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# Distributed Spectral Efficiency Maximization in Full-Duplex Cellular Networks 

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## Why full-duplex at the base station?



Half-Duplex


Full-Duplex

- Half-Duplex (HD) systems $\rightarrow$ Inefficient resource utilization
- Full-Duplex (FD) systems $\rightarrow \sim 2 \times$ spectral efficiency


## Outline

1. Introduction
2. System Model for Spectral Efficiency Maximization
3. Centralized Solution Based on Lagrangian Duality
4. Distributed Solution Based on Auction Theory
5. Numerical Results
6. Conclusions

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FD Characteristics in cellular networks


## Benefits

- Spectral efficiency: $\sim 2 \times$
- MAC layer: hidden terminal, collision avoidance, reduced end-to-end delay...

FD Characteristics in cellular networks


## Challenges

- Severe self-interference (SI)
- UE-to-UE interference
- User to frequency channels pairing and power allocation


## Research Gap in FD cellular networks

Need of distributed schemes

- Processing burden at the BS is high*
- Dense deployment of user
- New SI cancellation mechanisms
- Radio Resource Management

Lack of fair and efficient PHY procedures

- How to mitigate UE-to-UE interference and assess fairness?
- Pairing $\rightarrow$ UL and DL users to share the frequency resource
- Power allocation $\rightarrow$ mitigate interference
- Fairness $\rightarrow$ weighted sum spectral efficiency maximization

[^0]
## Contributions

- Study sum spectral efficiency maximization and fairness problem
- Joint pairing and power allocation $\rightarrow$ maximize weighted sum spectral efficiency


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- Lagrangian duality $\rightarrow$ optimal power allocation + optimal centralized assignment
- Provide distributed mechanisms for FD cellular networks
- Distributed auction algorithm $\rightarrow$ resource assignment to UL and DL users


## Contributions

- Study sum spectral efficiency maximization and fairness problem
- Joint pairing and power allocation $\rightarrow$ maximize weighted sum spectral efficiency
- Solve this MINLP problem
- Lagrangian duality $\rightarrow$ optimal power allocation + optimal centralized assignment
- Provide distributed mechanisms for FD cellular networks
- Distributed auction algorithm $\rightarrow$ resource assignment to UL and DL users
- Show spectral efficiency gains over HD with distributed schemes
- Realistic system simulations $\rightarrow$ Yes, $89 \%$ !


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## Definitions (1)



- Single-cell cellular system + only BS is FD-capable
- UL users $\rightarrow \mathrm{I}$; DL users $\rightarrow \mathrm{J}$; Frequency channels $\rightarrow \mathrm{F}$
- Effective path gain values $\rightarrow G_{i b}, G_{b j}, G_{i j}$
- SI cancellation coefficient $\rightarrow \beta$
- Assignment matrix $\rightarrow \mathbf{X} \in\{0,1\}^{I \times J}$

$$
x_{i j}= \begin{cases}1, & \text { if the } \mathrm{UL} \mathrm{UE}_{i} \text { is paired with the } \mathrm{DL} \mathrm{UE}_{j}, \\ 0, & \text { otherwise. }\end{cases}
$$

## Definitions (2)

- Power vectors $\rightarrow \quad \mathbf{p}^{\mathbf{u}}=\left[P_{1}^{u} \ldots P_{I}^{u}\right], \quad \mathbf{p}^{\mathbf{d}}=\left[P_{1}^{d} \ldots P_{J}^{d}\right]$
- SINR at the BS and at DL user

$$
\gamma_{i}^{u}=\frac{P_{i}^{u} G_{i b}}{\sigma^{2}+\sum_{j=1}^{J} x_{i j} P_{j}^{d} \beta}, \gamma_{j}^{d}=\frac{P_{j}^{d} G_{b j}}{\sigma^{2}+\sum_{i=1}^{I} x_{i j} P_{i}^{u} G_{i j}} .
$$

- Achievable spectral efficiency

$$
C_{i}^{u}=\log _{2}\left(1+\gamma_{i}^{u}\right), \quad C_{j}^{d}=\log _{2}\left(1+\gamma_{j}^{d}\right) .
$$

- Weights $\alpha_{i}^{u}, \alpha_{j}^{d}$
- $\alpha_{i}^{u}=\alpha_{j}^{d}=1, \forall i, j \rightarrow$ Sum spectral efficiency maximization
- $\alpha_{i}^{u}=G_{i b}^{-1}, \quad \alpha_{j}^{d}=G_{b j}^{-1} \rightarrow$ Path loss compensation


## Problem Formulation

- Weighted sum spectral efficiency maximization (P-OPT)

$$
\begin{aligned}
\underset{\mathbf{X}, \mathbf{p}^{u}, \mathbf{p}^{d}}{\operatorname{maximize}} & \sum_{i=1}^{I} \alpha_{i}^{u} C_{i}^{u}+\sum_{j=1}^{J} \alpha_{j}^{d} C_{j}^{d} \\
\text { subject to } & \gamma_{i}^{u} \geq \gamma_{\mathrm{th}}^{u}, \forall i, \\
& \gamma_{j}^{d} \geq \gamma_{\mathrm{th}}^{d}, \forall j, \\
& P_{i}^{u} \leq P_{\max }^{u}, \forall i, \\
& P_{j}^{d} \leq P_{\max }^{d}, \forall j, \\
& \sum_{i=1}^{I} x_{i j} \leq 1, \forall j, \\
& \sum_{j=1}^{J} x_{i j} \leq 1, \forall i, \\
& x_{i j} \in\{0,1\}, \forall i, j
\end{aligned}
$$

## Problem solution approaches



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## Lagrangian function

- Formulate partial Lagrangian function

$$
\begin{array}{r}
L\left(\boldsymbol{\lambda}^{u}, \boldsymbol{\lambda}^{d}, \mathbf{X}, \mathbf{p}^{u}, \mathbf{p}^{d}\right) \triangleq-\sum_{i=1}^{I} \alpha_{i}^{u} C_{i}^{u}-\sum_{j=1}^{J} \alpha_{j}^{d} C_{j}^{d}+ \\
+\sum_{i=1}^{I} \lambda_{i}^{u}\left(\gamma_{\mathrm{th}}^{u}-\gamma_{i}^{u}\right)+\sum_{j=1}^{J} \lambda_{j}^{d}\left(\gamma_{\mathrm{th}}^{d}-\gamma_{j}^{d}\right)
\end{array}
$$

- The dual function is

$$
g\left(\boldsymbol{\lambda}^{u}, \boldsymbol{\lambda}^{d}\right)=\inf _{\mathbf{x} \in \mathcal{X} \mathbf{p}^{u}, \mathbf{p}^{d} \in \mathcal{P}} L\left(\boldsymbol{\lambda}^{u}, \boldsymbol{\lambda}^{d}, \mathbf{X}, \mathbf{p}^{u}, \mathbf{p}^{d}\right)
$$

## Dual problem and closed-form solution for assignment

- Rewrite the dual as

$$
g\left(\boldsymbol{\lambda}^{u}, \boldsymbol{\lambda}^{d}\right)=\inf _{\mathbf{X} \in \mathcal{X} \mathbf{p}^{u}, \mathbf{p}^{d} \in \mathcal{P}} \sum_{n=1}^{N}\left(q_{i_{n}}^{u}\left(\mathbf{X}, \mathbf{p}^{u}, \mathbf{p}^{d}\right)+q_{j_{n}}^{d}\left(\mathbf{X}, \mathbf{p}^{u}, \mathbf{p}^{d}\right)\right),
$$

with

$$
\begin{aligned}
& q_{i_{n}}^{u}\left(\mathbf{X}, \mathbf{p}^{u}, \mathbf{p}^{d}\right) \triangleq \lambda_{i_{n}}^{u}\left(\gamma_{\mathrm{th}}^{u}-\gamma_{i_{n}}^{u}\right)-\alpha_{i}^{u} C_{i_{n}}^{u}, \\
& q_{j_{n}}^{d}\left(\mathbf{X}, \mathbf{p}^{u}, \mathbf{p}^{d}\right) \triangleq \lambda_{j_{n}}^{d}\left(\gamma_{\mathrm{th}}^{d}-\gamma_{j_{n}}^{d}\right)-\alpha_{j_{n}}^{d} C_{j_{n}}^{d} .
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& q_{j_{n}}^{d}\left(\mathbf{X}, \mathbf{p}^{u}, \mathbf{p}^{d}\right) \triangleq \lambda_{j_{n}}^{d}\left(\gamma_{\mathrm{th}}^{d}-\gamma_{j_{n}}^{d}\right)-\alpha_{j_{n}}^{d} C_{j_{n}}^{d} .
\end{aligned}
$$

- Closed-form expression for the assignment

$$
x_{i j}^{\star}= \begin{cases}1, & \text { if }(i, j)=\underset{i, j}{\arg \max }\left(q_{i_{n}}^{u, \max }+q_{j_{n}}^{d, \max }\right) \\ 0, & \text { otherwise }\end{cases}
$$

## Dual problem and optimal power allocation

- Analyse the dual problem

$$
\begin{array}{ll}
\underset{\boldsymbol{\lambda}^{u}, \boldsymbol{\lambda}^{u}}{\operatorname{maximize}} & g\left(\boldsymbol{\lambda}^{u}, \boldsymbol{\lambda}^{d}\right) \\
\text { subject to } & \lambda_{i}^{u}, \lambda_{j}^{d}, \geq 0, \forall i, j,
\end{array}
$$

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\text { subject to } & \lambda_{i}^{u}, \lambda_{j}^{d}, \geq 0, \forall i, j,
\end{array}
$$

- Turn our attention to the power allocation problem

$$
\begin{array}{ll}
\underset{\mathbf{p}^{u}, \mathbf{p}^{d}}{\operatorname{minimize}} & -\sum_{i=1}^{I} \alpha_{i}^{u} C_{i}^{u}-\sum_{j=1}^{J} \alpha_{j}^{d} C_{j}^{d} \\
\text { subject to } & \mathbf{p}^{u}, \mathbf{p}^{d} \in \mathcal{P} . \tag{1b}
\end{array}
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\text { subject to } & \mathbf{p}^{u}, \mathbf{p}^{d} \in \mathcal{P} . \tag{1b}
\end{array}
$$

- Optimal solution for (1) available*†
*A. Gjendemsj $\varnothing$, D. Gesbert, G. E. Øien and S. G. Kiani, "Binary Power Control for Sum Rate Maximization over Multiple Interfering Links," IEEE TWC, vol. 7, no. 8, pp. 3164-3173, August 2008
${ }^{\dagger}$ D. Feng, L. Lu, Y. Yuan-Wu, G. Y. Li, G. Feng and S. Li, "Device-to-Device Communications Underlaying Cellular Networks," IEEE TC, vol. 61, no. 8, pp. 3541-3551, August 2013


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## Problem Reformulation

- Reformulate closed-form assignment with optimal power allocation as

$$
\begin{aligned}
\underset{\mathbf{X}}{\operatorname{maximize}} & \sum_{i=1}^{I} \sum_{j=1}^{J} c_{i j} x_{i j} \\
\text { subject to } & \sum_{i=1}^{I} x_{i j}=1, \forall j, \\
& \sum_{j=1}^{J} x_{i j}=1, \forall i, \\
& x_{i j} \in\{0,1\}, \forall i, j .
\end{aligned}
$$

- Centralized solution $\rightarrow$ Hungarian Algorithm


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\end{aligned}
$$

- Centralized solution $\rightarrow$ Hungarian Algorithm
- Distributed solution $\rightarrow$ Auction Theory


## Distributed Auction

- Input: $c_{i j}$, and tolerance $\epsilon$


## Bidding Phase

- UL bids for a DL user that maximizes $c_{i j}-\hat{p}_{j}$
- Wait for acknowledgement on assignment or update price


## Assignment Phase

- BS is responsible for DL users
- BS selects the highest bid and update the prices $\hat{p}_{j}$
- Send updates and wait until the assignment matrix $X$ is feasible
- Messages exchanged using control channels, e.g., PUCCH or PDCCH


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## Simulation Parameters

- $F=I=J$ and $I+J=8, \ldots, 50$
- Path-loss compensation $\rightarrow \alpha_{i}^{u}=G_{i b}^{-1}, \quad \alpha_{j}^{d}=G_{b j}^{-1}$
- SI cancellation $\beta=[-70,-100] \mathrm{dB}$
- Proposed algorithm
- D-AUC: Dual solution with distributed Auction compared to
- P-OPT: Primal optimal from brute-force solution
- C-HUN: Centralized solution based on Duality and Hungarian algorithm
- R-EPA: Random assignment + equal power allocation
- HD: Traditional Half-Duplex scheme


## Optimality Gap Comparison with $\beta=-100 \mathrm{~dB}$



- Negligible difference between P-OPT, C-HUN and D-AUC


## Optimality Gap Comparison with $\beta=-100 \mathrm{~dB}$



- Negligible difference between P-OPT, C-HUN and D-AUC
- Possible to use distributed solutions without losing too much


## Sum Spectral Efficiency Comparison for different $\beta$



- $\beta=-110 \mathrm{~dB} \rightarrow$ UE-to-UE interference is the limiting factor
- $\beta=-70 \mathrm{~dB} \rightarrow$ residual SI is the limiting factor


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- Perform close to centralized schemes


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## Take home message

- Distributed algorithms to FD cellular networks
- Perform close to centralized schemes
- Fair and efficient
- Almost double spectral efficiency ( $89 \%$ gain)
- Trade-off: residual SI $\times$ UE-to-UE interference
- UE-to-UE interference is the limiting factor for low $\beta$
- Residual SI is the limiting factor for high $\beta$

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[^0]:    ${ }^{\text {* }}$ A. Osseiran et al., "Scenarios for 5 G mobile and wireless communications: the vision of the METIS project," IEEE Communications Magazine, vol. 52, no. 5, pp. 26-35, May 2014.

