Exercise session 6

11/12

(1) Let $\mathcal{F} = \{x \in \mathbb{R}^n \mid g_i(x) \leq 0,\ i = 1, \dots, m\}$ where g_i are convex functions on $\mathbb{R}^n$. Show that $\mathcal{F}$ is a convex set

(2) Let f be a convex function on some convex set C. Let $x_1, \dots, x_n \in C$ and $\lambda_1, \dots, \lambda_n$ s.t. $\lambda_i \geq 0$ $\forall i = 1, \dots, n$ and $\sum_{i=1}^{n} \lambda_i = 1$. Show Jensen's inequality

$$f\left(\sum_{i=1}^{n} \lambda_i x_i\right) \leq \sum_{i=1}^{n} \lambda_i f(x_i)$$

for $n = 3$. (Note Jensen's inequality holds for general n)

(3) Prove the arithmetic mean - geometric mean inequality for positive $x_1, \dots, x_n$, i.e.

$$\frac{x_1 + x_2 + \dots + x_n}{n} \geq \sqrt[n]{x_1 x_2 \cdot \dots \cdot x_n}.$$

Use Jensen's inequality and the convex function $-\log$

(4) Is $h(x) = |x| + \max\{e^x, 10 + 37x + x^6\}$ a convex function on $\mathbb{R}$?

You can use: 1) The sum of convex functions is a convex function

2) Maximum of a set of convex functions is a convex function

(5) Show that $g(x) = x^3$ is not a convex function on $\mathbb{R}$. Find a convex domain $C \subset \mathbb{R}$ s.t. $g(x) = x^3$ is a convex function on C.

(6) We did exercise 16.3 on page 128 in ASKS

(7) We did exercise 16.2 on page 128 in ASKS