

① Find an optimal solution to the following equality-constrained problem

$$\begin{aligned}
 (\text{NLP}_=) \quad & \min x_3 \\
 \text{s.t.} \quad & x_1^2 + x_2^2 + x_3^2 = 6 \\
 & x_1 + x_2 + x_3 = 4
 \end{aligned}$$

Motivate global optimality

Hint: Show that \mathcal{F} is compact.

② Solve

$$\begin{aligned}
 (\text{NLP}_{\leq}) \quad & \min cx_1 + x_2 \\
 \text{s.t.} \quad & (x_1 - 1)^2 + (x_2 - 1)^2 \leq 9 \\
 & (x_1 - 1)^2 + (x_2 + 1)^2 \leq 9 \\
 & (x_1 + 1)^2 + (x_2 - 1)^2 \leq 9 \\
 & (x_1 + 1)^2 + (x_2 + 1)^2 \leq 9
 \end{aligned}$$

Motivate global optimality. Use $c=1$ and $c=0$

③ Observe the following (NLP_{\leq})

$$\begin{aligned}
 (\text{NLP}_{\leq}) \quad & \min x_1 \\
 \text{s.t.} \quad & x_2 - x_1^3 \leq 0 \\
 & -x_2 - x_1^3 \leq 0
 \end{aligned}$$

a) Illustrate the feasible region and draw level curves of the objective function. Note that $\hat{x} = (0 \ 0)^T$ is a global minimizer

b) Show that \hat{x} does not satisfy the KKT-conditions for (NLP_{\leq}) . Explain why.