Exercise session 6

1. Let $\mathcal{F} = \{x \in \mathbb{R}^n \mid g_i(x) \leq 0, i = 1, \ldots, m\}$ where g_i are convex functions on $\mathbb{R}^n$. Show that $\mathcal{F}$ is a convex set.

OR

1. Let $g : C_1 \to \mathbb{R}$ and $f : C_2 \to \mathbb{R}$ be convex functions. The sets C_1 and C_2 are convex subsets of $\mathbb{R}^n$. Show that $f + g$ is a convex function.

2. Let f be a convex function on some convex set C. Let $x_1, \ldots, x_n \in C$ and $\lambda_1, \ldots, \lambda_n$ such that $\lambda_i \geq 0$ for all $i \in \{1, \ldots, n\}$ and $\sum_{i=1}^n \lambda_i = 1$. Show Jensen's inequality

$$f\left(\sum_{i=1}^n \lambda_i x_i\right) \leq \sum_{i=1}^n \lambda_i f(x_i)$$

for $n = 3$.

3. Prove the arithmetic-geometric mean inequality for positive $x_1, \ldots, x_n$, i.e.

$$\frac{x_1 + \ldots x_n}{n} \geq (x_1 \cdot x_n)^{1/n}.$$

Use Jensen's inequality and the convex function $-\log$.

4. Is $h(x) = |x| + \max\{e^x, 10 + 37x + x^6\}$ a convex function on $\mathbb{R}$?

5. a) Show that $g(x) = x^3$ is not a convex function on $\mathbb{R}$.
b) Find a convex domain $C \subset \mathbb{R}$ such that $g(x) = x^3$ is a convex function on C.

6. Let for some $n \in \mathbb{N}$

$$f(x) = \sum_{j=1}^n (x_j^4 - x_j^3 + x_j^2 - x_j)$$

where $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$.
a) Show that f is a convex function.
b) You want to minimize f. Make one iteration of Newton's algorithm and start in $x^{(0)} = (1, \ldots, 1)$.

7. Problem 3 from the exam 13-01-2016.