Assignment 2

- This assignment has to be handed in latest Friday, 13.10.2017, 10:00, preferably at the end of the lecture that day. Assignments handed in later will not be accepted.
- Assignments have to be handed in on paper, stapled, and the names of all members of a group should be written on each sheet of paper.
- Students can form groups of up to three people to hand in one assignment.
- If called upon, students must be able to explain their work.
- Students may work together in larger groups, but each group has to hand in their individually written solutions. Failure to do so may result in loss of points for this assignment.

The two assignments below involve usage of the software Quantlab. You can find information about installation and first steps for Quantlab on this site: `http://www.math.kth.se/matstat/finance/financelab/quickstart.html`. Specific information and **the necessary workspaces** for the two assignments can be found at: `https://people.kth.se/~aaurell/Teaching/SF2942_HT17/immunization2017.html` (first problem) and `https://people.kth.se/~aaurell/Teaching/SF2942_HT17/tracking2017.html` (second problem). If you are experiencing technical difficulties during installation or use of Quantlab, please contact Alexander Aurell at aaurell@kth.se.
Note that for each of the two problems, an additional software of your own choice has to be used for the final step. This could, for example, be Matlab, Excel or R.
The task is to write a report that addresses the points mentioned at the end of each problem. Screenshots, code snippets or software output may be added to illustrate results. Explain the steps towards your solution and motivate your decisions, but keep your report concise! You are not supposed to include the code (from R, Matlab, Excel ...) in your report, but may be asked for it later, during grading. Therefore, **please store all of your used code** and make sure that your results are reproducible. Grading will be based on correctness of the applied technique, motivation of the approach and presentation of the results.

Problem 1 - Immunization Portfolio (6 points)

Task

Your task is to find an immunization portfolio for a particular cash flow, similar to Example 3.16 from the book. That is, given a cash flow to be used as the liability, your task is to create a portfolio such that the net value of the portfolio and the liability is immune, in the sense discussed in class and in the textbook, to changes in the zero-rate curve. The liability consists of quarterly payments over 10 years. To construct your portfolio you can choose from a collection of Swedish government bonds.

Quantlab + other software

In Quantlab, you should select the bonds you want to use as hedging instruments, say j of them. You will then be able to construct a zero-rate curve for the involved cash flow times and maturities. To conduct a principal component analysis, you must chose a window length for the historical data to be used, as well as the increment over which changes in the zero-rate should be computed (quarterly in Example 3.16). This is done in your workspace and as output you will receive a correlation matrix and associated eigenvectors. You can choose the number of eigenvectors to display (they are sorted in accordance with their respective eigenvalues). Note that the components of all rate vectors correspond to the cash flow times of the liability and all chosen bonds *combined and in chronological order*. That is, the first component of a rate vector is the zero rate for the first time when a payment happens, either from the liability or a chosen bond, the second component is for the second payment time, and so on. Make sure to keep properly track of this. Note also that all displayed gradients do not include zeros for the components that correspond to interest rates that do not affect the underlying bond. Thus, you must match the reported sensitivities to the correct cash flow times (i.e., the correct component in the gradient); the cash flow times for the individual bonds and the liability can be found in the Quantlab workspace. With the present prices $P(\mathbf{r}) = P_L(\mathbf{r}), P_k(\mathbf{r}), k = 1, \ldots, j$, gradients $\nabla P_L(\mathbf{r}), \nabla P_k(\mathbf{r}), k = 1, \ldots, j$ and the eigenvectors from the PCA, you can now perform the immunization step and derive the corresponding bond weights (negative weights are allowed). Note that this is the only task that you cannot carry out in Quantlab.

Report

The report should cover the steps outlined above and provide the relevant numerical data. Describe briefly which bonds you chose as hedging instruments and why. Things that must be included: Instrument used to determine the zero rates, what bonds are used as hedging instruments, the window length and increment used in the PCA, used number of principal components, portfolio weights and the corresponding price of the portfolio.

Problem 2 - Index tracking (4 points)

Task

For this problem, the task is to replicate the behavior of the Swedish stock index OMXS30 with a portfolio that only consists of five stocks.

Quantlab + other software

The corresponding Quantlab workspace contains data of the OMXS30 and of the 30 stocks it consists of. Your task is to select five stocks and a period used for estimation and Quantlab will then determine estimators for the optimal hedging weights $h_1, \ldots, h_5$ according to Proposition 3.3 from the book (where the role of the liability

L is now played by the OMXS30), based on estimated variances and covariances. Negative weights are allowed.
In order to evaluate the performance of your hedge, you should then calculate the realized empirical standard deviation of the difference between your hedging portfolio and the index during a testing period $\{t_1, \ldots, t_n\}$, that is,

$$\hat{\sigma} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(\sum_{k=1}^{5} h_k S_{t_i}^k - I_{t_i} - \hat{\mu}\right)^2},$$

$$\text{where } \hat{\mu} = \frac{1}{n}\sum_{i=1}^{n}\left(\sum_{k=1}^{5} h_k S_{t_i}^k - I_{t_i}\right),$$

where I_{t_i} is the value of the OMXS30 at time t_i and $S_{t_i}^k$ is the price of the k-th stock in your portfolio at time t_i. The testing period should be 6 (calendar) months long and it should start after the end of the estimation period. The evaluation of empirical standard deviation has to be done in an external software, using the data provided from Quantlab. If the software of your choice has a built-in-function for calculating the empirical standard deviation, you may of course use it.

Report

For the report, name the five stocks that you chose and motivate your decision to do so. State also begin and end of your estimation and testing periods. Name the optimal weight for each stock, the price for your portfolio at the begin of the testing period and state the value of the empirical standard deviation of your portfolio during the testing period. If you like, you can compare two or three different choices of stocks with regard to their performance in the test period.