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Exploring rationality of prospect choices among decision-makers in a population

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Abstract: : The random utility model (RUM) is a fundamental notion in studies of human decision-making. However, RUM relies on the calibration of its choice function's weight parameter, usually interpreted as a rationality parameter, resulting in a case-dependence that undermines both interpretability and predictability of choices across experimental settings. We addressed this limitation by normalizing utilities in RUM and deriving a new choice parameter β, independent of case-specific prospects. Drawing from a novel interpretation of β in terms of the lowest perceived probability of unlikely events, we conducted an experimental survey in Swedish universities to infer β distributions, capturing the variability of probability perception among decision-makers. We tested these statistical models for β on two independent datasets exploring the framing effect. The results showed that the predictions align with the observed experimental data (Pearson's correlation greater than 94%), thereby indicating that the novel characterization of the choice parameter strengthens the predictive capabilities of RUM.

Keywords: unlikely events probability, choice parameter, RUM, prospects, decision-making.

1. INTRODUCTION

Predicting human choices is a challenging problem with implications well beyond behavioral economics, ranging from economics (Tversky and Kahneman (1992)) to politics (Fontan and Altafini (2021)), from transportation (Annaswamy and Yildiz (2020)) to lifestyle choices (Farjadnia et al. (2023)), and in its most general formulation the problem can be expressed in the form of the question *How do people make choices among different alternatives?* A standard hypothesis in the literature, consistent with, e.g., expected utility theory (Bernoulli (1953)) or random utility theory (McFadden (1976)), is that a decision-maker chooses the alternative among two prospects that maximizes their utility, usually expressed in monetary units. This decision-making process considers two types of variables: *exogenous* variables, which are observable, such as attributes on the alternatives that can be expressed in terms of outcomes and associated probabilities; and *endogenous* variables, which are not observable and vary among decision-makers (Ben-Akiva and Lerman (1985)).

A framework used to model discrete choices under the assumption of utility-maximizing behavior is given by the Random Utility Model (RUM), a cornerstone of behavioral economics for decisions on alternatives with uncertain outcomes (McFadden (1976)). The key assumption of RUM is that decision-makers' preferences can be described by a utility function depending on exogenous and endogenous attributes, the latter randomizing the actual choice; hence, RUM computes the probability of choosing a given alternative. In the standard RUM formulation, this probability is computed by a Logit function of the difference in utilities of the alternatives, scaled by a scalar parameter β, typically calibrated for the set of data at hand (Train (2016)).

We argue that the current treatment of β in the literature lacks coherency and limits the predictive power of RUM. First, there does not seem to be consensus in the notation. β is referred to in the literature in different ways, e.g., sensitivity parameter in Rieskamp (2008), steepness of S-shaped function in Wang and Busemeyer (2021), free sensitivity parameter in Glickman et al. (2019), precision parameter in McKelvey and Palfrey (1995), inverse rationality parameter in Rosenfeld and Kraus (2018), strength of preference parameter in Erev et al. (2002). Moreover, this diverse nomenclature indicates that β is understood as an endogenous parameter, even if in the common RUM formulation β is explicitly dependent on exogenous variables, i.e., the magnitude and units of the utilities. In other

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words, neither the range of β values nor its interpretation can be transferred from one experiment to another.

In this work, we claim that separation of exogenous and endogenous variables is crucial, not only for improving predictability of choices, but also for their interpretability. This can be achieved by adopting a normalization of the utilities (which can then be considered exogenous) in RUM (Train (2009)), thus obtaining a novel (dimensionless and endogenous) parameter β, which we name the choice parameter. The normalization also allows for a new interpretation of β as control parameter: $\beta = 0$ indicates indecision between two alternatives, equivalent to a probability of choice equal to 0.5, and $\beta \to +\infty$ indicates a sure choice of an alternative, or, equivalently, a sure disregard of the opposite option, with a probability of choice equal to 0.

Given this new interpretation of β, we hypothesize that suitable psychological evidence should be sufficient to infer a distribution for β. To this end, we ask the following question: If, as per RUM, sure disregard of a prospect is theoretically described by a probability of choice equal to 0, obtained when $\beta \to +\infty$, what is the value of β when we consider what humans perceive as near-zero probability, or, equivalently, the probability that an event is unlikely? There is ample evidence that people tend to mentally discount events they deem "too unlikely" to affect them (McClelland et al. (1993); Schade et al. (2012)); it is unclear however what numerical probability best represents our perceptions of unlikely events. Previous research on the subjective meaning of probability terms generally dealt with more moderate probabilities, typically restricting the scale to preclude very low probabilities (Stewart et al. (2006); Wallsten et al. (1986). In our study, we administered a survey to students in Swedish universities to estimate what probability best represents the subjective boundary for an unlikely event, as well as the distribution of this subjective boundary across the population.

Our main contributions can be stated as follows: (i) From an experimental survey on human perception of unlikely events, we infer distributions for the choice parameter β; (ii) We validate these models using independent data on framing effect experiments and show that they represent well variability of rationality among decision-makers across different experimental settings. We derive our concept of rationality from the work of De Martino et al. (2006) on the framing effect, associating the "most rational" behavior with complete indifference to framing manipulation.

The paper is organized as follows. Section 2 introduces technical preliminaries and the notation. Section 3 proposes the novel choice parameter. Sections 4 and 5 present the main results. Section 6 offers conclusive remarks.

2. TECHNICAL PRELIMINARIES

This section introduces the notions of subjective valuations (or utilities) of prospects (Section 2.1), choice probability (Section 2.2), and framing effect (Section 2.3).

2.1 Computation of valuation by Prospect Theory

We first explain how to compute utilities using Prospect Theory (PT) from prospect data, in a setting where a decision-maker chooses between two alternative prospects A and B (Tversky and Kahneman (1992) for details).

Let $\mathrm{A}: \{(Y_{\mathrm{A}_1}, \pi_{\mathrm{A}}), (Y_{\mathrm{A}_2}, 1-\pi_{\mathrm{A}})\}$ and $\mathrm{B}: \{(Y_{\mathrm{B}_1}, \pi_{\mathrm{B}}), (Y_{\mathrm{B}_2}, 1-\pi_{\mathrm{B}})\}$, where Y_{A_1} and Y_{A_2} are outcomes for A with probabilities π_{A} and $1-\pi_{\mathrm{A}}$, respectively, and similarly for B. A decision-maker chooses a prospect based on the subjective values of alternatives A and B, namely, V_{A} and V_{B}. As per PT, the utilities V_{A} and V_{B} are computed by:

$$V = \begin{cases} U(Y_1)\, w(\pi) + U(Y_2)\,(1 - w(\pi)), & \text{gain/loss} \\ U(Y_1)\, w(\pi) + U(Y_2)\, w(1-\pi), & \text{mixed} \end{cases} \tag{1a}$$

$$U(Y) = \begin{cases} (Y - Y_0)^{\delta^+} & \text{if } Y \geq Y_0 \\ -\lambda (Y_0 - Y)^{\delta^-} & \text{if } Y < Y_0 \end{cases} \tag{1b}$$

$$w(\pi) = (\pi^{\gamma})/(\pi^{\gamma} + (1-\pi)^{\gamma})^{1/\gamma}, \tag{1c}$$

where Y_0 is a reference value and V, Y_1, Y_2, and π pertain to A or B. Gain prospects are defined as $Y_1 > Y_2 \geq Y_0 \geq 0$, loss prospects as $Y_1 < Y_2 \leq Y_0 \leq 0$, and mixed prospects as $Y_1 < Y_0 < Y_2$. In this work, we use $Y_0 = 0$ and we consider mainly gain/loss prospects. The functions $U(Y)$ and $w(\pi)$ are called the utility and the weighting function, respectively, and we choose the functionals proposed in Tversky and Kahneman (1992). While the general formulation of $U(Y)$ includes two different parameters for gain (δ^+) and loss (δ^-) prospects, here we assume that $\delta^+ = \delta^- = \delta$. Selection of the positive PT parameters $(\lambda, \delta, \gamma)$ quantifies different risk and valuation biases. Specifically, $(\lambda, \delta, \gamma) = (1, 1, 1)$ implies a simple expected value without any biases, $(\lambda, \delta, \gamma) = (> 1, < 1, 1)$ implies an expected utility model without accounting for the risk bias, whereas $(\lambda, \delta, \gamma) = (> 1, < 1, < 1)$ includes both value and risk biases. In the PT literature, these parameters are inferred from experiments. We use the *standard PT parameters* $(\gamma, \delta, \lambda) = (0.65, 0.88, 2.25)$, from the calibration proposed in the classical work by Tversky and Kahneman (1992) [1].

2.2 Computation of choice probabilities from prospect data

The utility assigned to each alternative can be conceptualized as a random variable, depending both on observable and unobservable attributes, with a zero-mean disturbance term (Ben-Akiva and Lerman (1985)). Hence, while we are not able to model choices with certainty, we can compute a probability of the decision-maker choosing a given prospect. In this work, we calculate the expected utilities V_{A} and V_{B} for discrete prospects, A and B, respectively, using PT (Section 2.1), and we use the RUM to express the choice probability through a Logit function:

$$P_{\mathrm{A}} = \left(1 + \exp(-\beta(V_{\mathrm{A}} - V_{\mathrm{B}}))\right)^{-1}, \quad P_{\mathrm{B}} = 1 - P_{\mathrm{A}}, \tag{2}$$

where β is a weight or calibration parameter.

2.3 The Framing Effect

The framing effect describes the difference in behavior by decision-makers observed when the prospects are given in different frames. It is subject of many experimental studies, e.g., De Martino et al. (2006); Diederich et al. (2020); McDonald et al. (2021); Nabi et al. (2020). Our current interpretation of rationality follows De Martino et al. (2006), where the "most rational" decision-makers exhibit the lowest values of framing effect whereas the "least rational" decision-makers exhibit the highest values.

[1] The work identifies $\gamma = 0.61$ for loss prospects and $\gamma = 0.69$ for gain prospects. For simplicity and to keep a low no. of parameters, we use $\gamma = 0.65$, i.e., the average value, as the standard PT parameter.

The definition adopted in this work is the following. Let A and B indicate a risky and a sure prospect, respectively. The framing effect of a decision-maker is the difference in probabilities of choosing the risky prospect between the loss (L) and the gain (G) frame, i.e., $P_{\mathrm{A_L}} - P_{\mathrm{A_G}}$. Given a set of N trials in which a decision-maker needs to choose between a risky prospect A_i and a sure prospect B_i, $i = 1, \ldots, N$, the observed framing effect for a decision-maker is the percentage of trials in which the risky prospect is chosen in the loss vs the gain frame, while the estimated framing effect is defined as $\sum_{i=1}^{N}(P_{\mathrm{A}_{i\mathrm{L}}} - P_{\mathrm{A}_{i\mathrm{G}}})/N$.

3. CONTEXTUALIZATION BY MEANS OF NORMALIZATION IN RUM

In spite of its wide use, the expression in (2) of the logistic model for binary choices is limited by the context-dependent parameter β. There is no obvious value of β that can be used for predicting choices *across experimental settings*, where nominal values significantly vary. We claim that by using (L1) normalization in RUM we can propose a context-independent, i.e., endogenous, parameter β. Instead of the raw utilities, V_A and V_B in (2), we consider the normalized utilities $V_\mathrm{A}/(|V_\mathrm{A}| + |V_\mathrm{B}|)$ and $V_\mathrm{B}/(|V_\mathrm{A}| + |V_\mathrm{B}|)$, which we interpret as attitudes of a decision-maker towards prospect A and prospect B, respectively. In line with RUM, the probability by a decision-maker of choosing prospect A over prospect B can be rewritten as:

$$P_\mathrm{A} = \left(1 + \exp\left(-\beta \frac{V_\mathrm{A} - V_\mathrm{B}}{|V_\mathrm{A}| + |V_\mathrm{B}|}\right)\right)^{-1}, \tag{3}$$

and analogously $P_\mathrm{B} = 1 - P_\mathrm{A}$. Comparing (2) and (3), the novelty lies in the normalization as a method to draw comparisons between the attitudes towards the alternative prospects. Notably, in (3) $\beta \geq 0$ is a dimensionless scalar, which we name the *choice (or control) parameter*. β can be interpreted to quantify how sure a decision-maker is when making a choice: $\beta \to 0$ implies $P_\mathrm{A} = P_\mathrm{B} = \frac{1}{2}$ and $\beta \to +\infty$ implies $P_\mathrm{A} \to 1, P_\mathrm{B} \to 0$ or $P_\mathrm{A} \to 0, P_\mathrm{B} \to 1$.

3.1 The choice parameter

A sure choice of prospect A, defined as choice associated with sure probabilities $P_\mathrm{A} = 1$, $P_\mathrm{B} = 0$, is represented by $V_\mathrm{A}/(|V_\mathrm{A}| + |V_\mathrm{B}|) = 1$, $V_\mathrm{B}/(|V_\mathrm{A}| + |V_\mathrm{B}|) = 0$, yielding $\beta(V_\mathrm{A}/(|V_\mathrm{A}| + |V_\mathrm{B}|) - V_\mathrm{B}/(|V_\mathrm{A}| + |V_\mathrm{B}|)) = \beta$. Similarly, a sure choice of prospect B, defined as choice associated with sure probabilities $P_\mathrm{A} = 0$, $P_\mathrm{B} = 1$, is represented by $V_\mathrm{A}/(|V_\mathrm{A}| + |V_\mathrm{B}|) = 0$, $V_\mathrm{B}/(|V_\mathrm{A}| + |V_\mathrm{B}|) = 1$, yielding $\beta(V_\mathrm{A}/(|V_\mathrm{A}| + |V_\mathrm{B}|) - V_\mathrm{B}/(|V_\mathrm{A}| + |V_\mathrm{B}|) = -\beta$. That is, theoretically using (3), the interval $[-\beta, \beta]$ is associated with the interval $[0, 1]$ for the probability of choosing prospect A over B as $\beta \to +\infty$. However, in practice, the definition of zero probability depends on what humans *perceive* as near-zero probability or probability of an unlikely event.

Let P_0 be the observed probability of an unlikely event gathered from human-social probability perception evidence, and $(V_\mathrm{A} - V_\mathrm{B})/(|V_\mathrm{A}| + |V_\mathrm{B}|) = -1$ be the difference of the normalized utilities associated with $P_\mathrm{A} := P_0$. Inverting (3), one obtains the following expression for β as a function of P_0:

$$\beta = \log\left((1 - P_0)/P_0\right) \geq 0. \tag{4}$$

Given experimental data on P_0, we use (4) to infer a statistical model for β (Section 4).

3.2 Analysis of the framing effect for the normalized RUM

Before proceeding with the main results, namely, the design of the survey to estimate P_0 and infer β (Section 4), and the validation of the inferred models for β on datasets on variability of the framing effect (Section 5), we present an analysis of the framing effect within the normalized RUM. We first formulate the choice probabilities for the normalized RUM in the gain and loss frames (Theorem 1). Insights from the associated analysis of the framing effect are valuable to interpreting the collected data on the observed framing effect (Section 5).

Theorem 1. Let the prospects in a gain and loss frame be defined as follows, where $Y > 0$ and $\pi \in [0, 1]$:

$$\mathrm{A}: \begin{cases} \{(Y, \pi), (0, 1 - \pi)\}, & \text{gain frame} \\ \{(-Y, 1 - \pi), (0, 1 - \pi)\}, & \text{loss frame} \end{cases} \tag{5a}$$

$$\mathrm{B}: \begin{cases} \{(Y\pi, 1), (0, 0)\}, & \text{gain frame} \\ \{(-Y(1 - \pi), 1), (0, 0)\}, & \text{loss frame,} \end{cases} \tag{5b}$$

where the sure prospect B is created to match the expected value of the gamble A, depending on framing. Then, the probabilities of choosing the risky prospect A in the gain ($P_{\mathrm{A_G}}$) and loss ($P_{\mathrm{A_L}}$) frames are given by:

$$P_{\mathrm{A_G}} = \left(1 + \exp\left(-\beta \frac{w(\pi) - \pi^\delta}{w(\pi) + \pi^\delta}\right)\right)^{-1}, \tag{6a}$$

$$P_{\mathrm{A_L}} = \left(1 + \exp\left(-\beta \frac{-w(1 - \pi) + (1 - \pi)^\delta}{w(1 - \pi) + (1 - \pi)^\delta}\right)\right)^{-1}. \tag{6b}$$

Proof. We use PT ((1) with $\delta^- = \delta^+ = \delta$) to compute the valuations V_A and V_B for the prospects A and B in (5), in both frames. Calculations held:

$$(V_\mathrm{A}, V_\mathrm{B}) = \begin{cases} (Y^\delta w(\pi),\ Y^\delta \pi^\delta) & \text{gain} \\ (-\lambda Y^\delta w(1 - \pi),\ -\lambda Y^\delta (1 - \pi)^\delta) & \text{loss} \end{cases}$$

$$|V_\mathrm{A}| + |V_\mathrm{B}| = \begin{cases} Y^\delta \left(\pi^\delta + w(\pi)\right) & \text{gain} \\ \lambda Y^\delta \left(w(1 - \pi) + (1 - \pi)^\delta\right) & \text{loss.} \end{cases}$$

The normalized utilities (L1 normalization) are given by:

$$\Big(V_\mathrm{A}/(|V_\mathrm{A}| + |V_\mathrm{B}|),\ V_\mathrm{B}/(|V_\mathrm{A}| + |V_\mathrm{B}|)\Big) = \begin{cases} \left(\frac{w(\pi)}{w(\pi) + \pi^\delta},\ \frac{\pi^\delta}{w(\pi) + \pi^\delta}\right) & \text{gain} \\ \left(-\frac{w(1 - \pi)}{w(1 - \pi) + (1 - \pi)^\delta},\ -\frac{(1 - \pi)^\delta}{w(1 - \pi) + (1 - \pi)^\delta}\right) & \text{loss} \end{cases}$$

$$\frac{V_\mathrm{A} - V_\mathrm{B}}{|V_\mathrm{A}| + |V_\mathrm{B}|} = \begin{cases} \frac{w(\pi) - \pi^\delta}{w(\pi) + \pi^\delta}, & \text{gain} \\ -\frac{w(1 - \pi) - (1 - \pi)^\delta}{w(1 - \pi) + (1 - \pi)^\delta} & \text{loss.} \end{cases}$$

Therefore, using (3), the probabilities of choosing the risky prospect A in the gain ($P_{\mathrm{A_G}}$) and loss ($P_{\mathrm{A_L}}$) frames can be expressed as in (6). ■

Given the PT parameters δ, γ, using Theorem 1 we can compute the values of π associated with a positive framing effect (Fig. 1a). When $\gamma = 0.65$ and $\delta = 0.88$ (standard PT parameters), we obtain that $P_{\mathrm{A_L}} > 0.5 > P_{\mathrm{A_G}}$ for $\pi \in (0.245, 0.755)$ and $P_{\mathrm{A_L}} - P_{\mathrm{A_G}} > 0$ for $\pi \in (0.128, 0.872)$ (Fig. 1b). We can draw the following observations.

(i) The probabilities of choosing the risky prospect A in the gain ($P_{\mathrm{A_G}}$) and loss ($P_{\mathrm{A_L}}$) frames do not depend on the amount Y and are given by (6);
(ii) According to PT with standard PT parameters, the framing effect is positive for all $\pi \in (0.128, 0.872)$.

4. DERIVATION OF A STATISTICAL MODEL FOR THE CHOICE PARAMETER

According to the interpretation provided in Section 3.1, we use human-social probability perception evidence (Section 4.1) to infer a statistical model for β (Section 4.2).

4.1 Design of the survey to estimate P_0

The survey was administered to university students at Uppsala University and the Royal Institute of Technology in Sweden, in connection with regular lectures. Access to the survey was provided through a QR-code. Participation was entirely voluntary and no compensation was offered. Before starting the survey, respondents were given information on the subject and purpose of the survey, as well as ethical aspects such as anonymity and the voluntary nature of the survey. Respondents then answered three background questions regarding age, gender, and subject area of their university studies, before responding to the main question, described below. A total of 319 respondents participated in the survey, comprising 144 males, 171 females, and 4 who preferred not to disclose their gender. In terms of education, 268 respondents were from science and technology, 48 from social sciences, and 3 from humanities.

Respondents chose between six predetermined options of P_0, selected to allow for a wide distribution of possible values while representing meaningful points of comparison (1 in 10, 1 in 100, 1 in 1 000, 1 in 10 000, 1 in 100 000, and 1 in 1 000 000). To control for potential context effects, the following question was administered both without any specified context and with the additional instructions to consider what an unlikely event means in an economic, digital, weather-related, or medical context (see text inside brackets): *"With this question, we would like to know what you think counts as an unlikely event [when associated with economic collapse/ malware/ destructive weather/ illness]. You do not need to estimate the probability of any particular event, just what unlikely means to you [in this context]. Please choose the option that you think best represents the probability of an unlikely event [where you are financially affected by an economic collapse/ malware infects any of your digital devices/ you are directly affected by a destructive weather occurrence/ you develop a rare illness]."* Each respondent was randomized to one of the five conditions (no context, economic context, digital context, weather-related context, medical context).

The collected data on observed variability of P_0 (aggregating all contexts) is illustrated in Fig. 2 (left panel).

4.2 Inferring a distribution for the choice parameter

Variability in a population can be captured by assuming a statistical model for β with pdf defined on $(0, +\infty)$. In this work we consider two potential distributions. A log-normal distribution, i.e., $\beta \sim LN[\mu, \sigma]$ where $\mu > 0$ and $\sigma > 0$, or, equivalently, $\beta = \exp(\mu + \sigma Z)$ where $Z \sim \mathcal{N}[0, 1]$, and a normal distribution truncated from below, i.e., $\beta \sim \mathcal{N}_{(0,+\infty)}[\mu, \sigma]$ where $\mu > 0$ and $\sigma > 0$. The following distributions (illustrated in Fig. 2, right panel) are fit to the data using maximum likelihood estimation (with unbiased estimation of the σ parameter):

$$\beta \sim LN[2.0, 0.4], \quad \beta \sim \mathcal{N}_{(0,+\infty)}[8.3, 3.1]. \tag{7}$$

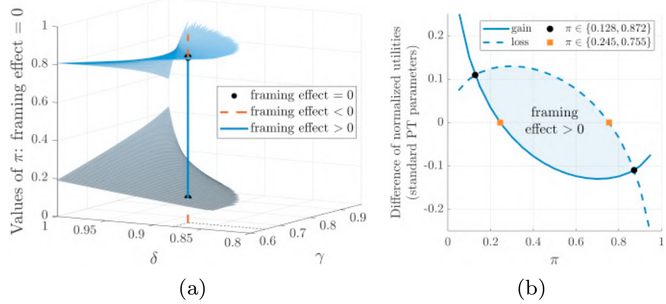


Fig. 1. (a): Values of π that satisfy $P_{A_L} - P_{A_G} = 0$ as a function of γ and δ. The values of π s.t. $P_{A_L} - P_{A_G} > 0$ (blue line) correspond to a positive framing effect for standard PT parameters ($\gamma = 0.65$, $\delta = 0.88$). (b): Difference in normalized utilities as a function of π (standard PT parameters). The framing effect is positive only when the curve corresponding to the gain frame is below the one for the loss frame.

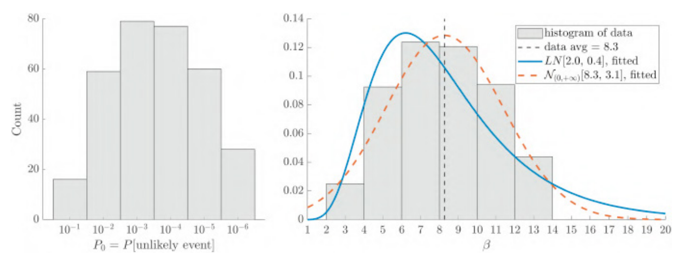


Fig. 2. Survey data. The left panel illustrates the histogram of the observed probability of unlikely events. The right panel presents the choice parameter calculated using (4), and associated fitted distribution (log-normal LN and truncated normal $\mathcal{N}_{(0,+\infty)}$).

5. APPLICATION: EXPERIMENTAL RESULTS ON DISCRETE CHOICES DATASETS

In this work we assume that the novel choice parameter β is endogenous, which suggests transferability across experimental settings. To validate this hypothesis, we test the models (7) for the choice parameter β using an independent set of data. In other words, we want to investigate whether the inferred distributions can capture choice heterogeneity among decision-makers in distinct experiments, involving different problem sets, contexts, and participant demographics than those examined in Section 4.1. To this end, we consider discrete choice experiments evaluating variability of framing effect across decision-makers (Section 5.1), and the findings are presented in Section 5.2.

5.1 Data description

We consider the studies by De Martino et al. (2006) and Diederich et al. (2020), which we denote by DS-FR1 and DS-FR2, respectively. For each experiment, the information collected includes two sets of problems and, for each problem, the description of each alternative structured as economic prospects (i.e., outcomes associated with probabilities of winning/losing) and the observed response of each decision-maker. The problems across the two sets are the same, but are posed in different frames (gain vs loss).

DS-FR1 The work of De Martino et al. (2006) presents experimental data that combine framing and heterogeneity effects, obtained by studying variation between 20 participants. The prospects presented during the experiments are defined according to (5), from the following 16 combinations of outcomes Y and probabilities π:

$$Y_i = (£25, £50, £75, £100)$$
$$\pi_j = (0.2, 0.4, 0.6, 0.8), \text{ for all } i, j = 1, 2, 3, 4.$$

The framing effect reported in De Martino et al. (2006) varies between 6.1% for the "least rational" to 38.4% for the "most rational" participant (Fig. 3 (left panels)).

DS-FR2 For completeness, we decided to test our models for β on the data collected by Diederich et al. (2020), which is a bigger dataset than DS-FR1, both in terms of number of subjects and set of problems. The number of participants is 54, but we exclude data from one participant due to a high number of undefined responses. In the second experiment described in Diederich et al. participants had to choose between a gamble and a sure prospect, with the sure prospect presented in either a gain or a loss frame, similarly to the study by De Martino et al. (2006). Four initial amounts, flanked by ± 1 point amounts, and four probabilities of winning the gamble were selected and paired together to form 48 combinations:

$$Y_i = (19\,€, 20\,€, 21\,€, 39\,€, 40\,€, 41\,€,$$
$$59\,€, 60\,€, 61\,€, 79\,€, 80\,€, 81\,€),$$
$$\pi_j = (0.3, 0.4, 0.6, 0.7), \quad \text{for all } i, j = 1, 2, 3, 4$$

Remark 2. Diederich et al. (2020) tested the influence of different experimental conditions on the framing effect, i.e., 2 frames (gain, loss), 2 time limits ($1s$, $3s$), 3 needs (0, 2500, 3500), which we group in 7 cases. In this work, we present results only for the 2nd case, whose data is depicted in Fig. 5a. The 2nd case is the one with the least no. of datapoints corresponding to negative framing effect, i.e., the least no. of datapoints that are not consistent with PT and thus with the approach taken in this work. Indeed, according to the analysis presented in Section 2.3, we expect $P_{A_L} - P_{A_G} > 0$ for all π_j, $j = 1, 2, 3, 4$, since $\pi_j \in [0.3, 0.7] \subset (0.128, 0.872)$ for all $j = 1, 2, 3, 4$. That is, observation of a positive framing effect (for each participant) is consistent with PT (Section 3.2). However, from the experimental study, some subjects exhibit a nonpositive framing effect which is not consistent with PT, even for the case we have selected (Fig. 5a, datapoints highlighted in red). We expect these datapoints to be more complex to explain/predict by our approach, which points to a limitation that we will explore in future work.

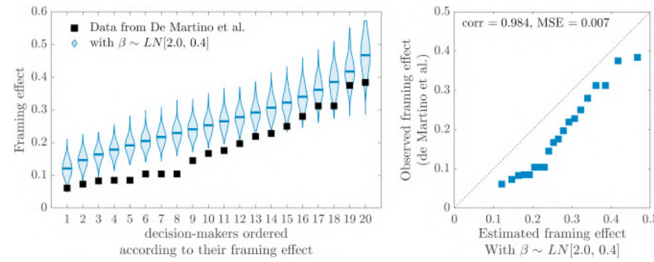
5.2 Testing the inferred models for the choice parameter

This section describes the findings obtained on the datasets DS-FR1 and DS-FR2.

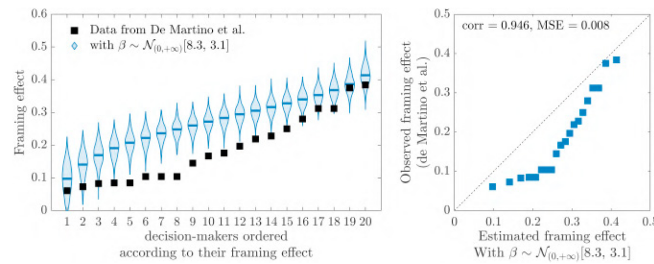
DS-FR1 Fig. 3 illustrates how the statistical models proposed in (7) reproduce the observed variability in framing effect, evaluated in terms of Pearson correlation coefficient and mean squared error (MSE). In both cases we obtain values of correlation $> 94\%$ and MSE < 0.01.

DS-FR2 Similar to DS-FR1, the statistical models for β in (7) reproduce the observed variability in the framing effect well (correlation $> 98\%$, MSE< 0.02, Fig. 5b).

To ensure a comprehensive analysis, we calibrate two



(a) $\beta \sim LN[2.0, 0.4]$



(b) $\beta \sim \mathcal{N}_{(0,+\infty)}[8.3, 3.1]$

Fig. 3. Testing the inferred models for the choice parameter β (7) on the dataset DS-FR1 (De Martino et al. (2006)). The left panels show the framing effect (observed and estimated) for each decision-maker. The right panels show observed vs estimated framing effect, and associated values of correlation and MSE.

additional distributions of the choice parameter (a log-normal and a truncated normal) on the data in DS-FR2, denoted by $\hat{\beta} \sim LN[\hat{\mu}, \hat{\sigma}]$ and $\hat{\beta} \sim \mathcal{N}_{(0,+\infty)}[\hat{\mu}, \hat{\sigma}]$. Then, we compare the goodness of fit obtained for $\hat{\beta}$ (calibrated on DS-FR2) and for β from (7) (calibrated on an independent set of data). To obtain $\hat{\beta}$, we partition the data in DS-FR2 into training and validation data (50%–50% partition). Training data is used to find the optimal parameters $(\hat{\mu}, \hat{\sigma})$ using mixed logit estimation (following Revelt and Train (1998)) and a 2-fold cross-validation procedure. Validation data is used to evaluate the predictive power of the calibrated statistical model $\hat{\beta}$, by computing goodness of fit. Goodness of fit is given in terms of correlation and MSE between observed and estimated framing effect across all decision-makers. The procedure is illustrated in Fig. 4. We obtain the following calibrated distributions:

$$\hat{\beta} \sim LN[1.1, 0.8], \quad \hat{\beta} \sim \mathcal{N}_{(0,+\infty)}[4.4, 4.1]. \tag{8}$$

The results along with goodness of fit of $\hat{\beta}$ from (8) and β from (7) are shown in Fig. 5c.

6. CONCLUSION

In this study, we proposed a novel interpretation of the weight parameter in the RUM for human choices under uncertainty. Specifically, by normalizing utilities in the RUM's choice function, we introduced a case-specific independent choice parameter, which we reframed in the context of human probability perception of unlikely events. To validate this reinterpretation, we first conducted a survey to experimentally infer statistical models explaining variability in probability perception across a population. Then, we tested these distributions on two distinct (and independent) datasets on the variability of the framing effect. Our approach showed strong predictive capabilities

(MSE< 0.02), thus demonstrating the efficacy of the proposed novel characterization of the choice parameter in RUM. Future works will include a meta-analysis of discrete choice experiments and sensitivity analysis to further validate the choice parameter and an extension of the experimental survey.

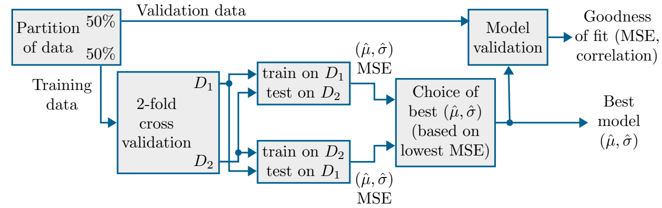
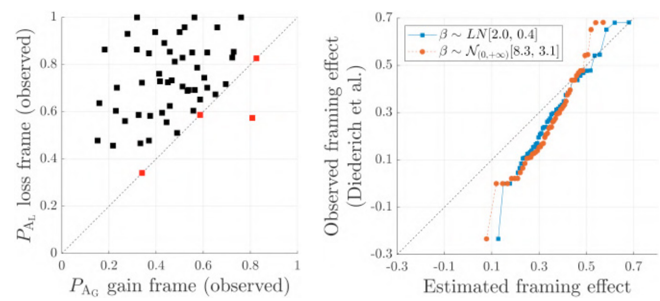
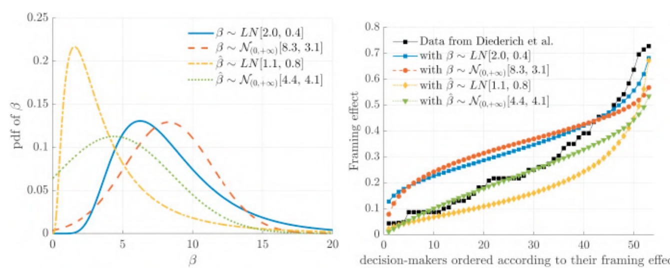


Fig. 4. Calibration of the choice parameter distribution on the dataset DS-FR2. This procedure is used twice, to calibrate a lognormal and a truncated normal distribution, i.e., $\hat{\beta} \sim LN[\hat{\mu}, \hat{\sigma}]$ and $\hat{\beta} \sim \mathcal{N}_{(0,+\infty)}[\hat{\mu}, \hat{\sigma}]$.



(a) (b)



pdf of β	mean	correlation	MSE
$\beta \sim LN[2.0, 0.4]$	8.38	0.989	0.009
$\beta \sim \mathcal{N}_{(0,+\infty)}[8.3, 3.1]$	8.31	0.941	0.013
$\hat{\beta} \sim LN[1.1, 0.8]$	4.42	0.980	0.011
$\hat{\beta} \sim \mathcal{N}_{(0,+\infty)}[4.4, 4.1]$	5.49	0.978	0.006

(c)

Fig. 5. Testing the statistical models for β (7) on the dataset DS-FR2. (a): Observed probabilities of risky choice in loss (P_{A_L}) and gain (P_{A_G}) frames for each decision-maker; the data are consistent with PT (see Section 3.2) if $P_{A_L} - P_{A_G} > 0$ (black datapoints above the dashed line). (b): Observed vs estimated framing effect using the inferred distributions for β (7). (c): Comparing inferred vs calibrated distributions of the choice parameter. The left panel shows the distributions, the right panel observed and estimated framing effect using validation data, and the table the corresponding values of correlation and MSE between observed and estimated framing effect.

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