

# FinanceLab project

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## Constant dividend $d$

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We get  $d = \frac{S_0 - G_T e^{-r_T T}}{e^{-r_t t}}$ .



## Proportional dividend $\alpha S_t$

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We get  $\alpha = \frac{S_0}{G_T} e^{r_T T} - 1$ .

# Summary

The constant dividend is given by

$$d = \frac{S_0 - G_T e^{-r_T T}}{e^{-r t}}.$$

The proportional dividend is given by  $\alpha S_t$  where

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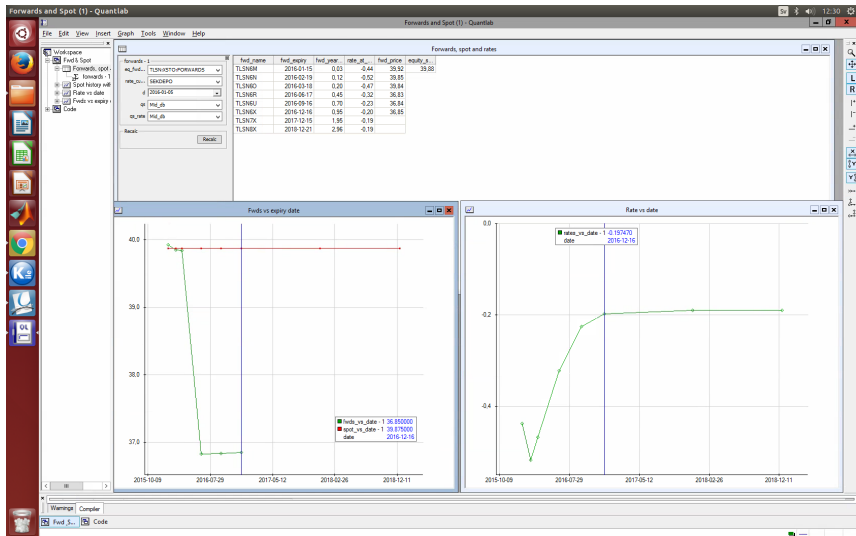
$$\alpha = \frac{S_0}{G_T} e^{r_T T} - 1.$$

We need the following market data to compute the dividend:

$$S_0, G_T, t, r_t, T, r_T$$

# Solution 1a)

Extract the market data.





## Solution 1a)

We read from Quantlab:

$$T = 0.45$$

$$r_T = -0.0032$$

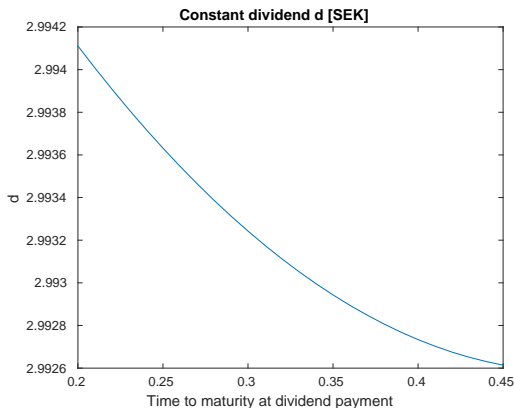
$$S_0 = 39.88$$

$$G_T = 36.83$$

It is given in the exercise that we should assume that the dividend is paid at 2016-04-15. What if we don't know this? The market indicates that the dividend is to be paid somewhere between 2016-03-18 and 2016-06-17. How to choose  $t$ ? Let's make a linear interpolation between the two forward contracts...

## Solution 1a)

Result:



Since the interest rate is approximately zero, we get  $d \approx S_0 - G_T \approx 2.993$ .

## Solution 1b)

Result:

$$\alpha \approx 0.0813$$

Since the interest rate is approximately zero, the dividend is

$$\alpha S_t \approx \alpha S_0 \approx 3.242$$

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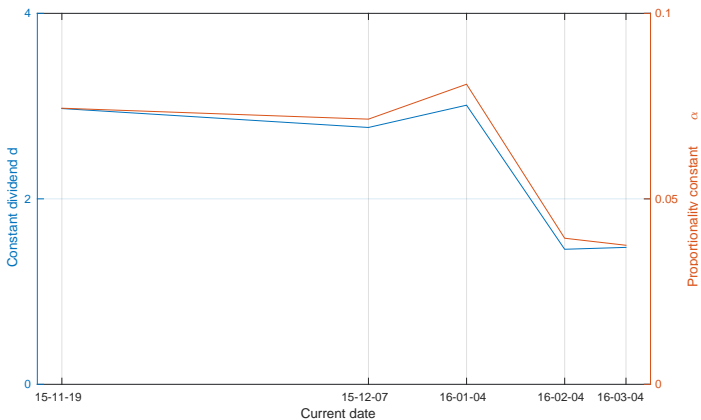
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Lets look up the dividend online...

# Solution 3)

Result:



## Discussion: errors

Do we have any sources of error in the analysis?

- ▶ Interpolation of interest rate
- ▶ Existence of small dividends?