$\boxed{c=0}$

$$\nabla f(x) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \Rightarrow \quad -\nabla f(x) = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

Observe the level-curves in the figure we can see that the optimal solution will be at point where $g_1(x) = g_3(x)$ and $x_1 = 0$

$$1^2 + (x_2 - 1)^2 = 9 \quad \Rightarrow \quad x_2 = 1 \pm \sqrt{8}$$

(look at OH pick $\ominus$ $\Rightarrow$ (ii) ✓)

$$\Rightarrow \hat{x} = (0 \quad 1-\sqrt{8})^T$$

if we pick $\oplus$ then $\hat{y}_2 = \hat{y}_4 < 0$

Check (i) - (iv) for this point

(ii) $g_1(\hat{x}) = g_3(\hat{x}) = 0 \qquad g_2(\hat{x}) = g_4(\hat{x}) = -7.31 < 0$

$$\Rightarrow \hat{y}_2 = \hat{y}_4 = 0 \quad \text{to satisfy (iv)}$$

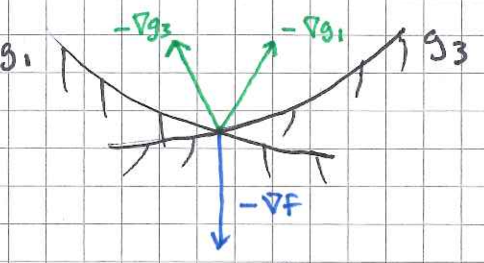
$$\text{(i)} \Rightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 2y_1 \begin{bmatrix} 0 & -1 \\ 1-\sqrt{8} & -1 \end{bmatrix} + 2y_3 \begin{bmatrix} 0 & +1 \\ 1-\sqrt{8} & -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{cases} 0 - 2y_1 + 2y_3 = 0 & \Rightarrow y_1 = y_3 \\ 1 - 2\sqrt{8}(y_1 + y_3) = 0 & \Rightarrow y_1 = y_3 = \frac{1}{4\sqrt{8}} \geq 0 \end{cases}$$

(iii) ✓

$\therefore$ $\hat{x} = (0 \quad 1-\sqrt{8})^T$ and $\hat{y} = \left(\frac{1}{4\sqrt{8}} \; 0 \; \frac{1}{4\sqrt{8}} \; 0\right)^T$

global minimizer to $(NLP_{\leq})$



$-\nabla f = (0 \quad -1)$
$-\nabla g_1 = 2(1 \quad \sqrt{8})$
$-\nabla g_3 = 2(-1 \quad \sqrt{8})$

Note: If $(NLP_{\leq})$ convex but $\hat{x}$ is not regular point then KKT-conditions are sufficient but not necessary for global optimality (Thm 21.4)

If $(NLP_{\leq})$ is regular and $\hat{x}$ point but $(NLP_{\leq})$ is not convex then KKT-conditions are necessary but not sufficient (Thm 20.11)

$$* \; -\hat{y}_1 \nabla g_1(\hat{x}) - \hat{y}_2 \nabla g_2(\hat{x}) = \frac{2}{4\sqrt{8}} \left[(1 \quad \sqrt{8}) + (-1 \quad \sqrt{8}) \right] =$$

$$= \frac{1}{2\sqrt{8}} (0 \quad 2\sqrt{8}) = (0 \quad 1)$$