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Influencing opinion dynamics to promote sustainable food choices

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Abstract: : Substantial shifts in contemporary diets are needed to address the growing burden of chronic diseases and the accelerating climate crisis. To facilitate these changes, policy-makers must develop effective strategies to advertise and encourage healthy dietary choices among the population. In this work, to capture changes in dietary behavior we propose an opinion dynamics model where prejudiced agents can interact with their neighbors in a social network. Additionally, an external entity, such as a policy-maker or a government, can influence the agents through sequential campaigns towards a desired collective opinion. We investigate the impact of this exogenous influence under budget constraints, limiting, for example, the number of agents that can be influenced per campaign, and show how to optimize influence allocation. We conclude with an application of the proposed approach within the context of sustainability goals on food consumption set by the UK Climate Change Committee. Using baseline food intake patterns derived from survey data on the UK population, we evaluate the impact of campaigns promoting dietary shifts and mitigating environmental impacts.

Keywords: opinion dynamics, prejudiced agents, influence allocation, sustainable behavior

1. INTRODUCTION

Guiding the collective opinion of a group of agents connected in a social network towards a desired state is a problem with significant applications well-beyond the control community, finding relevance not only in common marketing contexts (Morărescu et al. (2020); Varma et al. (2018)), but also public health (Ancona et al. (2022); Eustachio Colombo et al. (2021)). Recent years have seen a growing interest in the problem of inflencing opinion dynamics over social networks from diverse perspectives. These include pinning control problems to influence opinions (Ancona et al. (2023, 2022)), game-theoretical settings where multiple external entities compete to control consumers' opinions in the context of market networks under static (Varma et al. (2018)) or long-term campaigns (Varma et al. (2019)), dynamic game-theoretical settings with (Kareeva et al. (2023)) and without (Veetaseveera et al. (2021)) stubborn agents, and problems of optimal space-time budget allocation (Morărescu et al. (2020)).

In this work, we consider the problem of influencing the opinions of a group of agents connected in a social network over a sequence of campaigns, aiming to achieve convergence of the collective opinion toward a desired state, determined by an external entity. This entity, often a planner or policy-maker, strategically designs influence weights directed towards each agent in the social network, while accounting for campaign-specific budget constraints. Unlike Morărescu et al. (2020), we assume that the agents are prejudiced, implying a tendency to resist opinion change and anchor to their existing biases or beliefs. Drawing inspiration from the public health application in the context of dietary shifts, described below, we anticipate that a certain amount of time is needed before noticeable changes occur within the population. For this reason, we promote the idea that the external entity "acts", influencing the prejudices of the individuals in the population, only when the observed behavior of the social network is stationary.

For our modeling approach, we use a modified concatenated Friedkin-Johnsen (FJ) model (Tian and Wang (2018); Wang et al. (2023)) over a sequence of campaigns. Between consecutive campaigns the agents' opinions evolve according to a continuous-time FJ model with prejudiced agents (Altafini (2022); Proskurnikov and Tempo (2017)). The concatenation rule dictates that their initial opinions at the start of each campaign are determined by their final opinions at the end of the previous campaign. The individuals' prejudices remain constant between consecutive campaigns and only update at each campaign instant. To capture the influence of the external

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entity on opinion formation, we model the prejudice of each agent as a trade-off between their initial opinion and the desired opinion expressed by the external entity: Without external influence, the model reduces to the standard FJ model, where initial opinions serve as the sole source of prejudice. The trade-off between these terms is governed for each agent by a (design) weight parameter interpretable as the influence efforts of the external entity. It is assumed to be bounded, with the upper limit indicating, for instance, the maximum extent of influence that the individual is willing to accept or can tolerate.

As aforementioned, the external entity aims to guide the social network towards its desired opinion. Initially, we consider a baseline scenario where a non-strategic entity equally influences each agent in the network. However, the primary focus of this study is on investigating a strategic entity operating under budget constraints. In this main scenario, during each campaign, the strategic entity must optimize its allocation of influence (in terms of influence weight parameters) to a selected subgroup of agents to minimize a cost function proportional to the distance of the collective opinion, represented by the average, from the desired state. As a first contribution of this work, we demonstrate that the optimal allocation corresponds to prioritizing agents according to their distance from the desired opinion, weighted by their social power.

As a motivating example and novel illustrative application of this research, we consider the UK Climate Change Committee (2020) targets on consumption of meat, which aim for a specific percentage reduction in the average consumption of all meat and dairy products by 2030 and 2050. We use collected data from the National Diet and Nutrition Survey 2019 (Bates et al. (2019)) on food consumption to derive baseline dietary preferences, which we then map into the opinions of the agents representing the UK population. As a second contribution of this work, we apply the developed targeted allocation strategy under campaign-specific budget constraints to minimize the distance of the UK social network from a meat-free diet, thereby exploring the potential impact of influential campaigns promoting dietary shifts.

The paper is organized as follows. Section 2 introduces technical preliminaries and notation. Section 3 proposes the problem formulation. Sections 4 and 5 present the main results. Section 6 discusses the food consumption application. Finally, Section 7 offers conclusive remarks.

2. TECHNICAL PRELIMINARIES

This section introduces the notation used in the paper and useful notions (Section 2.1), and the Friedkin-Johnsen model of opinion dynamics (Section 2.2).

2.1 Notation, linear algebraic preliminaries, digraphs

$\mathbb{N}$ is the set of natural numbers; $\mathbb{1}_n$ is the n-dim vector whose elements are all 1s (when clear, we omit the subscript). $\odot$ indicates the Hadamard (element-wise) product.

Linear algebraic preliminaries Given a matrix $A \in \mathbb{R}^{n\times m}$, its (i,j)-th entry is denoted by a_{ij} or $[A]_{ij}$. $A \geq 0$ (resp. $A > 0$) means element-wise nonnegative (resp. positive). A is called: Hurwitz stable if all its eigenvalues have strictly negative real part; Schur stable if its spectral radius (the maximum of the absolute values of its eigenvalues) is strictly less than 1; Metzler if its off-diagonal elements are nonnegative; and an M-matrix if all its eigenvalues have a nonnegative real part and $-A$ is Metzler. $A \geq 0$ is row stochastic if $A\mathbb{1} = \mathbb{1}$.

Digraphs Let $\mathcal{G}(A) = (\mathcal{V}, \mathcal{E}, A)$ be the (weighted) digraph with vertex set $\mathcal{V}$ ($\text{card}(\mathcal{V}) = n$), edge set $\mathcal{E} = \mathcal{V}\times\mathcal{V}$, and adjacency matrix $A \in \mathbb{R}^{n\times n}$ s.t. $a_{ij} > 0$ iff $(j,i) \in \mathcal{E}$. A node i is said to be linked to j if there exists an edge sequence $(j,i_1),..,(i_{k-1},i_k),(i_k,i)$ picked from $\mathcal{E}$. We call $G(A)$ strongly connected if each pair of nodes in $\mathcal{V}$ is linked to each other. The Laplacian of $\mathcal{G}(A)$ is $L = \text{diag}\,(A\mathbb{1}) - A$; $\text{diag}\,(\cdot)$ creates a diagonal matrix with the elements of its argument as the diagonal entries, thus making $\text{diag}\,(A\mathbb{1})$ the diagonal matrix of in-degrees of $\mathcal{G}(A)$. By construction, $-L$ is Metzler, L is a singular M-matrix, and its 0 eigenvalue has multiplicity 1 when $\mathcal{G}(A)$ is strongly connected.

2.2 The FJ model of opinion dynamics in continuous-time

We describe the time evolution of the opinion of agent $i \in \mathcal{V}$ in a strongly connected graph $\mathcal{G}(A)$ by the following continuous-time (CT) version of the FJ model[1]:

$$\dot{x}_i(t) = \lambda_i \textstyle\sum_{j=1}^n a_{ij}(x_j(t) - x_i(t)) + (1-\lambda_i)(u_i - x_i(t)). \quad (1)$$

The parameter $\lambda_i \in [0,1]$ represents agent's i susceptibility to interpersonal influence, or, equivalently, $1-\lambda_i$ indicates attachment to their prejudice u_i. We say that an agent i is prejudiced, fully prejudiced, and fully susceptible if $\lambda_i \in (0,1)$, $\lambda_i = 0$, and $\lambda_i = 1$. In its standard formulation, the prejudice of each agent is given by their initial option, i.e., $u_i = x_i(0)$, and $1 - \lambda_i$ indicates stubborness; in general, u_i can account for other sources of exogenous influence. In its discrete-time version, the model (1) captures a process where each agent updates its opinion by minimizing a "dissonance" cost function consisting of an opinion disagreement cost with their neighbors in the network (i.e., a social cost) and with their own prejudice (i.e., an inertia cost) (Groeber et al. (2014)).

In compact form, the FJ model (1) can be rewritten as:

$$\dot{x}(t) = -(\Lambda L + I - \Lambda)x(t) + (I - \Lambda)u, \quad (2)$$

where $x(t) = [x_1(t) \cdots x_n(t)]^T$ is the vector of opinions, $u = [u_1 \cdots u_n]^T$ is the vector of prejudices, L is the graph Laplacian, and $\Lambda = \text{diag}\,(\lambda_1, \ldots, \lambda_n)$ is a diagonal matrix describing the susceptibility of the agents in the network. The convergence properties of the model (2) are presented in Altafini (2022); Proskurnikov and Tempo (2017), and are summarized in the following theorem, where we additionally assume strong connectivity of $\mathcal{G}(A)$.

Theorem 1. Consider the FJ model (2) on a strongly connected digraph $\mathcal{G}(A)$ with Laplacian L, and assume that $\Lambda \lneq I$. Then, the following conditions are equivalent:

(i) The matrix $-(\Lambda L + I - \Lambda)$ is Metzler and Hurwitz;
(ii) The system has a unique asymptotically stable equilibrium point $x(\infty) := \lim_{t\to\infty} x(t) = Vu$, where $V := (\Lambda L + I - \Lambda)^{-1}(I - \Lambda)$ is row-stochastic.

Under these conditions, $x(\infty)$ belongs to the convex hull of the prejudices' vector u.

In other words, Theorem 1 states that if there is at least one stubborn agent then the opinion of each agent converges in time to a fixed value, which in general is a disagreement state, i.e., $\exists\, i,j$ s.t. $x_i(\infty) \neq x_j(\infty)$.

[1] From Altafini (2022). For other CT versions of the FJ model see, e.g., Proskurnikov and Tempo (2017).

3. PROBLEM FORMULATION

This work aims to study the impact of influence campaigns to guide the network to a desired state or opinion d determined by an external entity (e.g., a policy-maker or planner). As a first step to include the campaign's influence on the model (2), an additional virtual node, i.e., an influencer agent or external entity, is linked to the agents in the network. Letting the opinion of the influencer agent set to d, we assume that the agents' prejudices are affected by the external entity as follows:

$$u_i = (1 - h_i\gamma_i)x_i(0) + h_i\gamma_i d = x_i(0) + h_i\gamma_i(d - x_i(0)),$$

which, in compact form, can be rewritten as:

$$u = (I - H\Gamma)x(0) + H\Gamma d\mathbb{1}. \quad (3)$$

Substituting (3) in the FJ model (2), the overall opinion dynamics with external influence becomes:

$$\dot{x}(t) = -(\Lambda L + I - \Lambda)x(t) + (I - \Lambda)((I - H\Gamma)x(0) + H\Gamma d\mathbb{1}) \quad (4)$$

where $H = \operatorname{diag}(h_1, \ldots, h_n)$ and $\Gamma = \operatorname{diag}(\gamma_1, \ldots, \gamma_n)$ are diagonal matrices describing the influence efforts of the external entity. The parameter $\gamma_i \in [0, \bar{\gamma}]$ models the influence (or control) efforts of the external entity towards each agent i, while the parameter $h_i \in [0, 1]$ represents how much an individual i can be persuaded by or is susceptible to the external entity. When $\Gamma = 0$, the external entity has no influence on the agents, and (4) reduces to the standard FJ model (2) where prejudices are equivalent to initial opinions. When $\lambda_i = 0$, γ_i captures the trade-off between stubbornness towards initial opinion and the influence of the external entity. The maximum value $\bar{\gamma}$ weighted by h_i represents, for instance, the maximum extent of influence that the agent i is willing to accept or can tolerate.

3.1 Repeated campaigns to achieve the desired opinion

In general, achieving the desired target opinion may prove challenging with just one campaign. To address this issue and effectively influence behavior we consider the case of repeated campaigns, adapting the concatenated FJ model (Tian and Wang (2018)) to include external influence.

Let $s \in \mathbb{N}$ indicate the campaign instants, where the (start and end of the) s-th campaign is captured by the interval $[s-1, s]$. Let the influence effort of the external entity towards each agent i at campaign s be represented by a parameter $\gamma_i(s) \geq 0$. To model repeated campaigns we concatenate the model (4) as follows:

$$\dot{x}(s, t) = (\Lambda L + I - \Lambda)x(s, t) + (I - \Lambda)u(s) \quad (5a)$$

$$u(s) = (I - H\Gamma(s))x(s, 0) + H\Gamma(s)d\mathbb{1} \quad (5b)$$

$$x(s, 0) = x(s-1, \infty) := \lim_{t\to\infty} x(s-1, t) \quad (5c)$$

where $\Gamma(s) = \operatorname{diag}(\gamma_1(s), \ldots, \gamma_n(s))$ is a diagonal matrix describing the influence efforts of the external entity at campaign s. Eq. (5a) means that during each campaign the opinion of each agent evolves according to the FJ model (4). Eq. (5b) specifies how the prejudices update at each campaign. Eq. (5c) indicates the concatenation rule between initial-final opinions of subsequent campaigns.

From Theorem 1, we know that the vector of final opinions at the end of each campaign is given by $x(s, \infty) = Vu(s)$ for all $s \in \mathbb{N}$. To simplify the notation, we use $y(s)$ to denote $x(s, \infty)$. A (simplified) reformulation of the system (5) focused only on the final opinions at the end of each campaign is given by:

$$y(s) = V\big((I - H\Gamma(s))y(s-1) + H\Gamma(s)d\mathbb{1}\big). \quad (6)$$

We indicate the solution of (6) at s with initial opinions $y(0) = x(0)$ using the notation $y(s|y(0))$ (or $y(s)$ when clear from context). It is easy to show that the model (6) satisfies the following properties: (i) if $y(0) = d\mathbb{1}$, then $y(s) = d\mathbb{1}$ for all s; (ii) if $y(0) \in [m, M]^n$ and $d \in [m, M]$, then $y(s) \in [m, M]^n$ for all s; (iii) if $y_0 \leq \bar{y}_0$, then $y(s|y_0) \leq y(s|\bar{y}_0)$ for all s.

3.2 Error dynamics and desired opinion

Let the (absolute value) difference from the target at campaign s be given by $e(s) = y(s) - d\mathbb{1}$, where the evolution of $y(s)$ is captured by (6). The difference or error dynamics evolves according to:

$$e(s) = V(I - H\Gamma(s))e(s-1). \quad (7)$$

In this work, we assume $d = 0$ and $x_i \in [0, 1]$, which, in turn, imply $y_i \in [0, 1]$ (according to the previous properties). Letting $d = 0$ and $y(0) \in [0, 1]^n$, we obtain that $e(s) \in [0, 1]^n$ for all s, and it can be shown that the function $\max_{i\in\{1,\ldots,n\}} e_i(s)$ is nonincreasing. The solution of (7) at campaign instant s is given by:

$$e(s) = \prod_{k=1}^{s} \big(V(I - H\Gamma(k))\big)e(0).$$

Remark 2. The choice d=0 and $x_i \in [0, 1]$ follows naturally from the dietary choices application we consider (details in Section 6), where opinions reflect preferences regarding the percentage of meat consumption in the agents' diets and the influence efforts of the external entity (e.g., a government) aim towards achieving a meat-free diet.

3.3 External influence scenarios

In this work, we investigate the impact of external influence on opinion dynamics, by considering different configurations for the external influence parameters $\Gamma(s)$, $s \in \mathbb{N}$.

As a first setting of our analysis, we study the impact of equal influence efforts among the agents, i.e., how the difference dynamics (7) evolves when $\Gamma(s) = \bar{\gamma}I$ for all $s \in \mathbb{N}$. This scenario represents a broadcasting situation where an external entity, without budget constraints, influences all agents equally (Section 4). The second setting we consider involves budget constraints and strategic decisions by the external entity. In this scenario, the external entity must optimally allocate its influence efforts within the limitations of its budget constraints (Section 5).

4. EQUAL INFLUENCE EFFORTS

The first setting we consider is that of a homogeneous (in time) broadcasting among the agents, i.e., $\Gamma(s) = \bar{\gamma}I$ for all $s \in \mathbb{N}$. In this case, the solution of the error dynamics (7) at campaign instant s is given by: $e(s) = (V(I - H\bar{\gamma}))^s e(0)$. Depending on the values assumed by the diagonal elements of the matrix H and on the number of prejudiced agents, the solution may take different forms, as shown in the following lemma. If there are no fully prejudiced agents, then the error converges to zero as the number of campaigns grows large (i.e., $s \to \infty$) when at least one prejudiced agent is susceptible to the external entity's influence. Otherwise, the error converges to zero only if all fully prejudiced agents are susceptible to the influence efforts of the external entity.

Lemma 3. Let $\mathcal{I}_1 = \{i : \lambda_i = 1\}$ and $\mathcal{I}_0 = \{i : \lambda_i = 0\}$ be the sets of fully susceptible and fully stubborn agents.

Assume that there are no fully stubborn agents, i.e., $\mathcal{I}_0 = \emptyset$. Then, the following conditions hold:

(i) If $h_i = 0$ for all $i \in \{1,\dots,n\} \setminus \mathcal{I}_1$, $\lim_{s\to\infty} e(s) = \mathbb{1}w^T e(0)$, where w is the left eigenvector of V corresponding to the eigenvalue 1;
(ii) If $\exists\, i \in \{1,\dots,n\} \setminus \mathcal{I}_1$ s.t. $h_i > 0$, $\lim_{s\to\infty} e(s) = 0$.

Assume that there is at least one fully stubborn agent, i.e., $\mathcal{I}_0 \neq \emptyset$. Then, the following conditions hold:

(iii) If $\exists\, i \in \mathcal{I}_0$ s.t. $h_i = 0$, then $\lim_{s\to\infty} e(s) = [v_1 \cdots v_{\mathrm{card}(\{i\in\mathcal{I}_0 : h_i=0\})}][w_1 \cdots w_{\mathrm{card}(\{i\in\mathcal{I}_0 : h_i=0\})}]^T e(0)$, where the vectors $v_i, w_i \in \mathbb{R}^n$ are the right and left eigenvectors, respectively, of $V(I - H\bar{\gamma})$ corresponding to the eigenvalues equal to 1;
(iv) If $h_i > 0$ for all $i \in \mathcal{I}_0$, then $\lim_{s\to\infty} e(s) = 0$.

Proof. After an adequate permutation, the matrix V (from Theorem 1) can be rewritten as:

$$V = \begin{bmatrix} 0_{n_1} & \star & \star \\ 0_{(n-n_1-n_0)\times n_1} & \star & \star \\ 0_{n_0\times n_1} & 0_{n_0\times(n-n_0-n_1)} & I_{n_0} \end{bmatrix}$$

where $n_1 = \mathrm{card}(\mathcal{I}_1)$ and $n_0 = \mathrm{card}(\mathcal{I}_0)$, and $\star$ indicates a nonnegative matrix of adequate dimension.

(i)-(ii). Assume $n_0 = 0$ or, equivalently, $\mathcal{I}_0 = \emptyset$. If $h_i = 0$ for all $i \in \{1,\dots,n\} \setminus \mathcal{I}_1$, then $V(I - \bar{\gamma}H) = V$ and $\lim_{s\to\infty} V^s = \mathbb{1}w^T$, where $\mathbb{1}$ and w are the right and left eigenvectors of V for the eigenvalue 1. However, if $\exists\, i \in \{1,\dots,n\} \setminus \mathcal{I}_1$ s.t. $h_i > 0$, then $V(I - \bar{\gamma}H)$ is a Schur stable matrix, which implies that $\lim_{s\to\infty} V^s = 0$.

(iii)-(iv). Assume $n_0 > 0$ or, equivalently, $\mathcal{I}_0 \neq \emptyset$. If $\exists\, i \in \mathcal{I}_0$ s.t. $h_i = 0$, then 1 is still an eigenvalue (and spectral radius) of $V(I-\bar{\gamma}H)$ of multiplicity $\mathrm{card}(\{i \in \mathcal{I}_0 : h_i = 0\})$, and $\lim_{s\to\infty} V^s$ assumes the structure shown in (iii). However, if $h_i > 0$ for all $i \in \mathcal{I}_0$, then $V(I - \bar{\gamma}H)$ is a Schur matrix, which implies that $\lim_{s\to\infty} V^s = 0$. ■

To address a wider range of scenarios, in the next section we incorporate budget constraints into the analysis.

5. INFLUENCE UNDER BUDGET CONSTRAINTS

In the second setting for the influence efforts, we examine the impact of budget constraints on campaigns. Specifically, we explore two situations: 1) Broadcasting influence with budget constraints: The external entity is able to reach all agents equally in each campaign, but a budget constraint limits the total number of campaigns where this influence can occur (Section 5.1); and 2) Targeted influence with budget constraints: For each campaign, a budget constraint limits the total number of agents that can be reached/influenced (Section 5.2). Based on these situations, a question arises: How can the external entity allocate its influence efforts in an optimal way?

In both scenarios, we adapt the technical approach taken in Morărescu et al. (2020), showing that optimal influence, or resource, allocation leads to water-filling solutions (Boyd and Vandenberghe (2004)).

5.1 Equal influence under budget constraints

The first strategy we consider is broadcasting across the community, i.e., $\gamma_i(s) = \gamma_j(s)$ for all $i, j = 1,\dots,n$ and $s \in \mathbb{N}$, under the assumption that the agents are homogeneous in their susceptibility to external influence, i.e., $h_i = 1$ for all $i = 1,\dots,n$. With some abuse of notation, we let $\gamma_i(s) = \gamma(s) \in [0, \bar{\gamma}]$ for all i and denote by $\{\gamma(k)\}_{k=1}^s := \{\gamma(1), \gamma(2), \dots, \gamma(s)\}$ a broadcasting sequence strategy.

We define the cost at campaign s associated with a broadcasting strategy as the (absolute value) difference between the average opinion and the desired opinion d:

$$J_s(\{\gamma(k)\}_{k=1}^s) = \left|\tfrac{1}{n}\mathbb{1}^T y(s) - d\right| = \tfrac{1}{n}\left|\mathbb{1}^T e(s)\right|, \qquad (8)$$

where the error evolves according to (7). Note that, since $e(s) \geq 0$, the absolute value in (8) can be disregarded. The aim of the external entity is to select an optimal sequence strategy $\{\gamma(k)\}_{k=1}^s$ that minimizes the cost (8), while respecting a total budget constraint B. This corresponds to solving the following optimization problem:

$$\underset{\{\gamma(k)\}_{k=1}^s}{\arg\min} \; J_s(\{\gamma(k)\}_{k=1}^s) = \mathbb{1}^T e(s)/n \qquad (9a)$$

$$\text{s.t. } 0 \leq \gamma(k) \leq \bar{\gamma}, \quad k = 1,\dots,s \qquad (9b)$$

$$\textstyle\sum_{k=1}^s \gamma(k) \leq \frac{B}{n}. \qquad (9c)$$

If the budget B is large, e.g., $B \geq s\,n\,\bar{\gamma}$, the optimal solution corresponds to selecting $\gamma(k) = \bar{\gamma}$ for all $k = 1,\dots,s$. Instead, let the budget B satisfy $B < sn\bar{\gamma}$. The following lemma presents an optimal broadcasting strategy that solves the constrained optimization problem (9). We follow the approach taken in Proposition 4.1 by Morărescu et al. (2020), but we omit the proof due to lack of space.

Lemma 4. The cost $J_s(\{\gamma(k)\}_{k=1}^s)$ in (8) at campaign s is minimized by adopting the broadcasting strategy:

$$\gamma(k) = \begin{cases} \bar{\gamma}, & \text{if } k \leq \lfloor \frac{B}{n\bar{\gamma}} \rfloor, \\ \frac{B}{n} - \bar{\gamma}\lfloor \frac{B}{n\bar{\gamma}} \rfloor, & \text{if } k = \lfloor \frac{B}{n\bar{\gamma}} \rfloor + 1, \\ 0, & \text{otherwise.} \end{cases} \qquad (10)$$

5.2 Targeted influence with budget constraints

Lemma 4 indicates that, when equal influence is possible, all resources should be used as soon as possible. Motivated by more realistic settings, the second strategy we consider is that of one step ahead targeted campaigns, where each diagonal matrix $\Gamma(s)$ is designed to minimize the cost at the s-th campaign, given the observed opinions at the end of the $(s-1)$-th campaign, and under a budget constraint imposed for that specific campaign and denoted by B_s.

Similarly to Section 5.1, we define the one-step-ahead cost at campaign instant s associated with the targeted strategy $\Gamma(s) = \mathrm{diag}\,(\gamma_1(s),\dots,\gamma_n(s))$ as:

$$J_1(\Gamma(s)|y(s-1)) = \left|\tfrac{1}{n}\mathbb{1}^T y(s) - d\right| = \tfrac{1}{n}\left|\mathbb{1}^T e(s)\right|,$$

which can be equivalently reformulated as follows:

$$J_1(\Gamma(s)|e(s-1)) = \tfrac{1}{n}\mathbb{1}^T V(I - H\Gamma(s))e(s-1). \qquad (11)$$

Define the vector $\rho := \frac{1}{n}V^T\mathbb{1}$; note that in the standard FJ model (2) with $u = x(0)$, each element ρ_i represents the social power of agent i, i.e., the mean weight of the initial opinion of agent i in the final opinion of the network of agents (Proskurnikov and Tempo (2017)).

As aforementioned, the external entity aims to select an optimal strategy $\Gamma(s)$ that minimizes the cost (11) given $e(s-1)$, while respecting the budget constraint B_s. This corresponds to solving the following optimization problem:

$$\underset{\Gamma(s)}{\arg\min} \; J_1(\Gamma(s)|e(s-1)) = \rho^T(I - H\Gamma(s))e(s-1) \qquad (12a)$$

$$\text{s.t. } 0 \leq \Gamma(s)\mathbb{1} \leq \bar{\gamma}\mathbb{1} \qquad (12b)$$

$$\textstyle\sum_{i=1}^n \gamma_i(s) \leq B_s. \qquad (12c)$$

If the budget is large ($B_s \geq n\bar{\gamma}$), the optimal solution corresponds to using $\gamma_i(s) = \bar{\gamma}$ for all $i = 1, \ldots, n$. Instead, let $B_s < n\bar{\gamma}$. The following theorem presents an optimal targeted strategy that solves the constrained optimization problem (12); in the proof, we build upon the approach taken in Proposition 4.2 of Morărescu et al. (2020).

Theorem 5. With $c = \rho \odot e(s-1) \geq 0$, i.e., $c_i = \rho_i e_i(s-1)$ for $i = 1, \ldots, n$, define the ordering $o : \{1, \ldots, n\} \mapsto \{1, \ldots, n\}$ such that $[Hc]_{o(1)} \geq [Hc]_{o(2)} \geq \cdots \geq [Hc]_{o(n)}$. The one-step-ahead cost $J_1(\Gamma(s)|e(s-1))$ at campaign s (11) is minimized by adopting the targeted strategy:

$$\gamma_{o(i)}(s) = \begin{cases} \bar{\gamma}, & \text{if } i \leq \lfloor \frac{B_s}{\bar{\gamma}} \rfloor, \\ B_s - \bar{\gamma}\lfloor \frac{B_s}{\bar{\gamma}} \rfloor, & \text{if } i = \lfloor \frac{B_s}{\bar{\gamma}} \rfloor + 1, \\ 0, & \text{otherwise.} \end{cases} \quad (13)$$

Proof. To ease the notation, we drop the time-dependence and write $\Gamma(s)\mathbb{1} =: \gamma = [\gamma_1 \cdots \gamma_n]^T$, where $\gamma_i \in [0, \bar{\gamma}]$ for all i. $J_1(\gamma|e(s-1))$ can be rewritten as

$$J_1(\gamma|e(s-1)) = \textstyle\sum_{i=1}^n (1 - h_i\gamma_i)c_i.$$

Finding a strategy γ that minimizes this cost is equivalent to solving the following constrained optimization problem:

$$\begin{aligned} \min_{\gamma} \quad & \left(\textstyle\sum_{i=1}^n (1 - h_i\gamma_i)c_i\right)^2 \quad (14) \\ \text{s.t.} \quad & 0 \leq \gamma_i \leq \bar{\gamma},\ i = 1, \ldots, n \\ & \textstyle\sum_{i=1}^n \gamma_i \leq B_s \end{aligned}$$

The cost function is convex and all the constraints are affine, which means that the KKT conditions can be applied and are necessary and sufficient for optimality. The Lagrangian associated with the problem (14) is defined as

$$\begin{aligned} \ell(\gamma, \lambda_1, \lambda_2, \lambda_3) =& ((\mathbb{1} - H\gamma)^T c)^2 \\ & - \lambda_1^T\gamma + \lambda_2^T(\gamma - \bar{\gamma}\mathbb{1}) + \lambda_3(\mathbb{1}^T\gamma - B_s), \end{aligned}$$

where $\lambda_1, \lambda_2 \in \mathbb{R}^n, \lambda_3 \in \mathbb{R}$ are the Lagrange multipliers associated with the inequality constraints (Boyd and Vandenberghe (2004)). Its gradient can be computed as

$$\nabla\ell(\gamma, \lambda_1, \lambda_2, \lambda_3) = -2((\mathbb{1} - H\gamma)^T c)Hc - \lambda_1 + \lambda_2 + \lambda_3\mathbb{1}.$$

Therefore the KKT conditions are given by:

$$\begin{gathered} -2((\mathbb{1} - H\gamma^*)^T c)Hc - \lambda_1^* + \lambda_2^* + \lambda_3^*\mathbb{1} = 0, \\ \lambda_1^*, \lambda_2^*, \lambda_3^* \geq 0, \quad 0 \leq \gamma^* \leq \bar{\gamma}\mathbb{1}, \quad \mathbb{1}^T\gamma - B_s \leq 0, \\ \lambda_1^* \odot \gamma^* = 0, \quad \lambda_2^* \odot (\gamma^* - \bar{\gamma}\mathbb{1}) = 0, \quad \lambda_3^*\left(\mathbb{1}^T\gamma^* - B_s\right) = 0. \end{gathered}$$

Consider γ^* as the solution proposed in (13). In the following, we show that the KKT conditions are satisfied for γ^*, which proves that γ^* is a solution to the minimization problem (14), thus concluding the proof. Since $\sum_{i=1}^n \gamma_i^* = \bar{\gamma}\lfloor B_s/\bar{\gamma} \rfloor + B_s - \bar{\gamma}\lfloor B_s/\bar{\gamma} \rfloor = B_s$, then the KKT conditions are satisfied with

$$\lambda^*_{1_{o(i)}} \begin{cases} = 0, & i = 1, \ldots, \lfloor B_s/\bar{\gamma} \rfloor + 1 \\ \geq 0, & i = \lfloor B_s/\bar{\gamma} \rfloor + 2, \ldots, n \end{cases}$$

$$\lambda^*_{2_{o(i)}} \begin{cases} \geq 0, & i = 1, \ldots, \lfloor B_s/\bar{\gamma} \rfloor \\ = 0, & i = \lfloor B_s/\bar{\gamma} \rfloor + 1, \ldots, n \end{cases}$$

$$\lambda_3^* = 2\Big(\textstyle\sum_{i=1}^n c_{o(i)}(1 - h_{o(i)}\gamma^*_{o(i)})\Big)[Hc]_{o\left(\lfloor \frac{B_s}{\bar{\gamma}} \rfloor + 1\right)} > 0,$$

where the inequalities hold by notion of the ordering o. ■

Note that in (12) $0 < h_i \ll 1$ could alternatively indicate that selected agents require more resources to change their habits; in the following, we assume for simplicity that $H = I$. Theorem 5 indicates that each campaign should direct resources towards agents with the highest social power, considering their proximity to the desired opinion.

Lastly, the constraint $B_s < \bar{\gamma}\,n$ is conservative when there are fully prejudiced agents whose initial opinion equals d, who hence maintain their opinion despite social influence. In this case, the budget constraint can be rewritten as $B_s < \bar{\gamma}\,(n - \text{card}(\{i \in \mathcal{I}_0 : e_i(0) = 0\}))$. Future work will explore in depth the roles of H and prejudiced agents.

Remark 6. In the food consumption context, these agents can be viewed as "fully stubborn" vegetarians, i.e., individuals who regardless of the influence of their peers will remain anchored to their meat-free diet. Essentially, tightening the budget constraint means that a reduced amount of resources (incentives/subsidies) need to be allocated.

6. APPLICATION: MEETING THE TARGET FOR MEAT INTAKE IN THE UNITED KINGDOM

As an illustrative application of this work, we consider the problem of influencing public opinion in the UK regarding meat consumption, involving influence campaigns to guide the UK population towards achieving the consumption target for meat set by the UK Climate Change Committee (2020) (CCC). The CCC's recommended Widespread Engagement Pathway represents a transition to net zero across all sectors of the economy and involves ambitious and substantial behavioral changes from consumers. One of the targets is to achieve a 35% percentage reduction in the average consumption of all meat products (and dairy products which we do not consider here) by 2030.

6.1 Data description

We derived dietary data from the National Diet and Nutrition Survey (NDNS) 2019 (Bates et al. (2019)), designed to collect comprehensive and quantitative information on the food consumption and nutrient intake of the general UK population [2]. The data presented here considers a sample of n=1841 individuals and intakes of four food groups, i.e., meat, meat alternatives, pulses, and vegetables.

6.2 Opinion dynamics modeling of food consumption

We frame dietary shifts in the UK population network as a problem of opinion formation with external influence, like that of governments or policy-makers. Here, agents are individuals in the UK population. Their opinion (y_i, i=1, . . . , n, in our notation) is indicated by a real number between 0 and 1, representing the preferred proportion of meat (in general, food group) consumption in relation to their total energy intake. Total energy intake is defined as the sum of the four above-mentioned food groups (meat, meat alternatives, pulses, and vegetables), and it is assumed to remain constant. Based on previous research (Bentham et al. (2020)) and for simplicity, we model only meat consumption opinion formation, presuming the other food groups adjust to maintain constant energy intake.

To model the evolution of opinions, we use the concatenated FJ model under repeated campaigns (6), with the survey data as initial opinions. We assume that vegetarians and carnivores (individuals whose percentage of meat consumption is 0% and 100%, respectively) are fully

[2] gov.uk/government/collections/national-diet-and-nutrition-survey

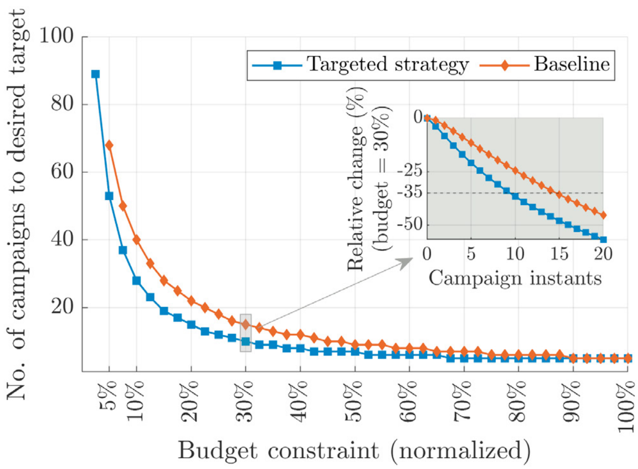

Fig. 1. No. of campaigns to reach the CCC's desired 35% reduction in meat consumption, using the optimal allocation strategy (13), under increasing campaign-specific budget constraints. The x-axis shows $\bar{B}/(n\bar{\gamma})$ with $\bar{\gamma} = 0.1$ and constant budget $B_s = \bar{B}$ for all $s \in \mathbb{N}$. The inset shows the average relative change for the particular case of normalized budget = 30%.

prejudiced ($\lambda_i = 0$), while omnivores (individuals that are neither carnivores nor vegetarians) are partially prejudiced ($\lambda_i \in (0, 1)$). From the survey data, we obtain 129 vegetarians and 31 carnivores. To model the social network, we assume an undirected graph where edges are included with a probability of 0.4. Edge weights are assigned by grouping agents based on socioeconomic status and age, resulting in a block adjacency matrix. Agents within the same group are linked by an edge of weight +1; those in different groups by an edge with nonnegative weight < 1, reflecting their distance on a linearly spaced grid that orders groups by age and socioeconomic status.

We compare the proposed optimal strategy (13) against a baseline strategy where the campaign-specific budget is distributed equally among the agents. The impact of influence campaigns is evaluated in terms of % average relative change with respect to the initial opinions, and no. of campaigns to reach an average 35% reduction in kcal meat consumption. Unsurprisingly, a small budget implies a high no. of campaigns is needed to reach the −35% goal, as illustrated in Fig. 1. Moreover, the effectiveness of the targeted strategy is shown by its requirement for fewer campaigns compared to the baseline strategy.

7. CONCLUSION

This work studies the problem of guiding the collective opinion of a social network with prejudiced agents toward a desired opinion, determined by an external entity, by means of repeated campaigns. Due to budget constraints, the external entity aims to optimally allocate its influence efforts to minimize a cost function, representing the average distance of the social network from the desired state. In the case of a targeted strategy under campaign-specific budget constraints, where the external entity chooses a subgroup of agents to influence, it is shown that the allocation follows a water-filling method. Here, the external entity prioritizes influencing agents with the highest social power weighted by their distance from the desired opinion.

As a concrete application, we numerically simulated the time evolution of opinions (i.e., dietary preferences) within a social network representing the UK population. We observed this process across a series of influence campaigns, aimed at achieving a sustainable meat-free diet.

Future directions will explore different network structures (relaxing the strong connectivity assumption) and assess the role and selection of prejudiced agents.

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