Presentation of the Paper: *"Individual Risk in Mean Field Control with Application to Automated Demand Response"*

by: Y.Chen, A.Busic and S.Meyn

G.Svensson

KTH - sci

November 3, 2015

Overview

1. Overview
   - Problem
   - Model
   - Contributions
2. Randomized Controls and Mean Field Models
   - Randomized Controls
   - Mean Field Model
   - Super-Sampling and example: Intelligent Pools
3. Individual Load Statistics
   - Measure of QoS
   - Variance of QoS
4. Numerical Experiment
5. Conclusions

Problem Introduction

- Regulation of power grids to ensure that supply matches demands.
- Balance has to be achieved on different time scales, corresponding to time scales of both supply and demand.
- The need for balancing regulation has increased due to the fickle nature of several renewable sources, such as wind power.
- To achieve balance great use can be made of the inherent flexibility certain systems of loads, such as electric vehicle charging, heating and ventilation, irrigation, pool maintenance, etc.
- The goal is to control a large set of loads to achieve balance.

Model Introduction

- An aggregate of loads can be modeled as a dynamical system.
- Design a signal to be broadcasted to the loads, based on a measurement of aggregate power output, such that the deviation in power consumption tracks a reference signal.
- Each load operates according to some randomized policy based on its internal state and a common signal ζ.
- In earlier works it has been shown that localized randomized policies can be designed so that control of the loads on grid level become easy.
- The analysis is based on a mean field limit and a LTI (Linear Time Invariant) approximation of the aggregate nonlinear model.

Contributions

The main contributions fall in two sub-categories, the first is the modelling and quatification of risk:

1. The average measure is not a sufficient measure of QoS (Quality of Service). It is found that the QoS follows an approximate gaussian distribution, hence each load will eventually recieve very poor service.
2. The variance can be estimated as a function of the randomized policies and the power spectral density.

The second sub-category treats the addition of a local control and how this can eliminate risk:

1. The distribution of QoS is then truncated through the addition of the local control.
2. This local bounding has minimal impact on the grid level performance.

Power Spectral Density

For continued signals that describe, for example, stationary physical processes, it makes sense to define a power spectral density (PSD), which describes how the power of a signal or time series is distributed over the different frequencies. In statistics one might study the variance of a set of data, but because of the analogy with electrical signals, it is customary to refer to it as the power spectrum even when it is not, physically speaking, power. The average power P of a signal $x(t)$ is the following time average:

$$P = \lim_{t \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)^2 dt$$

Cont.

In analyzing the frequency content of the signal $x(t)$, it is advantageous to work with a truncated Fourier transform $\hat{x}_T(\omega)$, where the signal is integrated only over a finite interval [0, T]:

$$\hat{x}_T(\omega) = \frac{1}{\sqrt{2\pi T}} \int_0^T x(t) e^{-i\omega t}\, dt.$$

Then the power spectral density can be defined as:

$$S_{xx}(\omega) = \lim_{T\to\infty} \mathbf{E}\left[|\hat{x}_T(\omega)|^2 \right].$$

Using such formal reasoning, one may already guess that for a stationary random process, the power spectral density $S_{xx}(\omega)$ and the autocorrelation function of this signal $\gamma(\tau) = \langle X(t)X(t+\tau)\rangle$ should be a Fourier transform pair.[Source:Wikipedia]

System Architecture

The system architecture under consideration is based on the following components:

1. There are N homogeneous loads that recieve a common signal from the BA, $\zeta_t \in \mathbb{R}$.
2. Each load evolves as a contolled Markov chain on a finite state space, $\mathcal{X} = \{x^1, ..., x^d\}$, were the transition probability is determined by the current state and the signal ζ_t. The common dynamics are defined by the controlled transition matrix $\{P_\zeta : \zeta \in \mathbb{R}\}$. So for the i:th load:

$$P\{X^i_{\tau+1} = y | X^i_\tau = x, \zeta_t = \zeta\} = P_\zeta(x, y), \quad \text{for each } x, y \in \mathcal{X}. \quad (1)$$

3. The BA has measurements of the normalized aggregate power consumption, $y \in \mathbb{R}$ and the desired normalized deviation in power consumption $r \in \mathbb{R}$.

Remark: time index t is used on grid level and τ for the loads due to supposed difference in sampling frequency.

Power Consumption

The power consumption from load i at time t is assumed to be a function of the state, denoted $\mathcal{U}(X_t^i)$. The normalized power consumption is denoted as:

$$y_t^N = \frac{1}{N} \sum_i \mathcal{U}(X_t^i) \tag{2}$$

and the nominal behaviour of each load is given by the dynamics for which $\zeta \equiv 0$, in which case the loads are independent.Further it is assumed that the Markov chains are ergodic and that P_0 has a unique invariant distribution π_0. The average nominal power usage becomes:

$$\bar{y}^0 = \sum_i \pi_0(x) \mathcal{U}(x) \tag{3}$$

Cont.

Combining the ergodic Markov chains with the LLN (Law of Large Numbers) for i.i.d. sequences, for large N and t, implies

$$y_t^N \approx \bar{y}^0, \quad \text{for } \zeta \equiv 0. \tag{4}$$

To track a signal ζ should be chosen such that $r_t \approx \tilde{y}_t$, $\forall t$ where $\tilde{y}_t = y_t - \bar{y}^0$ is the deviation from nominal. Then an error feed-back of the form:

$$\begin{cases} \zeta_t = G_c e_t \\ e_t = r_t - \tilde{y}_t \end{cases} \tag{5}$$

can be used, where G_c is a transfer function that depends on a linear approximation of the dependency of y on ζ. This can be obtained via a mean field construction.

Mean Field Model

Consider the empirical distributions,

$$\mu_t^N := \frac{1}{N} \sum_i \mathbb{I}\{X_t^i = x\}, \quad x \in \mathcal{X}. \tag{6}$$

Under some general conditions the empirical distributions converge to a solution of the nonlinear state space model equations,

$$\mu_{t+1} = \mu_t P_{\zeta_t}, \quad \text{where} \sum_{x \in \mathcal{X}} \mu_t = 1. \tag{7}$$

μ_t represents the fraction of loads in each state. The power output is then given by,

$$\begin{cases} y_t^N \to y_t, \quad \text{as } N \to \infty \\ y_t = \sum_{x \in \mathcal{X}} \mu_t(x) \mathcal{U}(x). \end{cases} \tag{8}$$

Linearization

The unique equilibrium with $\zeta \equiv 0$ is $\mu_t \equiv \pi_0$ and $y_t \equiv \bar{y}^0$. Linearization around this equilibrium gives the linear state space model,

$$\begin{cases} \Phi_{t+1} = A\Phi_t + B\zeta_t, \quad \text{where } A = P_0^T, B_j = \sum_x \pi_0(x)\xi(x, x^j) \\ \gamma_t = C\Phi_t, \quad \text{where } C_i = \mathcal{U}(x^i) \end{cases} \tag{9}$$

where $\xi = \frac{d}{d\zeta} P_\zeta \Big|_{\zeta=0}$ and,

$$\begin{cases} \Phi_t(i) \approx \mu_t(x^i) - \pi_0(x^i), \quad \text{for } i = 1, ..., d \\ \gamma_t \approx \tilde{y}_t. \end{cases} \tag{10}$$

Super-Sampling and Intelligent Pools

- The authors go on to talk about a method to keep track of times at both load level and grid level. Several loads are grouped together. This is called super-sampling.
- Then they apply the theory to power control for pool cleaning (in Florida).
  - A linear control is constructed of the form of (5) based on the mean field model. They show that it behaves as predicted by mean field approximation.
  - The distribution of QoS for the system illustrates the risk for individual loads (risk of under/over cleaning)

Mean Field Model for an Individual Load and QoS

Remark: In the mean field limit the aggregate dynamics are deterministic, following the discrete time nonlinear control model given by (7), while the individual loads remains probabilistic.

When looking at one load of many the pair (μ_t^N, ζ_t^N) can be replaced by their mean field limits (μ_t, ζ_t).

The analysis of a single load in state $X(t)$ is consitent with (1), where $X(t)$ denotes the Markov chain,

$$P\{X_{\tau+1} = y | X_\tau = x, r_0, ..., r_\tau\} = P_\zeta(x, y), \quad \text{for each } x, y \in \mathcal{X}. \tag{11}$$

With some manipulations it is found that the load evolves according to a random linear system, like the deterministic dynamics of (7),
where the state space has been lifted to a d-dimensional simplex, S, and for the i:th load at time τ, the element $\Gamma_\tau \in S$ is the degenerate distribution whose mass is concentrated at x if $X_\tau = x$; that is $\Gamma_\tau = \delta_x$.

Measure of QoS

Let the following measure of QoS for an individual load be,

$$\mathcal{L}_\tau = \sum_{k=0}^{\tau} \beta^k \ell(X_{\tau-k}) \tag{12}$$

where $\beta \in (0,1)$ is a discount factor and,

$$\ell = \begin{cases} \tau_s & \text{if the load was on for the period} \\ 0 & \text{if the load was off for the period.} \end{cases} \tag{13}$$

Where τ_s is the interval length. To eliminate local risk a simple "opt-out" mechanism is introduced at local level. A load ignores a command to switch states if $\mathcal{L}_{\tau+1} \notin [b_-, b_+]$, where $b_- and b_+$ are the lower and upper bounds on the allowed interval.

PSD of QoS

The main goal now is to estimate the statistics of $\{\mathcal{L}_\tau\}$ in steady state. For $\beta \approx 1$, a Gaussian approximation is possible hence it is sufficent to estimate the power spectral density, $S_{\mathcal{L}}$. The (PSD) is estimated through the following steps:

1. Consider the linearized state space approximation of Γ,

$$\Gamma_{\tau+1} = \Gamma_\tau P_0 + D_{\tau+1}. \tag{14}$$

2. The disturbance, D in (14), is approximated by the sum of two uncorrelated components,

$$D_{\tau+1} = B_\tau^T \zeta_\tau + (\Gamma_{\tau+1} - \Gamma_\tau P_{\zeta_\tau}), \quad \text{with } B_\tau^T = \Gamma_\tau \xi. \tag{15}$$

3. The approximation in (15) is based on analysis of input ζ compared to the Markov model from $\zeta \equiv 0$.
4. Given data for r obtain an approximation of its (PSD) and from that an estimate for the (PSD) of ζ can be obtained.

Variance of QoS

From the bove approximations the (PSD), $S_{\mathcal{L}}$, for $\mathcal{L}_\tau$ can be obtained via a series of z-transforms. This makes a stable factorization possible,

$$S_{\mathcal{L}}(\theta) = G_{\mathcal{L}}(e^{j\theta})G_{\mathcal{L}}(e^{-j\theta})^T, \tag{16}$$

where $G_{\mathcal{L}}$ is a stable spectral factor in the form of a row vector of transfer functions. Let $\{g_k\}$ denote its impulse response, $G(z) = \sum_{k=0}^{\infty} g_k z^k$, the variance of QoS is obtained as,

$$\sigma_{\mathcal{L}}^2 = \sum_{k=0}^{\infty} \| g_k \|^2 . \tag{17}$$

Can also be expressed in terms of an integral when appropiate.

Intelligent Pools

The authors present numerical results for the case of pool pumps with and without the option to opt-out. The results are in accordance with the ideas in the paper, with local control the risk of low QoS is basically eliminated.[include figures]

Conclusions

- The main contribution is the approximation of the QoS for an individual load, both first and second order statistics.
- Strict bounds on QoS can be guaranteed while still having near perfect tracking at grid level.
- It has been shown how useful the linear model can be for mean field like problems.

References

Y.Chen, A.Busic, S.Meyn (2014)
Individual Risk in Mean Field Control with Application to Automated Demand Response
53rd IEEE Conference on Decision and Control 6425 – 6432.

The End