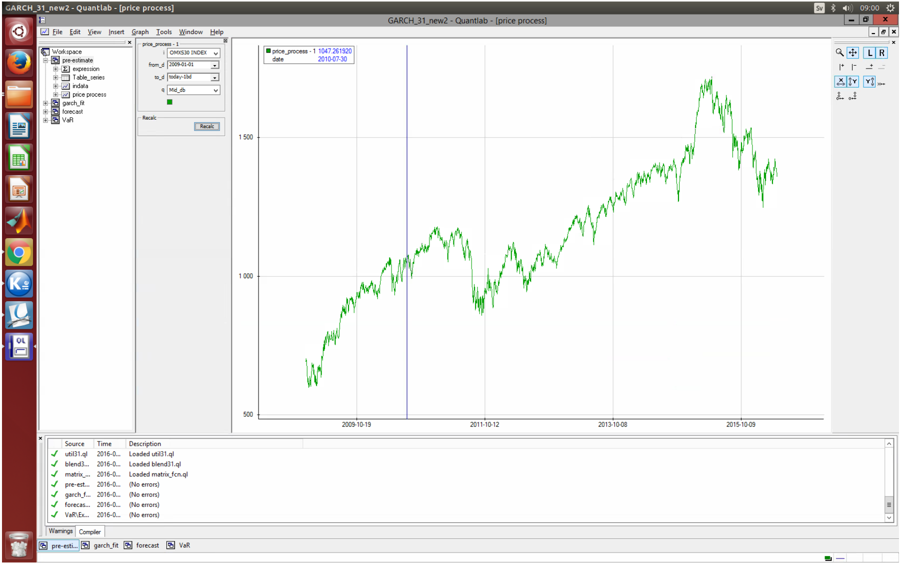
AKTIER | INDEX | OBLIGATIONER | OPTIONER & TERMINER | ETF/ETP | FONDER | NYHETER | UTBILDNING | OM OSS | Sök | FEEDBACK | SVENSKA

NASDAQ OMX Valueguard-KTH Boprisindex | Discontinued indexes

Namn	CCY	Senast	+/-	%	Köp	Sälj	Volym	Oms	Uppdaterad (CET)
ABB Ltd	SEK	169,7	-1,2	-0,7	169,5	169,7	1 840 201	311 910 995	18:00:00
Alfa Laval	SEK	126,6	-1,6	-1,25	126,5	126,7	1 187 042	150 707 407	18:00:00
ASSA ABLOY B	SEK	168,4	-3,5	-2,04	167,9	168,1	3 088 774	523 259 635	18:00:00
Atlas Copco A	SEK	207,8	-3,5	-1,66	207,7	207,9	2 077 204	432 591 141	18:00:00
Atlas Copco B	SEK	192,7	-3,7	-1,88	193,3	193,4	991 298	191 726 422	18:00:00
AstraZeneca	SEK	470	-4,8	-1,01	469,1	469,5	696 700	330 366 834	18:00:00
Boliden	SEK	140	-3,1	-2,17	140	140,1	2 545 280	356 911 409	18:00:00
Electrolux B	SEK	233	0,3	0,13	232,6	232,8	2 232 872	522 190 660	18:00:00
Ericsson B	SEK	65	-1,9	-2,84	65	65,1	12 277 754	804 180 159	18:00:00
Fingerprint Cards B	SEK	481	-10,1	-2,06	481,6	481,8	1 909 888	939 241 109	18:00:00
Getinge B	SEK	169,8	-3,7	-2,13	169,6	169,8	767 541	130 906 873	18:00:00
Hennes & Mauritz B	SEK	285,5	-4,6	-1,59	285,4	285,6	3 607 418	1 030 548 309	18:00:00
Investor B	SEK	294,7	-9,3	-3,06	295,1	295,2	1 636 090	485 850 454	18:00:00
Kinnevik B	SEK	231	-9	-3,75	231,4	231,5	1 424 214	331 956 962	18:00:00
Lundin Petroleum	SEK	150,4	-0,2	-0,13	150,2	150,4	1 006 819	151 646 962	18:00:00
Nordea Bank	SEK	77,95	-2	-2,5	77,85	77,95	9 934 752	779 060 691	18:00:00
Nokia Oyj	SEK	47,32	-1,06	-2,19	47,33	47,4	1 277 253	60 918 983	18:00:00
Sandvik	SEK	82,4	-4,1	-4,74	82,2	82,3	5 477 165	451 246 878	18:00:00
SCA B	SEK	252,9	-2,8	-1,09	252,9	253,1	1 931 148	489 726 581	18:00:00
SEB A	SEK	76,7	-2	-2,54	76,7	76,75	11 068 250	855 349 177	18:00:00
Securitas B	SEK	126,8	-3	-2,31	126,9	127	1 512 108	192 455 207	18:00:00
Sv. Handelsbanken A	SEK	106,9	-2	-1,84	107	107,1	3 784 243	406 101 906	18:00:00
Skanska B	SEK	176,6	-3,5	-1,94	176,4	176,6	1 021 889	180 711 725	18:00:00
SKF B	SEK	147,9	-4,1	-2,7	147,5	147,7	3 252 226	482 314 735	18:00:00
SSAB A	SEK	33,74	-1,4	-3,98	33,74	33,76	3 718 716	127 296 484	18:00:00
Swedbank A	SEK	173,2	-2,4	-1,37	173,2	173,4	2 943 089	510 374 878	18:00:00
Swedish Match	SEK	254,7	-21,8	-7,88	254,6	254,8	963 335	246 059 033	18:00:00
Tele2 B	SEK	76,6	-0,5	-0,65	76,55	76,65	3 462 809	264 833 481	18:00:00
Telia Company	SEK	38,35	-0,66	-1,69	38,32	38,35	9 391 994	361 599 662	18:00:00
Volvo B	SEK	94	-3,35	-3,44	94,3	94,35	10 403 245	986 225 658	18:00:00

OMXS30

OMXS30 price from 01-01-2009 until today:



Problem: risk estimation

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What risk are we exposed to if we buy one share of OMXS30 and hold it for 10 days?

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Let S_t be the value of the index at day t. A typical thing to look at is the quantiles in the left tail of $S_{10} - S_0$. This gives an estimate of the worst case return (loss of money) in a certain percentage of all possible scenarios.

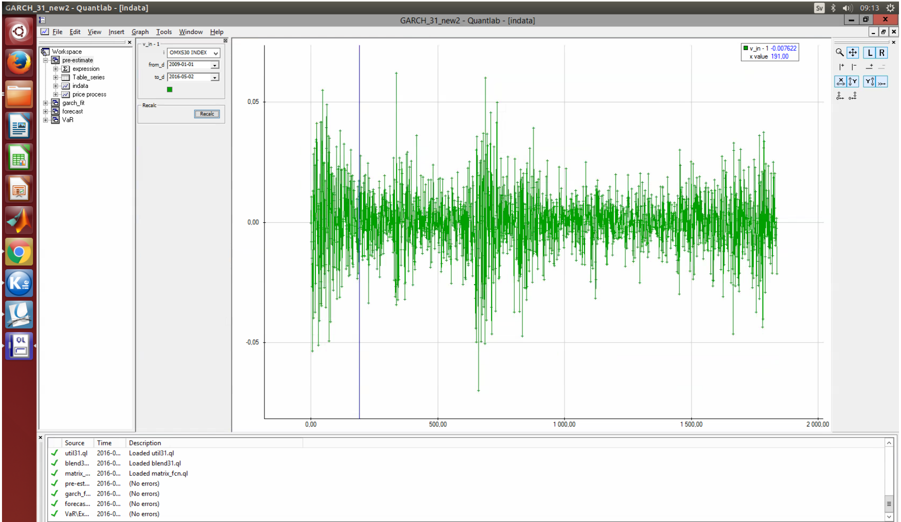
We will model the returns in two ways; a naive approach based on fitting a normal distribution and with a GARCH process.

Log-returns

Transforming past index values $(S_t)_{t=-N}^{0}$ into its log-reurns,

$$X_t = \ln(S_t / S_{t-1})$$

yields the following time series:



Naive approach

Assume that the log-returns are IID $\mathcal{N}(\mu, \sigma)$. If we estimate (μ, σ) from past index data with $(\hat{\mu}, \hat{\sigma})$, we may write

$$S_{10} - S_0 = S_0 \left(e^{X_1 + \cdots + X_{10}} - 1 \right) \overset{d}{\approx} S_0 \left(e^{10\hat{\mu} + \sqrt{10}\hat{\sigma} Z} - 1 \right)$$

where $Z \sim \mathcal{N}(0, 1)$.

Naive approach

By sampling from Z we may calculate empirical quantiles of $-(S_{10} - S_0)$. Even better, there is analytical formula for the quantile of $-(S_{10} - S_0)$

$$F^{-1}_{S_0 - S_{10}}(0.05) = S_0 \left(1 - e^{10\hat{\mu} + \sqrt{10}\hat{\sigma}\Phi^{-1}(0.05)}\right)$$

and for the density of $S_{10} - S_0$

$$f_{S_{10}-S_0}(x) = \left|\frac{1}{\sqrt{2\pi}S_0\sqrt{10}\hat{\sigma}(1 + x/S_0)}\right| \exp\left(-\frac{(\ln(1 + x/S_0) - 10\hat{\mu})^2}{2 \cdot 10\hat{\sigma}^2}\right)$$

GARCH approach

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From simulations, calculate the empirical quantile of

$$-(S_{10} - S_0) = S_0 \left(1 - \exp\left(Y_1 + \cdots + Y_{10}\right)\right)$$

Simulations in Quantlab

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GARCH - local, naive - global

Lets analyze some data in Quantlab...