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Spectral Efficiency and Fairness Maximization in Full-Duplex Cellular Networks

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Abstract

Future cellular networks, the so-called 5G, are expected to provide explosive data volumes and data rates. To meet such a demand, the research communities are investigating new wireless transmission technologies. One of the most promising candidates is in-band full-duplex communications. These communications are characterized by that a wireless device can simultaneously transmit and receive on the same frequency channel. In-band full-duplex communications have the potential to double the spectral efficiency when compared to current half duplex systems. The traditional drawback of full-duplex was the interference that leaks from the own transmitter to its own receiver, the so-called self-interference, which renders the receiving signal unsuitable for communication. However, recent advances in self-interference suppression techniques have provided high cancellation and reduced the self-interference to noise floor levels, which shows full-duplex is becoming a realistic technology component of advanced wireless systems.

Although in-band full-duplex promises to double the data rate of existing wireless technologies, its deployment in cellular networks is challenging due to the large number of legacy devices working in half-duplex. A viable introduction in cellular networks is offered by three-node full-duplex deployments, in which only the base stations are full-duplex, whereas the user- or end-devices remain half-duplex. However, in addition to the inherent self-interference, now the interference between users, the user-to-user interference, may become the performance bottleneck, especially as the capability to suppress self-interference improves. Due to this new interference situation, user pairing and frequency channel assignment become of paramount importance, because both mechanisms can help to mitigate the user-to-user interference. It is essential to understand the trade-offs in the performance of full-duplex cellular networks, specially three-node full-duplex, in the design of spectral and energy efficient as well as fair mechanisms.

This thesis investigates the design of spectral efficient and fair mechanisms to improve the performance of full-duplex in cellular networks. The novel analysis proposed in this thesis suggests centralized and distributed user pairing, frequency channel assignment and power allocation solutions to maximize the spectral efficiency and fairness in future full-duplex cellular networks. The investigations are based on distributed optimization theory with mixed integer-real variables and novel extensions of Fast-Lipschitz optimization. The analysis sheds lights on two fundamental problems of standard cellular networks, namely the spectral efficiency and fairness maximization, but in the new context of full-duplex communications. The results in this thesis provide important understanding in the role of user pairing, frequency assignment and power allocation, and reveal the special behaviour between the legacy self-interference and the new user-to-user interference. This thesis can provide input to the standardization process of full-duplex communications, and have the potential to be used in the implementation of future full-duplex in cellular networks.

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José Mairton B. da Silva Jr.
Stockholm, April 2017

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List of Acronyms

3-DAP	3-Dimensional Assignment Problem
3GPP	3rd Generation Partnership Project
5G	5th Generation
BFD	Bidirectional Full-Duplex
BS	Base Station
CDF	Cumulative Distribution Function
D2D	Device-to-Device
DL	Downlink
eMBB	Enhanced Mobile BroadBand
EPA	Equal Power Allocation
FD	Full-Duplex
FDD	Frequency Division Duplex
FL	Fast-Lipschitz
HD	Half-Duplex
JAFM	Joint Assignment and Fairness Maximization
JAFMA	Joint Assignment and Fairness Maximization Algorithm
JASEM	Joint Assignment and Spectral Efficiency Maximization
LOS	Line of Sight
LP	Linear Programming
LTE	Long Term Evolution
MAC	Medium Access Control

MIMO	Multiple Input Multiple Output
MINLP	Mixed Integer Nonlinear Programming
mMTC	Massive Machine Type Communications
NLOS	Non-Line of Sight
NP	Non-Deterministic Polynomial-Time
PC	Power Control
PDCCH	Power Downlink Control Channel
PUCCH	Power Uplink Control Channel
QoS	Quality of Service
RF	Radio-Frequency
RFD	Relaying Full-Duplex
SE	Spectral Efficiency
SINR	Signal-to-Interference-Plus-Noise Ratio
SI	Self-Interference
SISO	Single Input Single Output
SNR	Signal-to-Noise Ratio
SWIPT	Simultaneous Wireless Information and Power Transfer
TDD	Time Division Duplex
TNFD	Three-Node Full-Duplex
UE	User Equipment
UL	Uplink
URLLC	Ultra-Reliable Low Latency Communications

Part I

Thesis Overview

Chapter 1

Introduction

Current wireless communication systems are witnessing an explosive increase in data traffic [1]. For the incoming 5^{th} generation (5G) of these systems, the international telecommunication union radiocommunication sector expects peaks of data rates in the order of tens of $\mathrm{Gbit/s}$ [2]. To meet these demands, wireless network operators seek to enhance the spectral efficiency – the rate of information a system delivers over a given bandwidth– in lower-frequency bands [1], and to exploit higher-frequency bands such as the millimiter waves (mmWave). Figure 1.1 shows the three main use cases envisioned for 5G [3], enhanced mobile broadband, ultra-reliable low latency communications, and massive machine type communications. The enhanced mobile broadband expresses the extended support of conventional mobile broadband services through improved peak/average/cell-edge data rates, capacity and coverage. ultra-reliable low latency communications represents the requirement for network services with extreme demand on availability, latency and reliability. massive machine type communications is necessary to support the envisioned 5G scenarios with tens of billions of network-enable devices [4]. The research and standardization communities are currently studying physical layer technologies, including massive multiple input multiple output (MIMO) systems, spectrum sharing in mmWave networks, new waveforms, non-orthogonal multiple access technologies, and full-duplex communications [3, 5–7]. 5G networks will also have to feature large numbers of close-by nodes wanting to exchange data [8], and one relevant technological component to meet these requirements is full-duplex (FD) communications [9], as shown in Figure 1.1.

Traditional cellular networks operate in half-duplex (HD) transmission mode, in which a user equipment (UE) or the base station (BS) either transmits or receives on given frequency channels. Recently, in-band FD has been proposed as a key enabling technology to increase the spectral efficiency of conventional wireless transmission modes. Full-Duplex communications overcome the assumption that it is not possible for radios to transmit and receive simultaneously on the same time-frequency resource. In-band FD transceivers are expected to improve the attainable spectral efficiency of traditional wireless networks operating with HD transceivers by a factor close to two [6, 10]. In addition to the spectral efficiency gains, full-duplex can provide gains in the medium access

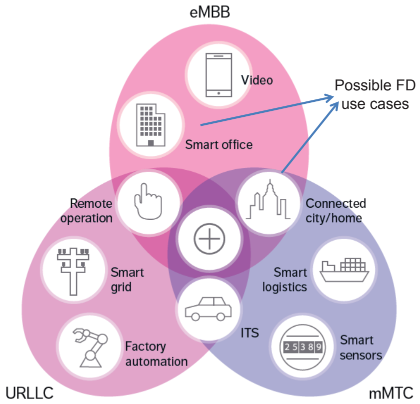


Figure 1.1: *The three main use cases in 5G, enhanced mobile broadband (eMBB), ultra-reliable low latency communications (URLLC), and massive machine type communications (mMTC) [3]. Notice that possible candidates for the application of FD communications could be in smart offices (small cells), and connected city/home (use of relays to improve coverage).*

control layer, on which problems such as the hidden/exposed nodes and collision detection can be mitigated [11–13].

Until recently, in-band FD was not considered as a solution for wireless networks due to the inherent interference created from the transmitter to its own receiver, the so called self-interference (SI). However, recent advancements in mitigating SI have been successful [14–20]. To cancel the SI created by full-duplex communications, we need to rely on different techniques that can be classified as passive and active suppression [13]. The first step is to consider passive suppression of the SI using antenna techniques and after that active SI cancellation schemes with analog and digital cancellation to suppress the residual SI. Overall, the recent advances imply that FD is becoming a realistic technology component of advanced wireless – including cellular – systems, especially in the low transmit power regime [17, 18].

Although in-band FD promises to double the data capacity of existing technology, its deployment in wireless local area and cellular networks is challenging due to the large number of legacy devices and wireless access points. A viable introduction of FD technology in cellular networks is offered by three-node full-duplex (TNFD) deployments, in which only the wireless access points or BSs, typically equipped with multiple antennas, implement FD transceivers to support the simultaneous downlink (DL) and uplink (UL) communication with two distinct UEs on the same frequency channel [12]. In TNFD networks, in addition to the inherently present SI, the users experience the UE-to-UE interference, which may become the performance bottleneck, especially as the capability of FD transceivers to suppress SI improves. It is crucial to understand the trade-offs between UL and DL performance of full-duplex in cellular systems, specially TNFD, in the design

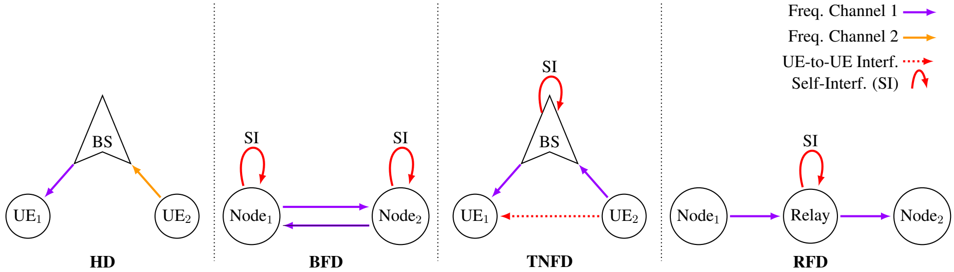


Figure 1.2: *We can divide in-band FD schemes in three configurations, bidirectional full-duplex, three-node full-duplex, relaying full-duplex, and the additional HD mode. Some configurations are more suitable to cellular networks with the BS as access points, whereas others are also suited for general wireless networks.*

of efficient and fair medium access control protocols, and also coordination mechanisms that help to realize the FD potential even for legacy UEs.

1.1 Background

In this section we overview some fundamental aspects of FD communications, including its history and recent developments in SI cancellation, cellular networks, and distributed solutions for power control and assignment of users.

1.1.1 Full-Duplex Communications

In-band FD in wireless networks is not recent, the concept of transmission and reception in the same frequency channel has been used since 1940 in radar systems [11]. The SI was already a key challenge on which the first circuits to null the SI were proposed and provided low levels of cancellation. Such mild SI cancellation levels limited transmit power, which provided a reduced range of detectable targets, which is desirable in radar systems. In the last decade, wireless broadband systems, such as WiFi and cellular, started to take into account in-band FD communications. To the best of our knowledge, the first application of FD in such context considers it for relaying purposes, where the relay could be used to improve the sum rate, area coverage or in areas where it is prohibitive to implement an operational BS [11, 21].

With respect to the transmission configurations, we can categorize the in-band FD in three: bidirectional full-duplex (BFD), TNFD, and relaying full-duplex (RFD) [12, 22]. Figure 1.2 shows the standard half-duplex system and the aforementioned configurations, as well as the interference scenarios they face. In BFD, two FD-capable nodes (either a UE or the BS) transmit and receive on the same time-frequency resource, which creates SI for both nodes. In contrast, TNFD involves three nodes, but only one requires FD capability. The FD-capable node transmits to its receiver node while receiving from another transmitter node on the same frequency channel, in which SI is present only at the FD-capable node. In cellular networks, the BS is FD-capable and transmits in the DL and receives in the UL from the HD UEs (Figure 1.2). In RFD, one node transmits to a FD-capable relay and then retransmits the signal to the second node, where all transmissions occur in the same time-frequency. Since the relay is the only FD-capable node, SI is present only at the relay.

These new configurations extend the design options and allow for a higher spectral usage of the already available frequency resource. For example, in BFD the nodes can ideally double the spectral efficiency, provided that a good SI cancellation is performed at each node. The same idea applies to the other configurations, but now with a broader application range. Figure 1.3 shows some FD applications in which all three configurations have already been studied or envisioned, where the blue color represents BFD, the green color represents TNFD, and the purple color the RFD. Notice that FD has already a broad range of applications, ranging from device-to-device (D2D) communications, going to simultaneous wireless information and power transfer (SWIPT) networks, and also reaching mmWaves, which show an evolution and broad application range of the technology. For some application areas, more than one configuration can be applied at the same time, which is the case of MIMO – and massive MIMO –, SWIPT, and cognitive radio. However, the common drawback of FD in all the technologies and applications in Figure 1.3 remains the SI, which highlights that SI cancellation is crucial.

1.1.2 Self-Interference Cancellation for Full-Duplex Communications

The driving concept of FD communication is to allow simultaneous transmission and reception for a node, and for this to happen, the FD nodes require a transmit and receive radio-frequency (RF) chain. To separate these two RF chains, there are two methods: separated and shared antennas [22]. The first method consists of separating physically the RF chains, i.e., when the number of antennas is greater than 2, use a part of the set of antennas to transmit and the other part to receive. In this situation, there is a loss in the degree of freedom in the spatial domain. The second method shares all the antennas in the RF chains, and to isolate the receiver from the transmitter, a special duplexer named circulator is used. The circulator idea comes from radar technology [11, 22]; it is a three-port device that prevents the transmitted signal from one RF chain to leak into the receiver signal of the receiver RF chain. With shared antennas, it is possible to have a single antenna to transmit and receive, which is advantageous in single input single output (SISO) systems.

Irrespectively of the methods to separate the RF chains, the simultaneous transmission and reception in the same frequency resource makes the transmitted signal to loop back to the receiving antenna, and this leaked signal is the SI. Depending on the transmitter power, the SI needs to be cancelled by more than $100\,\mathrm{dB}$ to reduce its value to the noise floor [11, 16], which is an extremely high level to be cancelled. To have a better understanding of the impact of SI in the system, we need to cancel a signal that is approximately 1 trillion times higher than the received signal [18]. To cancel SI, we need to rely on two steps that can be classified as *passive* and *active* suppression [13].

Usually, the first step is *passive* suppression of the SI using the antenna techniques

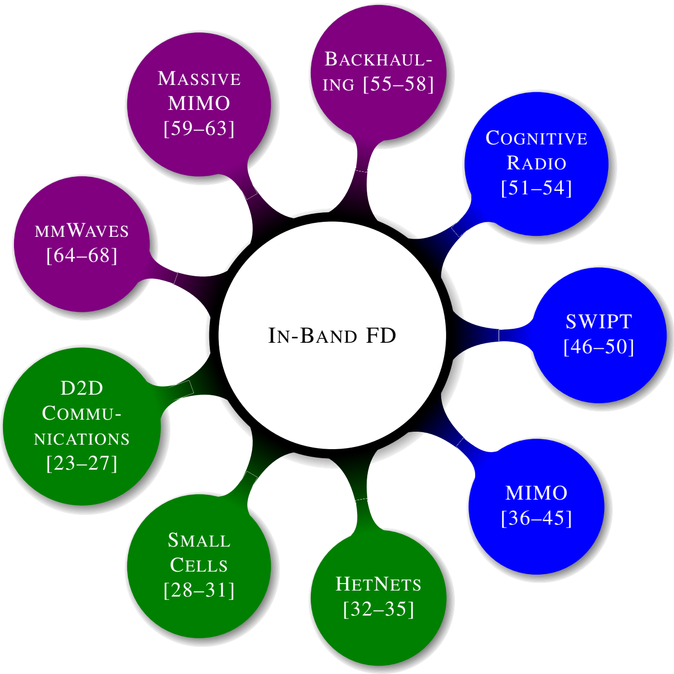


Figure 1.3: *A range of areas in which FD has been envisioned, irrespective of the configuration in Figure 1.2. We show in blue some applications for BFD, in green for TNFD, and in purple for RFD. In some cases, more than one configuration can be applied at the same time, such as D2D communications, MIMO, and SWIPT.*

mentioned above, and then *active* SI cancellation schemes with analog and/or digital cancellation to suppress the residual SI. Besides the physical separation between the transmitting and receiving antenna, passive suppression can also be accomplished by setting a directional beam towards the receiving antenna and a lobe to the transmitting antenna, which is called *directional SI suppression* [69–71]. The passive suppression is suggested to be used before the SI enters the receive RF chain circuit [11, 72, 73], and it was shown experimentally that up to 65 dB of SI can be suppressed with omni-directional antennas [73].

On the other hand, active suppression is subdivided into analog and digital cancellation techniques. Analog cancellation is usually the first step in the active suppresion, and it is performed before the signal enters the analog-to-digital converter. Usually, analog cancellation subtracts an estimate of the SI signal after performing a passive suppression [22], but it can also be used to tap the transmit signal into the digital domain (to apply adjustments digitally), and then convert it back to the analog domain to cancel the self-interference [11, 14, 15, 73, 74]. To further reduce the SI, passive suppression is also used along with analog cancellation, and some authors report up to $80\,\mathrm{dB}$ using both techniques [75].

To further reduce SI, digital cancellation is performed, where an estimate of the SI signal is treated in the digital domain. The goal of this cancellation is to mitigate any residual self-interference signal. Since it is easier to perform heavy calculations in the digital domain, the digital cancellation can model linear and nonlinear distortions of the signal, and then cancel these components in the main signal. Digital cancellation can also be used along with passive suppression and analog cancellation to further suppress the SI, but it was noted that digital cancellation should not be used when analog cancellation performs well [15]. In Duarte et al. [15] the authors show that using separated antennas along with analog and digital cancellation, the system is able to suppress 74 dB of the SI. The best SI suppression appears to have been achieved by Bharadia et al. [16], where 110 dB of SI suppression was obtained with analog and digital cancellation in Wi-Fi networks with bandwidth of 20 MHz. According to the authors, such performance can also achieved in current long term evolution (LTE) systems regardless of the frequency band. For a more detailed analysis of all cancellation methods, [6, 22] provide tables with different methods of passive and active suppression, as well as advantages and disadvantages of each method. More recently, SI suppression has achieved high levels also for mobile devices [20, 68], FD MIMO relays [76], and also in mmWaves [68]. As an example, researchers from Stanford University have founded a startup company, Kumu Networks, to develop practical full-duplex radios [77].

With these high levels of SI cancellation circuits, many works assume that either the SI is fully cancelled [26, 78], or some residual value is left [15, 23, 28, 79, 80]. Provided that some residual value is left, some authors consider three different types of residual SI: fixed value independent of the transmitter power [79], fixed value dependent of the transmitter power in a linear function [28, 80], and a random variable following a Rician distribution dependent on the transmitter power in a linear function [15, 23]. Throughout this thesis, we consider that the residual SI is fixed and depends on the transmitter power in a linear manner.

We notice that the historical drawback of FD, the SI interference, remains important and needs to be mitigated, but the recent developments show the maturity and feasibility of FD communications in practical scenarios.

1.1.3 Full-Duplex Applications in Cellular Networks

With the recent research in SI cancellation, and considering the different in-band FD configurations in Figure 1.2, a viable introduction of FD technology in cellular networks is offered by TNFD. This configuration requires that, at least the wireless access points or BSs, implement FD transceivers to support the simultaneous DL and UL communication with two distinct UEs on the same frequency channel [12]. Although recent research have pointed to practical FD mobile users [20, 68], its implementation in modern phones and standards has still a long road to travel. Due to this reason, the most common assumption in TNFD cellular networks is that only the BS is full-duplex capable, whereas the majority of users remain HD.

As shown in Figure 1.2, TNFD also suffers from the UE-to-UE interference, which

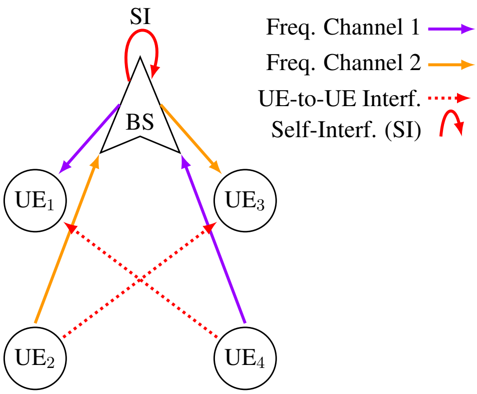


Figure 1.4: *An example of TNFD in a cellular network with 4 users. Notice that it is advantageous to pair users far apart, such as* UE_1*-*UE_4 *and* UE_2*-*UE_3*, and then assign these pairs the same frequency channel.*

is present because the UL user is transmitting in the same frequency resource as the one the DL user is receiving from the BS. The UE-to-UE interference depends on the users location and propagation effects, but also on the transmit powers of UL users. In addition, in cellular SISO networks there are inherent constraints of orthogonality within the same transmission direction. That is, every UL user – as well as the BS in the DL – must transmit in a different frequency channel. To cope with this orthogonality, another challenge appears, which is how to *pair* UL and DL such that the assignment of both sets of users to frequency channels maximize the desired performance indicator, e.g., the spectral efficiency. Figure 1.4 highlights a situation with four users and two frequency channels, where the pairing and assignment of users to frequency channels needs to be carefully performed. To deal with these new challenges, coordination mechanisms that take into account *pairing*, *frequency assignment* – to which both are usually named assignment– and *power allocation* are crucial for FD cellular networks.

1.1.4 Power Allocation and Assignment for Full-Duplex Cellular Networks

Coordination mechanisms – such as power allocation and assignment – are important to reduce the impacts of the different sources of interference in TNFD cellular networks, and also to optimize a desired performance indicator. Typical and natural objectives for many physical layer procedures for FD cellular networks are maximizing the sum spectral efficiency and fairness. In order to achieve the two main goals, the coordination mechanisms can use power allocation [81–83], assignment [28,84,85] or a joint mechanism that takes both into account [29, 32, 79, 86–88]. The main motivation for considering joint aspects is to further improve the desired performance indicator.

Most of the works in FD cellular networks aim to maximize the spectral efficiency and analyse the theoretical doubling that FD provides. In [81, 82], the authors analyse the rate gain region of TNFD and BFD considering only power allocation, whereas in [83] the authors analyse power allocation only in the UL. On the other hand, scheduling was also used to improve the spectral efficiency [28] in a cellular environment, as well as the pairing UL and DL of users [84, 85]. Some works also took into account both aspects, power allocation and assignment, to maximize the spectral efficiency. In [32], the authors use this joint approach in a heterogeneous network, whereas [29, 79, 86–88] have a similar approach but towards small cellular networks. However, some of the above works tackle the assignment problem from a subcarrier or scheduling perspective, make simplifications to the model, or provide heuristic solutions.

Another important objective is to improve the fairness and per-user quality of service (QoS) of FD cellular networks, but little has been done as emphasized in [12,80]. The work in [12] emphasizes the importance of fairness and that it may degrade by a factor of two compared with HD communications. However, the authors do not provide power allocation and assignment schemes that are developed with such objectives in mind. In [80], a QoS provisioning framework within bidirectional FD configuration is proposed, but without considering the implications of TNFD transmissions. We notice here a research gap that has not been tackled, and which may have great impact in the application of FD in cellular networks.

1.1.5 Distributed Algorithms for Full-Duplex Cellular Networks

In future cellular networks, there is a need to move from a fully centralized architecture towards a more decentralized architecture [8]. The objective is to use the infrastructure of the BS to help the UEs to communicate in a distributed manner, reducing the processing burden at the BS and the latency. In TNFD, the burden of the BS is further increased by the SI cancellation circuits, which increases the need of distributed solutions. Although the TNFD configuration remains centralized, the functions – such as power allocation and assignment– may be distributed. Few works have developed distributed algorithms that are applicable in TNFD networks [89, 90]. The authors of [89] have tackled the problem of the UE-to-UE interference from an information theoretic perspective, without relating to resource allocation and power control. In [90], the authors have proposed a distributed power control for general wireless networks using approximation techniques, but they have not taken into account the specific aspects of TNFD in cellular networks. Therefore, there is a need for distributed solutions in TNFD for cellular networks.

1.2 Problem Formulation

In this thesis we take into account a joint formulation of power allocation and assignment – pairing and/or frequency assignment– in order to maximize the performance indicator of interest for TNFD in cellular networks. We can pose this objective in a joint formulation of a mixed integer nonlinear programming (MINLP) problem as

$$\begin{aligned} \underset{\mathbf{X},\mathbf{p}^u,\mathbf{p}^d}{\text{maximize}} \quad & f_0(\mathbf{X},\mathbf{p}^u,\mathbf{p}^d) && \text{(1.1a)} \\ \text{subject to} \quad & \mathbf{f}_i(\mathbf{X},\mathbf{p}^u,\mathbf{p}^d) \leq \mathbf{b}_i,\ \forall i \in \mathcal{I}, && \text{(1.1b)} \\ & \mathbf{h}_j(\mathbf{X}) \leq \mathbf{c}_j,\ \forall j \in \mathcal{J}, && \text{(1.1c)} \\ & \mathbf{X} \in \{0,1\}^S. && \text{(1.1d)} \end{aligned}$$

The main optimization variables are $\mathbf{p}^u$, $\mathbf{p}^d$ and $\mathbf{X}$, where $\mathbf{p}^u$, $\mathbf{p}^d$ represent the transmit power vectors for I UL and J DL users, and $\mathbf{X}$ is the assignment matrix. Notice that the assignment matrix may vary its size S depending on the fading the environment is experiencing, where for frequency selective fading $\mathbf{X}$ has three dimensions, the first two for UL and DL users and the third for the frequency channel. For block fading environments, $\mathbf{X}$ has only the dimensions for UL and DL users. The objective function (1.1a) of the problem depends on the three variables, and it is nonconvex for the applications in this thesis. Constraint (1.1b) represent a series of inequality constraints that depend on either two or three optimization variables. The inequality function vector $\mathbf{f}_i(\mathbf{X},\mathbf{p}^u,\mathbf{p}^d)$ is usually nonconvex if it takes into account all three variables, but it is convex if it considers only the transmit power vectors $\mathbf{p}^u$, $\mathbf{p}^d$. This constraint may represent minimum QoS requirement per UL and DL user, as well as maximum transmitting power for UL users and the BS. The other constraint function vector in Eq. (1.1c) represents the binary inequalities, and embed the orthogonality between UL/DL users and frequency channels. This constraint may represent that a UL user can be associated to only one DL user and frequency channel, and it is applied for all users and frequency channels. The last constraint (1.1d) requires the assignment matrix to be binary. In addition, the set of constraints $\mathcal{I}$ and $\mathcal{J}$ are complementary, the inequality function vectors $\mathbf{f}_i(\mathbf{X},\mathbf{p}^u,\mathbf{p}^d)$ and $\mathbf{h}_j(\mathbf{X})$ represent different functions, and all inequalities in Eqs. (1.1b)-(1.1d) are component-wise. Optimization problem (1.1) is difficult to solve because it has binary and real variables intertwined in a nonconvex problem. In fact, we show that for some applications the problem is non-deterministic polynomial-time (NP)-hard, i.e., that no polynomial time solution – in the sense of optimality – for the problem is known. With this in mind, we will use different optimization techniques to provide an approximated or close-to-optimal solution, either centralized or distributed, for the maximization of two practical objectives: spectral efficiency and fairness.

1.2.1 Spectral Efficiency Maximization

One of the main promises of FD is to theoretically double the spectral efficiency by the transmission and reception in the same frequency channel. With this, one of the most important performance objectives is the spectral efficiency. In the sequel, we pose a spectral maximization problem that jointly takes into account frequency assignment of UEs in the UL and DL to frequency channels and the transmitting powers of UL users and the BS.

To this end, we need to first define some parameters. Let the number of UEs in the UL and DL be denoted by I and J, respectively, which are constrained by the total number of frequency channels in the system F, i.e., $I \leq F$ and $J \leq F$. Let G_{ib} denote the effective path gain between transmitter UE i and the BS, G_{bj} denote the effective path gain between the BS and the receiving UE j, and G_{ij} denote the interfering path gain between the UL transmitter UE i and the DL receiver UE j, and β as the SI cancellation coefficient. The vector of transmit power levels in the UL by UE i is denoted by $\mathbf{p^u} = [P_1^u \dots P_I^u]$, whereas the DL transmit powers by the BS is denoted by $\mathbf{p^d} = [P_1^d \dots P_J^d]$. Accordingly, we define the assignment matrix, $\mathbf{X} \in \{0,1\}^{I \times J \times F}$, such that $x_{ij}f = 1$ if the UL UE$_i$ is paired with the DL UE$_j$ and assigned to frequency channel f, and $x_{ij} = 0$ otherwise. The signal-to-interference-plus-noise ratio (SINR) at the BS of transmitting user i and the SINR at the receiving user j of the BS are given by

$$\gamma_i^u = \frac{P_i^u G_{ib}}{\sigma^2 + \sum_{j=1}^{J} x_{ij} P_j^d \beta}, \; \gamma_j^d = \frac{P_j^d G_{bj}}{\sigma^2 + \sum_{i=1}^{I} x_{ij} P_i^u G_{ij}}. \tag{1.2}$$

The achievable spectral efficiency for each user is given by the Shannon equation (in bits/s/Hz) for the UL and DL as $C_i^u = \log_2(1 + \gamma_i^u)$ and $C_j^d = \log_2(1 + \gamma_j^d)$, respectively.

Our goal is to devise the pairing and assignment of UEs in the UL and DL to frequency channels, that maximize the sum spectral efficiency over all users. The problem can be formulated as

$$\begin{aligned}
\underset{\mathbf{X}, \mathbf{p}^u, \mathbf{p}^d}{\text{maximize}} \quad & \sum_{i=1}^{I} C_i^u + \sum_{j=1}^{J} C_j^d && (1.3a)\\
\text{subject to} \quad & P_i^u \leq P_{\max}^u, \; \forall i, && (1.3b)\\
& P_j^d \leq P_{\max}^d, \; \forall j, && (1.3c)\\
& \sum_{j=1}^{J} \sum_{f=1}^{F} x_{ijf} \leq 1, \; \forall i, && (1.3d)\\
& \sum_{i=1}^{I} \sum_{f=1}^{F} x_{ijf} \leq 1, \; \forall j, && (1.3e)\\
& \sum_{i=1}^{I} \sum_{j=1}^{J} x_{ijf} \leq 1, \; \forall f, && (1.3f)\\
& x_{ijf} \in \{0,1\}, \; \forall i, j, f. && (1.3g)
\end{aligned}$$

The main optimization variables are $\mathbf{p}^u$, $\mathbf{p}^d$ and $\mathbf{X}$. Constraints (1.3b) and (1.3c) limit the transmit powers per-user and per-channel DL power constraint, whereas constraints (1.3d)-(1.3f) assure that at most one UE in the DL can share the frequency resource with a UE in the UL and vice-versa. Problem (1.3) belongs to the category of MINLP, and in addition, problem (1.3) belongs to the category of 3-D nonlinear assignment problems. We may also have slightly different problem formulations in which we require a minimum SINR for UL and DL users, and also use weights in the spectral efficiency for the UL and DL users. In this thesis, we provide a close-to-optimal solution with distributed power control for problem (1.3), and for a slightly modified problem with reduced complexity, we provide an approximate distributed solution.

1.2.2 Fairness Maximization

Provided that we have an operational usage of FD in cellular networks, we need to ensure some level of fairness for all the users in the system. However, fairness can be defined in many ways [91], such as a minimum QoS guarantee for all users, a high average spectral efficiency for all users, or we can guarantee a high spectral efficiency for the user with minimum spectral efficiency.

In this thesis, fairness maximization is understood as the maximization of the minimum spectral efficiency of all users, i.e., max-min spectral efficiency. To this end, we propose a joint frequency assignment of UEs in the UL and DL to frequency channels and power allocation to increase the fairness of the system. We formulate the problem as

$$
\begin{aligned}
\underset{\mathbf{X}^u,\mathbf{X}^d,\mathbf{p}^u,\mathbf{p}^d}{\text{maximize}} \quad & \min_{\forall i,j}\{C_i^u, C_j^d\} && \text{(1.4a)}\\
\text{subject to} \quad & \sum_{f=1}^{F} \gamma_{if}^u \geq \gamma_{\text{th}}^u,\ \forall i, && \text{(1.4b)}\\
& \sum_{f=1}^{F} \gamma_{jf}^d \geq \gamma_{\text{th}}^d,\ \forall j, && \text{(1.4c)}\\
& P_i^u \leq P_{\text{max}}^u,\ \forall i, && \text{(1.4d)}\\
& P_j^d \leq P_{\text{max}}^d,\ \forall j, && \text{(1.4e)}\\
& \sum_{i=1}^{I} x_{if}^u \leq 1,\ \forall f, && \text{(1.4f)}\\
& \sum_{f=1}^{F} x_{if}^u \leq 1,\ \forall i, && \text{(1.4g)}\\
& \sum_{j=1}^{J} x_{jf}^d \leq 1,\ \forall f, && \text{(1.4h)}\\
& \sum_{f=1}^{F} x_{jf}^d \leq 1,\ \forall j, && \text{(1.4i)}\\
& x_{if}^u,\ x_{jf}^d \in \{0,1\},\ \forall i,j,f. && \text{(1.4j)}
\end{aligned}
$$

The optimization variables are $\mathbf{p}^u$, $\mathbf{p}^d$, $\mathbf{X}^u$, and $\mathbf{X}^d$. Constraints (1.4b) and (1.4c) ensure a minimum SINR to be achieved in the DL and UL, respectively. Constraints (1.4d) and (1.4e) limit the transmitting power and constraints (1.4f)-(1.4i) assure that only one UE in the UL and DL can use frequency channel f and that any given frequency channel is used by at most one UE in the UL and DL. Differently from the spectral efficiency maximization in Section 1.2.1, this problem is MINLP and belongs to the category of multi-level nonlinear bottleneck assignments, which is also extremely complex. In this thesis, we provide an approximate solution for problem (1.4). In addition, we provide a centralized solution to a multi-objective problem that takes into account both spectral efficiency and fairness maximization.

1.3 Contributions of the Thesis

In this thesis we analyse key performance measures to further optimize full-duplex communications in cellular networks. In the second part of the thesis, each chapter is based on a published or submitted manuscript. We present below the publications each chapter is based on:

[C1] José Mairton B. da Silva Jr., Yuzhe Xu, Gábor Fodor, Carlo Fischione, "Distributed Spectral Efficiency Maximization in Full-Duplex Cellular Networks", in *Proc. IEEE International Conference on Communications* (ICC'16), May 2016.

[J1] José Mairton B. da Silva Jr., Gábor Fodor, Carlo Fischione, "Fast-Lipschitz Power Control and User-Frequency Assignment in Full-Duplex Cellular Networks," submitted to *IEEE Transactions on Wireless Communications*, February 2017.

[J2] José Mairton B. da Silva Jr., Gábor Fodor, Carlo Fischione, "Spectral Efficient and Fair User Pairing for Full-Duplex Communication in Cellular Networks", *IEEE Transactions on Wireless Communications*, Vol. 15, No. 11, pp. 7578-7593, Nov. 2016.

[C2] José Mairton B. da Silva Jr., Gábor Fodor, Carlo Fischione, "On the Spectral Efficiency and Fairness in Full-Duplex Cellular Networks", in *Proc. IEEE International Conference on Communications* (ICC'17), May 2017, accepted.

In Table 1.1 we show the main coordination mechanisms, design aspects and performance measures present in the publications. We consider user pairing, frequency channel assignment, and power allocation as coordination mechanisms, which are necessary to mitigate the effect of the SI and UE-to-UE interference in the spectral efficiency of the system. All publications consider user pairing and power allocation, whereas only [J1] and [J2] consider frequency assignment, i.e., the environment is frequency selective and the users need to also decide on which frequency they will transmit/receive. Since distributed mechanisms are needed, we propose a distributed user pairing in [C1] and a distributed power control in [J2]. The two objective functions considered in this thesis are the sum spectral efficiency (SE), either weighted or with weights equal to one, and the minimum spectral efficiency of the system, aiming at a fair allocation of resources.

Table 1.1: *Coordination mechanisms and performance objectives considered in the included articles of this thesis.*

	[C1]	[J1]	[J2]	[C2]
User Pairing	✓	✓	✓	✓
Freq. Assignment	-	✓	✓	-
Power Alloc./Control	✓	✓	✓	✓
Distributed	✓	✓	-	-
Objective	Sum SE	Sum SE	Minimum SE	Sum SE + Minimum SE

1.3.1 Distributed Assignment Solution for Spectral Efficiency Maximization

This chapter is based on [C1] and studies the sum spectral efficiency maximization problem through a distributed approach. We propose a joint problem of user pairing, and UL/DL power control that is a MINLP problem, whose objective is to maximize the overall weighted spectral efficiency of the system. Due to the complexity of the MINLP problem proposed, the solution approach uses Lagrangian duality. We provide an initial solution to the user pairing problem that can be solved centrally, and prove the optimal power allocation for any given pair of UL/DL users (i, j). However, such centralized and demanding solutions may increase the burden on the BS, and since the cellular environment is network-controlled by the BS, it is possible to use its resources to provide a distributed solution for the pairing. The pairing solution is expressed as an assignment problem, whereas we use the theory of combinatorial auction to develop the distributed solution that provides a bounded number of iterations and optimality. The numerical results in Figure 1.5 show the sum spectral efficiency in a path loss compensation modelling. We notice that the distributed solution proposed, named D-AUC, improves the sum spectral efficiency when compared to current HD modes when the SI cancelling level is high.

1.3.2 Distributed Power Control for Spectral Efficiency Maximization

This chapter is based on [J1] and we study the sum spectral efficiency maximization problem, but aiming at a distributed power control solution. The focus of the problem is on joint user-frequency channel assignment and power control, and the problem is formulated as a MINLP optimization. The joint problem is NP-hard, and to find an approximated solution, we decompose it into two parts that correspond to the user-frequency channel assignment problem and power control problem, respectively. However, the assignment problem is also NP-hard. Then, we propose a greedy algorithm with guaranteed performance with respect to the optimal solution. To solve the power control problem, we develop a novel power control mechanism especially suited for FD networks. The proposed power control scheme is a distributed algorithm that sets the SINR targets at each receiver such that the achieved sum rate is close-to-optimal. The sum rate maximization problem is non-convex and I therefore proposed to use Fast-Lipschitz (FL)

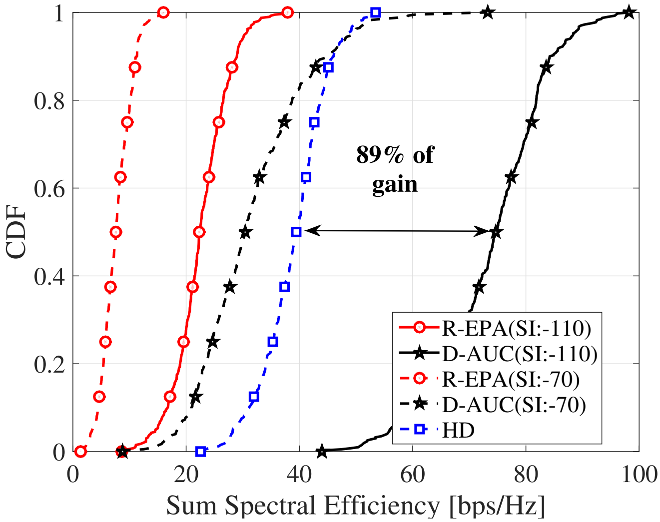
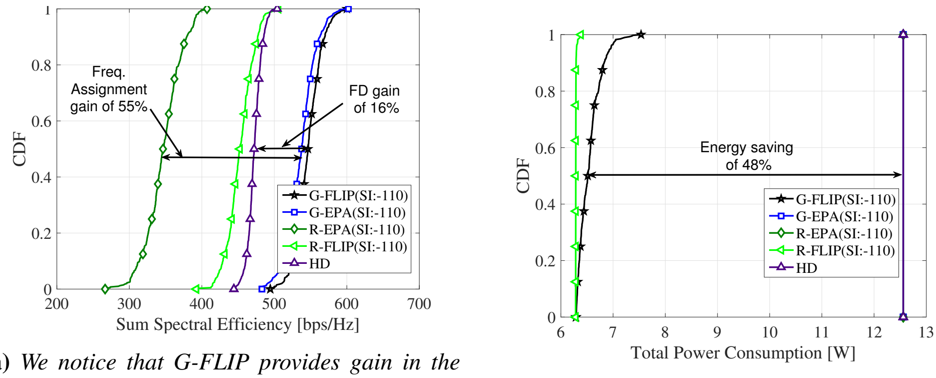


Figure 1.5: *Cumulative distribution function of the sum spectral efficiency in a system with 25 UL, DL users, and frequency channels, using different SI cancelling levels, and with path loss compensation as weights. The proposed distributed solution D-AUC improves the sum spectral efficiency in scenarios with high SI cancellation when compared to HD systems and a naive full-duplex solution using random pairing and equal power allocation, named R-EPA.*

optimization [92] to solve it in a distributed and fast manner. The numerical results in Figure 1.6a show the sum spectral efficiency, whereas in Figure 1.6b we show the total power consumption. We notice that our proposed solution, named G-FLIP, outperforms existing methods in the literature for interference-limited scenarios, providing gains in terms of energy saving, as well as in the sum spectral efficiency gains.

1.3.3 Fairness Maximization for Full-Duplex Cellular Networks

This chapter is based on [J2] and studies fairness in FD for cellular networks. We propose a joint problem of user-frequency channel assignment and transmit power allocation, formulated as a mixed integer nonlinear optimization. The objective of our proposed problem is to maximize the spectral efficiency of the user with the lowest achieved spectral efficiency while respecting minimum QoS constraints, aiming at a fair and efficient communication. The proposed problem is NP-hard, implying that no optimal solution in polynomial time can be obtained. Thus, we apply Lagrangian duality, allowing us to provide the optimal power allocation and an initial solution for the assignment. However, the assignment cannot be solved efficiently, because this problem is also NP-hard, but now with respect to the assignment variables. Then, we propose an approximated greedy solution to the assignment problem, and prove that the duality gap between the greedy



(a) *We notice that G-FLIP provides gain in the sum spectral efficiency when compared to HD systems.*

(b) *We notice that using FL power control solution we have large energy saving gains.*

Figure 1.6: *Cumulative distribution functions of the sum spectral efficiency and total power consumption for an interference-limited system. The system has 25 UL, DL users, and frequency channels, whereas the self-interference cancelling level is* $-110\,\mathrm{dB}$.

solution with optimal power allocation and the primal solution is bounded and diminishes as the number of frequency channels increases. The numerical results in Figure 1.7 show the ratio of users that fulfil the minimum QoS constraint in the system. We notice that our proposed solution, named JAFMA, improves the spectral efficiency of the users with low spectral efficiency in a wide range of scenarios. The results also indicated that the optimization of the assignment and power allocation should be approximately solved jointly; otherwise, a random allocation with equal power allocation (EPA) achieves a similar performance.

1.3.4 Joint Spectral Efficiency and Fairness Maximization for Full-Duplex Cellular Networks

This chapter is based on [C2] and we study the interplay between fairness and weighted sum spectral efficiency maximization in FD cellular networks. We propose a multi-objective optimization problem to maximize simultaneously both the weighted sum spectral efficiency and the minimum spectral efficiency of all users. We use Lagrangian duality and the associated centralized algorithm based on the dual problem to decompose it into parts: the power allocation problem and UL/DL assignment problem. We proved the optimal power allocation between UL/DL users, but it depends on the Lagrangian multiplies of both UL and DL users. Then, we proved an initial solution for the assignment problem, but due to the intertwined optimal power allocation between Lagrangian multiplies of both UL and DL users, this solution is still too complex. Based on this, we reformulated the dual problem and proposed a fast centralized solution for the assignment problem. The numerical results in show that proposed solution, named C-HUN, increases the sum spectral efficiency (see Figure 1.8a) and fairness (see Figure 1.8b) among the users.

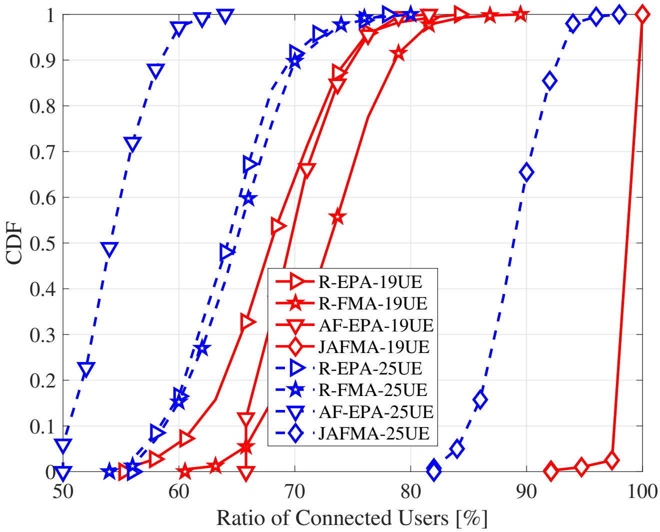


Figure 1.7: *Cumulative distribution function of the ratio of connected users in a system with 25 frequency channels and 19 or 25 UL and DL users. Our proposed JAFMA solution guarantees connection to a high ratio of connected users although the system is completely loaded, and for different SI cancellation levels.*

An important feature of the proposed solution is that there is no need to consider weights in the sum spectral efficiency, be it equal weights (SR), or for path-loss compensation (PL). This is advantageous because defining the weights in a weighted sum objective function is typically cumbersome and difficult in practice.

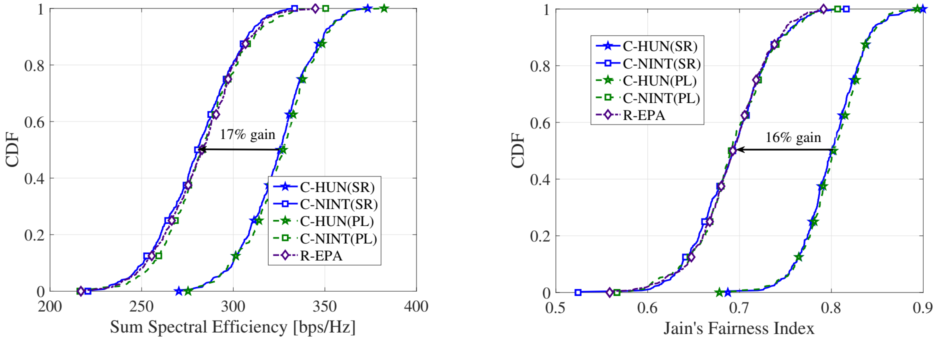
1.3.5 Contributions not Covered in the Thesis

The following publications are not covered in the thesis, but contain related materials and applications:

[J3] Gábor Fodor, S. Roger, N. Rajatheva, S. B. Slimane, T. Svensson, P. Popovski, José Mairton B. da Silva Jr., S. Ali, "An Overview of Device-to-Device Communications Technology Components in METIS", *IEEE Access*, Vol. 4, pp. 3288-3299, 2016.

[BC1] Z. Li, F. S. Moya, Gábor Fodor, José Mairton B. da Silva Jr., K. Koufos, "Device-to-Device (D2D) Communications", in A. Osseiran, J.F. Monserrat, P. Marsch(eds.), "5G Mobile and Wireless Communications Technology", Cambridge University Press, pp. 107-136, 2016.

[J4] José Mairton B. da Silva Jr., Gábor Fodor, "A Binary Power Control Scheme for D2D Communications," *IEEE Wireless Communications Letters*, Vol. 4, No.



(a) *We notice that C-HUN improves the sum spectral efficiency, and the performance is similar for different weights.*

(b) *We notice that C-HUN improves Jain's index and achieves similar performance regardless of the weights used.*

Figure 1.8: *Cumulative distribution function of the sum spectral efficiency and Jain's fairness index in a system with 25 UL, DL users, and frequency channels, with* $\mu = 0.9$ *–coefficient to tune between both objectives–, and using different weights.*

6, pp. 669-672, Dec. 2015. **IEEE Best Readings Topics on Device-to-Device Communications**

1.4 Conclusions and Future Works

1.4.1 Conclusions

Full-duplex communications have the potential to almost double the spectral efficiency of traditional wireless systems operating in HD, and due to the recent advances in SI suppression techniques, FD is becoming a realistic technology component of cellular systems. However, in addition to the inherent SI, the UE-to-UE interference may further degrade the performance, which calls for coordination mechanisms such as user pairing, frequency assignment and power control.

In this thesis we analysed different performance measures along with resource allocation solutions for full-duplex in cellular networks. We analysed the fundamental problem of weighted sum spectral efficiency maximization for two situations: with/without frequency selective fading. With the frequency selective fading, we had to take into account both frequency assignment and user pairing. We proposed a greedy solution to approximate the assignment problem, and developed a novel distributed solution using Fast-Lipschitz optimization. The results showed improvements in the spectral efficiency of the users with in an interference-limited regime, achieved higher spectral efficiency than HD transmissions, and showed large energy saving gains. The results also indicated that a smart assignment solution achieves more gains in interference-limited regime than smart power control solution, whereas the opposite happens in a SI-limited regime.

Without the frequency selective fading, users have no preference of which resource they share. However, user pairing is a concern, since it has a great impact on the individual users' performance. We developed a distributed solution for the user pairing using combinatorial auction and proved the optimal power allocation for users sharing a frequency channel. The numerical results showed that the proposed distributed solution increases the weighted sum spectral efficiency with weights based on path loss compensation when compared to current HD modes with high SI cancelling level. Furthermore, the impact of the UE-to-UE interference is severe when the SI cancelling level is high, and conversely, when the SI cancelling level is low, a proper management of the UE-to-UE interference is not enough to bring gains to FD cellular networks. With respect to the sum spectral efficiency, we showed with these works that for sum spectral efficiency maximization the assignment and power control solutions play different roles that depend on the level of SI cancelling level. Also, we showed that distributed solutions, either for the assignment or power control, can be performed and provide gains with respect to HD systems.

Besides sum spectral efficiency, we were also interested in fairness for full-duplex cellular networks. We tackled this important performance indicator through the maximization of the minimum spectral efficiency achieved in the system. We proposed an approximated solution for the assignment and proved the optimal power allocation for a system with minimum QoS constraints, as well as the minimum SI cancelling level that allow the pair of users to achieve at least the minimum SINR. The numerical results showed that our solution improved the minimum spectral efficiency and achieved the highest ratio of connected users in a wide range of scenarios. Also, the results indicated that the optimization of the assignment and power allocation should be solved jointly; otherwise, a random allocation with EPA achieves a similar performance.

Since both sum spectral efficiency and fairness are key performance indicators, we also studied the interplay between them through the multi-objective optimization problem of maximizing the weighted sum spectral efficiency and minimum spectral efficiency achieved in the system. Due to its high complexity, we proposed a low-complexity centralized solution that can be implemented at the cellular base station. The numerical results showed that using a proper weight between the two objectives, the sum spectral efficiency and fairness are improved regardless of the individual weights on the sum spectral efficiencies of UL and DL users. Furthermore, the UE-to-UE interference has to be taken into account irrespectively of how the assignment and power allocation are performed. When it is not, the performance in terms of sum spectral efficiency and fairness will be close to a random assignment and equal power allocation among users. Regarding fairness in full-duplex for cellular networks, we showed that it is possible to achieve a fair and spectral efficient resource allocation, provided smart assignment and power allocation solutions are employed.

1.4.2 Future Works

Although we propose some future directions in the second part of this thesis, there are other directions that we also wish to take, and they are listed as follows:

- *Interplay between more performance indicators*: In [C2] we analysed the relation between spectral efficiency and fairness, whereas in [J2] we showed a solution that maximized the spectral efficiency and achieved high energy saving gains. Another possibility is to directly analyse the interplay between spectral and energy efficiency in full-duplex for cellular networks. By proposing an optimization problem that aims at both objectives, we may further increase the gains of FD communications.

- *Channel estimation and pilot design for FD-MIMO*: A natural extension is to consider FD with multiple antennas at both transmitter and receiver, and this has been studied recently. However, most of the works assume perfect or partial channel knowledge, and without taking into account a proper method for channel estimation and the design of pilots. The pilot-data trade-off is a well-studied topic in standard MIMO, but it has not been addressed in the context of full-duplex communications.

- *Beamforming design for FD-MIMO in mmWaves*: With the growing of mmWave and the intensive development in hardware, FD in mmWave is becoming a viable technology component. To use fully the benefits of mmWave, we need a proper beamforming design with digital and analog architectures, i.e., a hybrid beamforming. However, none of these aspects have been considered in the context of full-duplex communications, implying that the combined gains may help to further increase the rate and spectral efficiency of future cellular systems.

Chapter 2

Preliminaries

This chapter summarizes the background theory used in the thesis. In particular, Section 2.1 introduces the Hungarian algorithm, a centralized solution to assignment problems. Section 2.2 introduces the fundamentals of Auction theory, a solution to assignment problem that we use in the thesis to pair UL and DL users. Then, in Section 2.3 we discuss the framework of Fast-Lipschitz optimization.

2.1 Hungarian Algorithm

In this section, we summarize an efficient centralized method for solving assignment problems optimally and in polynomial time [93–95]. Let us suppose there are n agents that need to be assigned to n locations in a one-to-one basis. For each association, we define the costs a_{ij} to represent the cost expenses of assigning agent i to location j, which are collected in the matrix $\mathbf{A} \in \mathbb{R}$. The objective of this problem is to minimize the overall cost of transportation, i.e., the association between agents to locations need to be performed such that the overall cost of transportation is minimized. We can formulate this assignment problem as a linear optimization with binary variables as

$$\begin{aligned} \underset{\mathbf{X}}{\text{minimize}} \quad & \sum_{i=1}^{I}\sum_{j=1}^{J} a_{ij}x_{ij} && \text{(2.1a)}\\ \text{subject to} \quad & \sum_{i=1}^{I} x_{ij} = 1,\ \forall j, && \text{(2.1b)}\\ & \sum_{j=1}^{J} x_{ij} = 1,\ \forall i, && \text{(2.1c)}\\ & x_{ij} \in \{0,1\},\ \forall i,j, && \text{(2.1d)} \end{aligned}$$

where the assignment matrix $\mathbf{X} \in \{0,1\}^{n\times n}$ is the optimization variable. This assignment problem can also be seen as a 2D assignment problem, which represents the two dimensions that we have to assign. The Hungarian algorithm is a polynomial time centralized solution for this problem [93, 94], and it was named by Kuhn [93] in honor of the work of König and Egerváry on which it is based. The algorithm assigns in an optimal manner the agents to the locations in $O(n^3)$ operations [95].

For the sake of simplicity, we present below the summarized steps of the Hungarian algorithm in matrix form [95]:

Step 1 Subtract the smallest $a_{ij_{\min}}$ in each row of $\mathbf{A}$ from all the entries of its row.

Step 2 Similarly to **Step 1**, subtract the smallest entry $a_{i_{\min}j}$ in each column of $\mathbf{A}$ from all the entries of its column.

Step 3 Cover rows and columns with the minimum number pf lines so that all the zero entries of matrix $\mathbf{A}$ are covered. For simplicity, let us denote this minimum number as l.

Step 4 If $l = n$, we have an optimal assignment of zeros and we are finished. However, $l < n$, an optimal assignment of zeros is not possible yet, and we proceed to the next step.

Step 5 Find the smallest entry $\bar{a}_{ij}$ not covered by any line. Subtract this entry from each uncovered row, and then add it to each covered column. Go back to **Step 3**.

We use the Hungarian algorithm as a benchmarking solution for the pairing of UL and DL users in [C1] and [C2].

2.2 Distributed Auction Theory

In this section, we summarize an efficient method for solving assignment problems in a distributed manner. The method is based on Auction theory, which is centralized [96], and its extension to the distributed case was proposed by Yuzhe et al. [97]. Let us suppose there are n UL users (persons in [96]) and n DL users (objects in [96]) that we have to pair on a one-to-one basis. In our application, this pairing represents the UL and DL users that will transmit in the same frequency channel, and the DL users can also be represented by the BS. The UL user i benefits c_{ij} for the association with DL user j. Differently from Section 2.1, our objective now is to assign DL to UL users so as to maximize the total sum utility, instead of minimize it. This assignment problem is formulated as a linear optimization problem with binary variables as

$$\underset{\mathbf{X}}{\text{maximize}} \quad \sum_{i=1}^{I}\sum_{j=1}^{J} c_{ij}x_{ij} \tag{2.2a}$$

$$\text{subject to} \quad \sum_{i=1}^{I} x_{ij} = 1,\ \forall j, \tag{2.2b}$$

$$\sum_{j=1}^{J} x_{ij} = 1,\ \forall i, \tag{2.2c}$$

$$x_{ij} \in \{0,1\},\ \forall i,j, \tag{2.2d}$$

where the assignment matrix $\mathbf{X} \in \{0,1\}^{n\times n}$ is the optimization variable. The auction algorithm is a method to solve the assignment problem (2.2) by an iterative procedure that can be performed in a distributed manner between UL and DL users.

For the correct operation of the auction algorithm, we require the *ϵ-complementary slackness* (ϵ-CS), which relates a partial assignment $\mathcal{S}$ and a price vector $\mathbf{p} = [p_1 \dots p_n]$. We can view this variable p_j as the price the UL user is willing to pay to be associated with DL user j. Also, it is useful to define the non-empty set of DL users to which UL user i can be paired as $\mathcal{A}(i)$. The couple $\mathcal{S}$ and $\mathbf{p}$ satisfy ϵ-CS if for every pair $(i,j) \in \mathcal{S}$, DL user j is within ϵ of being the best candidate pair for UL user i [96], i.e.,

$$c_{ij} - p_j \geq \max_{k\in\mathcal{A}A(i)} \{c_{ik} - p_k\} - \epsilon, \forall (i,j) \in \mathcal{S}. \tag{2.3}$$

For the sake of clarity we define $c_{ij} - p_j$ as the utility that UL user i can obtain from DL user j. The auction algorithm is iterative, where each iteration starts with a partial assignment and the algorithm terminates when an assignment for all users is obtained. This iterative process consists of two phases: the *bidding phase* and the *assignment phase*. In the bidding phase, each UL user bids for a DL user that maximizes the associated utility $(c_{ij} - p_j)$, and the DL evaluates the bid received from the UL users. In the assignment phase, the DL users select the UL user with the highest bid and updates the prices.

The bidding phase is performed by the UL users, and can be summarized from a UL user i as:

Step 1 Evaluate the maximum utility v_i for all DL users, which is defined as

$$v_i = \max_{j\in\mathcal{A}(i)} \{c_{ij} - p_j\}. \tag{2.4}$$

Step 2 Select the DL user j_i that maximizes its maximum utility v_i

$$j_i = \arg\max_{j\in\mathcal{A}(i)} v_i. \tag{2.5}$$

Step 3 Evaluate the utility w_i of the best DL user, without considering user j_i

$$w_i = \max_{j\in\mathcal{A}(i), j\neq j_i} \{c_{ij} - \hat{p}_j\}. \tag{2.6}$$

Step 4 The bid of UL user i to DL user j_i, b_i, is defined as

$$b_{ij_i} = c_{ij} - w_i + \epsilon. \tag{2.7}$$

When the DL users receive the bids, the assignment phase starts. For the sake of simplicity, let us denote $\mathcal{P}(j)$ as the set of UL users from which DL user j received a bid. In the assignment phase the prices are updated based on the highest bid received, which is given by

$$p_j = \max_{i\in\mathcal{P}(j)} \{b_{ij}\}. \tag{2.8}$$

Then, remove from the partial assignment $\mathcal{S}$ any pair that has the DL user j assigned to, and include to $\mathcal{S}$ the new pair with the UL user i_j that attained the maximum b_{ij}.

This distributed iterative process is repeated until each UL user has been assigned to a DL user. The iterative process has a finite and bounded number of iterations, and the auction algorithm provides an approximated solution for the assignment problem [96] that is within $n\epsilon$ of being optimal. We use the Auction algorithm to develop a distributed solution for the pairing of UL and DL users in [C1].

2.3 Fast-Lipschitz Optimization

In this section, we give an introductory formal definition of the Fast-Lipschitz optimization framework, which was introduced in [92]. For a thorough discussion of Fast-Lipschitz problems we refer the interested reader to the following articles [92, 98, 99].

Definition 1. A problem is said to be on *Fast-Lipschitz form* if it can be written

$$\begin{array}{ll} \underset{\mathbf{x}}{\text{maximize}} & \mathbf{f}_0(\mathbf{x}) \\ \text{subject to} & x_i \le f_i(\mathbf{x}) \quad \forall i \in \mathcal{A} \\ & x_i = f_i(\mathbf{x}) \quad \forall i \in \mathcal{B}, \end{array} \tag{2.9}$$

where

- $\mathbf{f}_0(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}^m$ is a differentiable scalar ($m=1$) or vector valued ($m \geq 2$) function.
- $\mathcal{A}$ and $\mathcal{B}$ are complementary subsets of $\{1, \ldots, n\}$, i.e., $\mathcal{A} \cup \mathcal{B} = \{1, \ldots, n\}$ and $\mathcal{A} \cap \mathcal{B} = \emptyset$
- The functions $f_i : \mathbf{f}_0(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$ are all differentiable.

From the individual constraint functions, we form the vector valued function $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^n$ as

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} f_1(\mathbf{x}) & \cdots & f_n(\mathbf{x}) \end{bmatrix}^T .$$

One characteristic of Fast-Lipschitz optimization is that each variable x_i is associated with one constraint $f_i(\mathbf{x})$, although this does not reduce the generality of the approach because one can always introduce redundant constraints satisfying such characteristic. The form $x \le f(\mathbf{x})$ is general because any constraint on canonical form $g(\mathbf{x}) \le 0$ can be written as $x \le x - \gamma g(\mathbf{x})$ for some $\gamma > 0$.

Definition 2. We restrict our attention to a *bounding box* $\mathcal{D} = \{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{a} \le \mathbf{x} \le \mathbf{b}\}$. We assume $\mathcal{D}$ contains all candidates for optimality and that $\mathbf{f}$ maps $\mathcal{D}$ into $\mathcal{D}$, $\mathbf{f} : \mathcal{D} \to \mathcal{D}$. This box arises naturally in practice, since any real-world decision variable, such as transmitting power, must be bounded.

Definition 3. A problem is said to be *Fast-Lipschitz* when it can be written on Fast-Lipschitz form and admits a unique Pareto optimal solution $\mathbf{x}^*$, defined as the unique solution to the system of equations

$$\mathbf{x}^* = \mathbf{f}(\mathbf{x}^*). \tag{2.10}$$

A problem written on FL form is not automatically Fast-Lipschitz. We present below the first qualifying conditions proposed in [92] that a problem in Fast-Lipschitz form needs to fulfil. Many other qualifying conditions have been proved since [92], and for this reason we will refer to the following conditions as old.

Old Qualifying Conditions. For all $\mathbf{x} \in \mathcal{D}$, $\mathbf{f}_0(\mathbf{x})$ and $\mathbf{f}(\mathbf{x})$ must fulfil at least one of the following cases, either (0) and (i) or (0) and (ii):

(0) $\nabla \mathbf{f}_0(\mathbf{x}) > \mathbf{0}$

AND **(i. a)** $\nabla \mathbf{f}(\mathbf{x}) \geq \mathbf{0}$

(i. b) $|||\nabla \mathbf{f}(\mathbf{x})||| < 1$

OR **(ii. a)** $\mathbf{f}_0(\mathbf{x}) = c\mathbf{1}^T\mathbf{x}$

(ii. b) $\nabla \mathbf{f}(\mathbf{x}) \leq \mathbf{0}$, or more generally, $\nabla \mathbf{f}(\mathbf{x})^2 \geq \mathbf{0}$

(ii. c) $|||\nabla \mathbf{f}(\mathbf{x})|||_\infty < 1$

OR **(iii. a)** $\mathbf{f}_0(\mathbf{x}) \in \mathbb{R}$

(iii. b) $|||\nabla \mathbf{f}(\mathbf{x})|||_\infty < \frac{\bar{\delta}}{\bar{\delta}+\bar{\Delta}}$, where $\bar{\delta} \triangleq \min_i \min_{\mathbf{x}\in\mathcal{D}} \nabla_i \mathbf{f}_0(\mathbf{x})$, and $\bar{\Delta} \triangleq \max_i \max_{\mathbf{x}\in\mathcal{D}} \nabla_i \mathbf{f}_0(\mathbf{x})$.

Theorem 1 (Fischione [92])**.** A problem in Fast-Lipschitz form that fulfils any pair of the Old Qualifying Conditions is Fast-Lipschitz, i.e., it has a unique Pareto optimal point given by $\mathbf{x}^* = \mathbf{f}(\mathbf{x}^*)$. Furthermore, $\mathbf{x}^*$ can be found as the limit of the iterations $\mathbf{x}^{k+1} = \mathbf{f}(\mathbf{x}^k)$.

Therefore, once we know the problem is Fast-Lipschitz, to obtain the solution we only need to solve the system of equations (2.10). In general, solving this system of equations is much easier solving an optimization problem using standard Lagrangian duality. Specifically, the qualifying conditions assure than $\mathbf{f}(\mathbf{x})$ is contractive on $\mathcal{D}$, which implies that the iterations $\mathbf{x}^* := \mathbf{f}(\mathbf{x}^*)$ converge geometrically to the optimal point $\mathbf{x}^*$ starting from any point $\mathbf{x}^0 \in \mathcal{D}$. We use the Fast-Lipschitz framework to develop a distributed power control solution in [J1].

Part II

Included Papers

Paper A

Distributed Spectral Efficiency Maximization in Full-Duplex Cellular Networks

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Distributed Spectral Efficiency Maximization in Full-Duplex Cellular Networks

José Mairton B. da Silva Jr., Yuzhe Xu, Gábor Fodor, and Carlo Fischione

Abstract

Three-node full-duplex is a promising new transmission mode between a full-duplex capable wireless node and two other wireless nodes that use half-duplex transmission and reception respectively. Although three-node full-duplex transmissions can increase the spectral efficiency without requiring full-duplex capability of user devices, inter-node interference – in addition to the inherent self-interference – can severely degrade the performance. Therefore, as methods that provide effective self-interference mitigation evolve, the management of inter-node interference is becoming increasingly important. This paper considers a cellular system in which a full-duplex capable base station serves a set of half-duplex capable users. As the spectral efficiencies achieved by the uplink and downlink transmissions are inherently intertwined, the objective is to device channel assignment and power control algorithms that maximize the weighted sum of the uplink-downlink transmissions. To this end a distributed auction based channel assignment algorithm is proposed, in which the scheduled uplink users and the base station jointly determine the set of downlink users for full-duplex transmission. Realistic system simulations indicate that the spectral efficiency can be up to 89 % better than using the traditional half-duplex mode. Furthermore, when the self-interference cancelling level is high, the impact of the user-to-user interference is severe unless properly managed.

A.1 Introduction

Traditional cellular networks operate in half-duplex (HD) transmission mode, in which a user equipment (UE) or the base station (BS) either transmits or receives on any given frequency channel. However, the increasing demand to support the transmission of unprecedented data quantities has led the research community to investigate new wireless transmission technologies. Recently, in-band full-duplex (FD) has been proposed as a key enabling technology to drastically increase the spectral efficiency of conventional wireless transmission modes. Due to recent advances in antenna design, interference cancellation algorithms, self-interference (SI) suppression techniques and prototyping of FD transceivers, FD transmission is becoming a realistic technology component of advanced wireless – including cellular – systems, especially in the low transmit power regime [17, 18].

In particular, in-band FD and three-node full-duplex (TNFD) transmission modes can drastically increase the spectral efficiency of conventional wireless transmission modes since both transmission techniques have the potential to double the spectral efficiency of traditional wireless systems operating in HD [12,16]. TNFD involves three nodes, but only

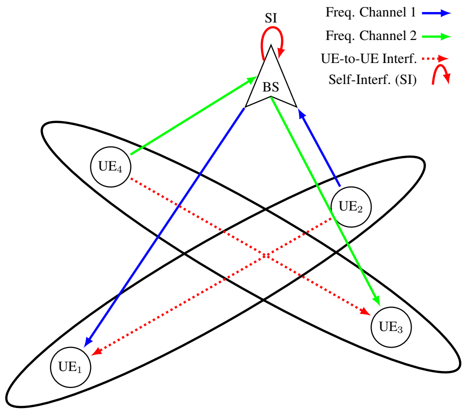

Figure A.1: *A cellular network employing FD with two UEs pairs. The BS selects pairs of UEs, represented by the ellipses, and jointly schedules them for FD transmission by allocating frequency channels in the UL and DL. To mitigate UE-to-UE interference, it is advantageous to co-schedule DL/UL users for FD transmission that are far apart, such as* UE_1*-*UE_2 *and* UE_3*-*UE_4*.*

one of them needs to have FD capability. The FD-capable node transmits to its receiver node while receiving from another transmitter node on the same frequency channel.

As illustrated in Figure A.1, FD operation in a cellular environment experiences new types of interference, aside from the inherently present SI. Because the level of UE-to-UE interference depends on the UE locations and their transmission powers, coordination mechanisms are needed to mitigate the negative effect of the interference on the spectral efficiency of the system [10]. A key element of such mechanisms is UE *pairing* and frequency channel selection that together determine which UEs should be scheduled for simultaneous uplink (UL) and downlink (DL) transmissions on specific frequency channels. Hence, it is crucial to design efficient and fair medium access control protocols and physical layer procedures capable of supporting adequate pairing mechanisms. Furthermore, in future cellular networks the idea is to move from a fully centralized to a more distributed network [8], where the infrastructure of the BS can be used to help the UEs to communicate in a distributed manner and reduce the processing burden at the BS, which is further increased by SI cancellation.

To the best of our knowledge, the only work to consider a distributed approach for FD cellular networks is reported in [89]. However, the authors tackle the problem of the UE-to-UE interference from an information theoretic perspective, without relating to resource allocation and power control. Conversely, some works consider the joint subcarrier and power allocation problem [79] and the joint duplex mode selection, channel allocation, and power control problem [32] in FD networks. The cellular network model in [79] is applicable to FD mobile nodes rather than to networks operating in TNFD mode. The work reported in [32] considers the case of TNFD transmission mode in a cognitive femto-cell context with bidirectional transmissions from UEs and develops sum-rate optimal resource allocation and power control algorithms. However, none of these two works consider a distributed approach for FD cellular networks.

In this paper we formulate the joint problem of user pairing (i.e. co-scheduling of UL and DL simultaneous transmissions on a frequency channel), and UL/DL power control as a mixed integer nonlinear programming (MINLP) problem, whose objective is to maximize the overall spectral efficiency of the system. Due to the complexity of the MINLP problem proposed, our solution approach relies on Lagrangian duality and a distributed auction algorithm in which UL users offer bids on desirable DL users. In this iterative auction process, the BS – as the entity that owns the radio resources – accepts or rejects bids and performs resource assignment. This algorithm is tested in a realistic system simulator that indicates that the bidding process converges to a near optimal pairing and power allocation.

A.2 System Model and Problem Formulation

A.2.1 System Model

We consider a single-cell cellular system in which only the BS is FD capable, while the UEs served by the BS are only HD capable, as illustrated by Figure A.1. In Figure A.1, the BS is subject to SI and the UEs in the UL (UE_2 and UE_4) cause UE-to-UE interference to co-scheduled UEs in the DL, that is to UE_1 and UE_3 respectively. The number of UEs in the UL and DL is denoted by I and J, respectively, which are constrained by the total number of frequency channels in the system F, i.e., $I \leq F$ and $J \leq F$. The sets of UL and DL users are denoted by by $\mathcal{I} = \{1, \dots, I\}$ and $\mathcal{J} = \{1, \dots, J\}$ respectively.

We consider frequency flat and slow fading, such that the channels are constant during the time slot of a scheduling instance and over the frequency channels assigned to co-scheduled users. Let G_{ib} denote the effective path gain between transmitter UE i and the BS, G_{bj} denote the effective path gain between the BS and the receiving UE j, and G_{ij} denote the interfering path gain between the UL transmitter UE i and the DL receiver UE j. To take into account the residual SI power that leaks to the receiver, we define β as the SI cancellation coefficient, such that the SI power at the receiver of the BS is βP_j^d when the transmit power is P_j^d.

The vector of transmit power levels in the UL by UE i is denoted by $\mathbf{p^u} = [P_1^u \dots P_I^u]$, whereas the DL transmit powers by the BS is denoted by $\mathbf{p^d} = [P_1^d \dots P_J^d]$. As illustrated in Figure A.1, the UE-to-UE interference is much dependent on the geometry of the co-scheduled UL and DL users, i.e., the *pairing* of UL and DL users. Therefore, UE pairing is a key functions of the system. Accordingly, we define the assignment matrix, $\mathbf{X} \in \{0, 1\}^{I \times J}$, such that

$$x_{ij} = \begin{cases} 1, & \text{if the UL UE}_i \text{ is paired with the DL UE}_j, \\ 0, & \text{otherwise.} \end{cases}$$

The signal-to-interference-plus-noise ratio (SINR) at the BS of transmitting user i and the SINR at the receiving user j of the BS are given by

$$\gamma_i^u = \frac{P_i^u G_{ib}}{\sigma^2 + \sum_{j=1}^{J} x_{ij} P_j^d \beta}, \ \gamma_j^d = \frac{P_j^d G_{bj}}{\sigma^2 + \sum_{i=1}^{I} x_{ij} P_i^u G_{ij}}, \tag{A.1}$$

respectively, where x_{ij} in the denominator of γ_i^u accounts for the SI at the BS, whereas x_{ij} in the denominator of γ_j^d accounts for the UE-to-UE interference caused by UE$_i$ to UE$_j$.

Thus, the achievable spectral efficiency for each user is given by the Shannon equation (in bits/s/Hz) for the UL and DL as $C_i^u = \log_2(1 + \gamma_i^u)$ and $C_j^d = \log_2(1 + \gamma_j^d)$, respectively. In addition to the spectral efficiency, we consider weights for the UL and DL users, which are denoted by α_i^u and α_j^d, respectively. The idea behind weighing is that it allows the system designer to choose between the commonly used sum rate maximization and important fairness related criteria such as the well known *path loss compensation* typically employed in the power control of cellular networks [100]. For the weights α_i^u and α_j^d, we can account for sum rate maximization with $\alpha_i^u = \alpha_j^d = 1$ and for path loss compensation with $\alpha_i^u = G_{ib}^{-1}$ and $\alpha_j^d = G_{bj}^{-1}$.

A.2.2 Problem Formulation

Our goal is to jointly consider the assignment of UEs in the UL and DL (*pairing*), while maximizing the weighted sum spectral efficiency of all users. Specifically, the problem is formulated as

$$\begin{aligned}
\underset{\mathbf{X},\mathbf{p}^u,\mathbf{p}^d}{\text{maximize}} \quad & \sum_{i=1}^{I} \alpha_i^u C_i^u + \sum_{j=1}^{J} \alpha_j^d C_j^d && \text{(A.2a)}\\
\text{subject to} \quad & \gamma_i^u \geq \gamma_{\text{th}}^u, \ \forall i, && \text{(A.2b)}\\
& \gamma_j^d \geq \gamma_{\text{th}}^d, \ \forall j, && \text{(A.2c)}\\
& P_i^u \leq P_{\text{max}}^u, \ \forall i, && \text{(A.2d)}\\
& P_j^d \leq P_{\text{max}}^d, \ \forall j, && \text{(A.2e)}\\
& \sum_{i=1}^{I} x_{ij} \leq 1, \ \forall j, && \text{(A.2f)}\\
& \sum_{j=1}^{J} x_{ij} \leq 1, \ \forall i, && \text{(A.2g)}\\
& x_{ij} \in \{0, 1\}, \ \forall i, j. && \text{(A.2h)}
\end{aligned}$$

The main optimization variables are $\mathbf{p}^u$, $\mathbf{p}^d$ and $\mathbf{X}$. Constraints (A.2b) and (A.2c) ensure a minimum SINR to be achieved in the DL and UL, respectively. Constraints (A.2d) and (A.2e) limit the transmit powers whereas constraints (A.2f)-(A.2g) assure that only one UE in the DL can share the frequency resource with a UE in the UL and vice-versa. Note that constraints (A.2b)-(A.2c) require that the SINR targets for both UL and DL be defined a priori.

Problem (A.2) belongs to the category of MINLP, which is known for its high complexity and computational intractability. Thus, to solve problem (A.2) we will rely on Lagrangian duality, which is described on Section A.3. We develop the optimal power allocation for a UL-DL pair and the optimal closed-form solution for the assignment, which can be solved in a centralized manner. However, in future cellular networks the idea is to move from a fully centralized to a more distributed network [8], offloading the burden on the BS. With this objective, in Section A.4 we use the optimal power allocation to create a distributed solution for the assignment between UL and DL users.

A.3 A Solution Approach Based on Lagrangian Duality

From problem (A.2), we form the *partial* Lagrangian function by considering constraints (A.2b)-(A.2c) and ignoring the integer (A.2f)-(A.2h) and power allocation constraints (A.2d)-(A.2e). To this end, we introduce Lagrange multipliers $\boldsymbol{\lambda}^u$, $\boldsymbol{\lambda}^d$, where the superscript u and d denote the dimensions of I and J, respectively. The partial Lagrangian is a function of the Lagrange multipliers and the optimization variables $\mathbf{X}, \mathbf{p}^u, \mathbf{p}^d$ as follows:

$$L(\boldsymbol{\lambda}^u, \boldsymbol{\lambda}^d, \mathbf{X}, \mathbf{p}^u, \mathbf{p}^d) \triangleq -\sum_{i=1}^{I} \alpha_i^u C_i^u - \sum_{j=1}^{J} \alpha_j^d C_j^d + \sum_{i=1}^{I} \lambda_i^u \Big(\gamma_{\text{th}}^u - \gamma_i^u\Big) + \\ + \sum_{j=1}^{J} \lambda_j^d \Big(\gamma_{\text{th}}^d - \gamma_j^d\Big). \tag{A.3}$$

Let $g(\boldsymbol{\lambda}^u, \boldsymbol{\lambda}^d)$ denote the dual function obtained by minimizing the partial Lagrangian (A.3) with respect to the variables $\mathbf{X}, \mathbf{p}^u, \mathbf{p}^d$. That is, the dual function is

$$g(\boldsymbol{\lambda}^u, \boldsymbol{\lambda}^d) = \inf_{\mathbf{X} \in \mathcal{X}\ \mathbf{p}^u, \mathbf{p}^d \in \mathcal{P}} L(\boldsymbol{\lambda}^u, \boldsymbol{\lambda}^d, \mathbf{X}, \mathbf{p}^u, \mathbf{p}^d), \tag{A.4}$$

where $\mathcal{X}$ and $\mathcal{P}$ are the set where the assignment and power allocation constraints are fulfilled, respectively. Notice that we can rewrite the dual as

$$g(\boldsymbol{\lambda}^u, \boldsymbol{\lambda}^d) = \inf_{\mathbf{X} \in \mathcal{X}\ \mathbf{p}^u, \mathbf{p}^d \in \mathcal{P}} \sum_{n=1}^{N} \left(q_{i_n}^u(\mathbf{X}, \mathbf{p}^u, \mathbf{p}^d) + q_{j_n}^d(\mathbf{X}, \mathbf{p}^u, \mathbf{p}^d) \right), \tag{A.5}$$

where we assume $N = I = J$ is the maximum number of UL-DL pairs, i_n and j_n are the UL and DL users of pair n, respectively. Moreover,

$$q_{i_n}^u(\mathbf{X}, \mathbf{p}^u, \mathbf{p}^d) \triangleq \lambda_{i_n}^u \left(\gamma_{\text{th}}^u - \gamma_{i_n}^u\right) - \alpha_i^u C_{i_n}^u, \tag{A.6a}$$

$$q_{j_n}^d(\mathbf{X}, \mathbf{p}^u, \mathbf{p}^d) \triangleq \lambda_{j_n}^d \left(\gamma_{\text{th}}^d - \gamma_{j_n}^d\right) - \alpha_{j_n}^d C_{j_n}^d. \tag{A.6b}$$

We can find the infimum of (A.5) if we maximize the SINR of the N UL-DL pairs. Thus, we can write a closed-form expression for the assignment x_{ij} as follows:

$$x_{ij}^{\star} = \begin{cases} 1, & \text{if } (i,j) = \arg\max_{i,j} \left(q_{i_n}^{u,\max} + q_{j_n}^{d,\max} \right) \\ 0, & \text{otherwise,} \end{cases} \tag{A.7}$$

where for simplicity we denoted an ordinary pair as (i, j). Notice that $x_{ij}^{\star}$ and equation (A.7) uniquely associate an UL user with a DL user. However, the solutions are still tied through the SINRs γ_i^u and γ_j^d, i.e., the solution to the assignment problem is still complex and – through (A.7) – is intertwined with the optimal power allocation.

Since the SINRs on the UL are not separable from those on the DL, we cannot analyse them independently. Consequently, we need to find the powers that jointly minimize (A.5). To this end, we first analyse the dual problem, given by

$$\underset{\boldsymbol{\lambda}^u,\, \boldsymbol{\lambda}^u}{\text{maximize}} \quad g(\boldsymbol{\lambda}^u, \boldsymbol{\lambda}^d) \tag{A.8a}$$

$$\text{subject to} \quad \lambda_i^u,\ \lambda_j^d, \geq 0, \forall i, j, \tag{A.8b}$$

where recall that $g(\boldsymbol{\lambda}^u, \boldsymbol{\lambda}^d)$ is the solution of problem (A.4). Notice that if constraints (A.2b)-(A.2c) are fulfilled in the inequality or equality, $\lambda_i^u\left(\gamma_{\text{th}}^u - \gamma_i^u\right)$ and $\lambda_j^d\left(\gamma_{\text{th}}^d - \gamma_j^d\right)$ will be either negative or zero. If the terms are negative, then λ_i^u or λ_j^d will be zero. Thus, the terms with λ_i^u and λ_j^d will not impact $g(\boldsymbol{\lambda}^u, \boldsymbol{\lambda}^d)$. Therefore, the dual is easily solved by assigning zero to λ_i^u or λ_j^d whose corresponding UL and DL user fulfils the inequalities (A.2b)-(A.2c). If there are users that do not fulfil the inequalities, the problem is unbounded.

Therefore, we now turn our attention to the power allocation problem, and – based on the above considerations on λ_i^u or λ_j^d–, we formulate the power allocation problem as:

$$\underset{\mathbf{p}^u, \mathbf{p}^d}{\text{minimize}} \quad -\sum_{i=1}^{I} \alpha_i^u C_i^u - \sum_{j=1}^{J} \alpha_j^d C_j^d \tag{A.9a}$$

$$\text{subject to} \quad \mathbf{p}^u, \mathbf{p}^d \in \mathcal{P}. \tag{A.9b}$$

From Gesbert et al. [101], the optimal transmit power allocation will have either P_i^u or P_j^d equal to P_{max}^u or P_{max}^d, given that i and j share a frequency channel and form a pair. Moreover, from Feng et al. [102, Section III.B], the optimal power allocation lies within the admissible area for pair (i, j), where we do not show the explicit expressions for the optimal power allocation here due to space limit.

Therefore, with the optimal transmit powers for any given pair (i, j), and with the closed-form solution for the assignment in Eq. (A.7), we can solve the dual problem (A.8). To compute the optimal assignment as given by Eq. (A.7) requires checking $N!$ assignments [103, Section 1], or we could apply the Hungarian algorithm in a fully centralized manner [103, Section 3.2], that has worst-case complexity of $O(N^3)$.

However, we are not interested in such centralized and demanding solutions that would increase the burden on the BS. Since we are in a network-controlled environment with the BS, we use its resources to provide a distributed solution for the assignment, whereas the power allocation would remain centralized, because distributed power allocation schemes require too many iterations to converge. Therefore, in the next section we reformulate the closed-form solution in Eq. (A.7) and propose a fully distributed assignment based on Auction Theory [96].

A.4 Distributed Auction Solution

With the optimal power allocation for a pair (i, j) at hand, and as mentioned in Section A.3, we are interested in a distributed solution for the closed-form solution for the assignment in Eq. (A.7) in order to reduce the burden on the BS by supporting a more distributed system. In Section A.4.1 we reformulate the closed-form expression as an asymmetric assignment problem, whereas Section A.4.2 introduces the fundamental definitions necessary to propose the distributed auction algorithm in Section A.4.3. Furthermore, we give one of the core results in this paper in Section A.4.4, where we show that the number of iterations of the algorithms is bounded and that the feasible assignment provided at the end is within a bound of desired accuracy around the optimal assignment.

A.4.1 Problem Reformulation

We can rewrite the closed form expression (A.7) as an asymmetric assignment problem, given by

$$\underset{\mathbf{x}}{\text{maximize}} \quad \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij} x_{ij} \tag{A.10a}$$

$$\text{subject to} \quad \sum_{i=1}^{I} x_{ij} = 1, \ \forall j, \tag{A.10b}$$

$$\sum_{j=1}^{J} x_{ij} = 1, \ \forall i, \tag{A.10c}$$

$$x_{ij} \in \{0, 1\}, \ \forall i, j, \tag{A.10d}$$

where $c_{ij} = \alpha_i^u C_i^u + \alpha_j^d C_j^d$ for a pair (i, j) assigned to the same frequency and it can be understood as the benefit of assigning UL user i to DL user j. Constraint (A.10b) ensures that the DL users are associated with one UL user. Similarly, constraint (A.10c) ensures that all the UL users need to be associated with a DL user. To solve this problem in a distributed manner, we use Auction Theory.

A.4.2 Fundamentals of the Auction

We consider the assignment problem (A.10), where we want to pair I UL and J DL users on a one-to-one basis, where the benefit for pairing UL user i to DL user j is given by c_{ij}. Initially, we assume that $J = I$ and $J \leq F$, whereas the set of DL users to which UL user i can be paired is non-empty and is denoted by $\mathcal{A}(i)$. We define an assignment $\mathcal{S}$ as a set of UL-DL pairs (i, j) such that $j \in \mathcal{A}(i)$ for all $(i, j) \in \mathcal{S}$ and for each UL and DL user there can be at most one pair $(i, j) \in \mathcal{S}$, respectively. The assignment $\mathcal{S}$ is said to be *feasible* if it contains I pairs; otherwise the assignment is called *partial* [96]. Lastly, in terms of the assignment matrix $\mathbf{X}$, the assignment is feasible if constraints (A.10b)-(A.10c) are fulfilled for all $i \in \mathcal{I}$ and $j \in \mathcal{J}$.

An important notion for the correct operation of the auction algorithm is the *ϵ-complementary slackness* (ϵ-CS), which relates a partial assignment $\mathcal{S}$ and a price vector $\hat{\mathbf{p}} = [\hat{p}_1 \dots \hat{p}_J]$. In practice, the DL user j that supports more interference from a UL user i will get a higher price, i.e., the prices reflect how much a UL user i is willing pay to connect to DL user j. The couple $\mathcal{S}$ and $\hat{\mathbf{p}}$ satisfy ϵ-CS if for every pair $(i, j) \in \mathcal{S}$, DL user j is within ϵ of being the best candidate pair for UL user i [96], i.e.,

$$c_{ij} - \hat{p}_j \geq \max_{k \in \mathcal{A}(i)} \{c_{ik} - \hat{p}_k\} - \epsilon, \forall (i, j) \in \mathcal{S}. \tag{A.11}$$

For the sake of clarity we define $c_{ij} - \hat{p}_j$ as the utility that UL user i can obtain from DL user j. The auction algorithm is iterative, where each iteration starts with a partial assignment and the algorithm terminates when a feasible assignment is obtained.

The iteration process consists of two phases: the *bidding* and the *assignment*. In the bidding phase, each UL user bids for a DL user that maximizes the associated utility ($c_{ij} - \hat{p}_j$), and the BS evaluates the bid received from the UL users. In the assignment phase, the BS (responsible for the transmission to DL users), selects the UL user with the highest bid and updates the prices. In fact, this bidding and assignment process implies that the UL users select the DL users which they will be paired with. However, the information exchange occurs between UL users and the BS, rather than between the UL-DL users. Therefore, we propose a forward auction in which the UL users and the BS determines the pairing of UL-DL users in a distributed manner, where the UL is responsible for bidding and the BS for the assignment phase.

In the following, we present some necessary definitions for the iteration process. First, we define v_i as the maximum utility achieved by UL user i on the set of possible DL users $\mathcal{A}(i)$, which is given by

$$v_i = \max_{j \in \mathcal{A}(i)} \{c_{ij} - \hat{p}_j\}. \tag{A.12}$$

The selected DL user j_i is the one that maximizes v_i, which is given by

$$j_i = \arg\max_{j \in \mathcal{A}(i)} v_i. \tag{A.13}$$

The best utility offered by other DL users than the selected j_i is denoted by w_i and is given by

$$w_i = \max_{j \in \mathcal{A}(i), j \neq j_i} \{c_{ij} - \hat{p}_j\}. \tag{A.14}$$

The bid of UL user i on resource j_i is given by

$$b_{ij_i} = c_{ij} - w_i + \epsilon. \tag{A.15}$$

Let $\mathcal{P}(j)$ denote the set of UL users from which DL user j received a bid. In the assignment phase the prices are updated based on the highest bid received, which is given by

$$\hat{p}_j = \max_{i \in \mathcal{P}(j)} \{b_{ij}\}. \tag{A.16}$$

Subsequently, the BS adds the pair (i_j, i) to the assignment $\mathcal{S}$, where i_j refers to the UL user $i \in \mathcal{P}(j)$ that maximizes b_{ij} in eq. (A.16) for DL user j. At UL user i, $\hat{\mathbf{P}}_i = [\hat{P}_{i1} \dots \hat{P}_{iJ}]$ denotes the price vector of associating with DL user j and it is informed by the BS. Differently from $\hat{P}_{ij}$, $\hat{p}_j$ is the up-to-date maximum price of DL user j.

Summarizing, in the bidding phase the UL users need to evaluate v_i, j_i, w_i and b_{ij_i}, whereas in the assignment phase the BS receives the bids and decides to update the prices $\hat{p}_j$ or not. In the next section, we propose the distributed auction that is executed in an asynchronous manner, where the UL users perform the bidding, and the BS performs the assignment.

A.4.3 The Distributed Auction Algorithm

Algorithm 1 and Algorithm 2 show the steps of the iterative process of the bidding and assignment phases, where the bidding is performed at each UL user and the assignment at the BS. We define messages M1, M2, M3 and M4 that enable the exchange of information between the UL users and the BS. Message M1 informs UL user i that the bid was accepted. Message M2 informs that the bid is not high enough and it also contains the most updated price $\hat{p}_j$ of the demanded DL user j_i. Message M3 informs that a feasible assignment was found, which allows the auction algorithm to terminate. Message M4 informs the UL and DL users their respective pairs and transmitting powers. Notice that all these messages can be exchanged between the BS and the UL/DL users using control channels, such as power uplink control channel (PUCCH) and power downlink control channel (PDCCH) [104].

UL user i requires as inputs the benefits $\mathbf{c}_i$ and the ϵ for the bidding phase (see line 1 on Algorithm 1). Then, the price vector is initialized with zero, as well as the associated DL user is initially empty and the set of DL users it can associate with is the set $\mathcal{J}$ (see line 2 in Algorithm 1). The auction algorithm at the UL users will continue until message M3 is received (see line 3 on Algorithm 1). If message M2 is received, UL user i disconnects from the previously associated DL user j_i and set it to $\emptyset$. Next, the price received from the BS is updated (see lines 5-6 on Algorithm 1). Notice that message M2 implies that either the previously associated DL user has a new association or the bid was lower than the current price (the bid was placed with an outdated price).

Subsequently, if UL user i is not associated with a DL user, the bidding phase starts. In this phase, the necessary variables – v_i, j_i, w_i and b_{ij_i} – are evaluated (see lines 9-12 on Algorithm 1). UL user i reports the selected DL user and bid to the BS and wait for the response on line 13 on Algorithm 1. If the response is message M1, then the association to the selected DL user j_i is stored.

Algorithm 1 Distributed Auction Bidding at UL user i

Input: $\mathbf{c}_i, \epsilon$
Define $\mathbf{P}_i = 0, i_j = \varnothing$ and $\mathcal{A}(i) = \mathcal{J}$
while Message M3 is not received **do**
  if Message M2 is received **then**
    Disconnect from previous DL user j_i and set $j_i = \emptyset$
    Update prices $\hat{P}_{ij} = \hat{p}_j$
  end if
  Bidding Phase at the UL users
  if $j_i = \emptyset$ **then**
    Evaluate v_i according to Equation (A.12)
    Select the DL user j_i according to Equation (A.13)
    Evaluate w_i according to Equation (A.14)
    Evaluate the bid b_{ij_i} according to Equation (A.15)
    Report the selected DL user j_i and the bid b_{ij_i} and **wait** response
    if Message M1 is received **then**
      Store the assigned DL user j_i
    end if
  end if
end while
Message M4 is received with the assigned DL user j_i and the power p_i

Algorithm 2 Distributed Auction Assignment at the BS

Estimate the channel gains G_{ib}, G_{bj} and G_{ij} for all UL and DL users
Evaluate the optimal power allocation $\mathbf{p}^u, \mathbf{p}^d$ for every pair (i, j) based on the solution of problem (A.9)
Evaluate c_{ij} and then send to all UL users its respective row ($\mathbf{c}_i = [c_{i1} \dots c_{iJ}]^T$) along with ϵ
Initialize the selected UL users $i_j = \varnothing, \ \forall j$ and $\mathcal{P}(j) = \mathcal{I}, \ \forall j$
Initialize the prices $\hat{p}_j = 0, \ \forall j$ and the assignment matrix $\mathbf{X} = \mathbf{0}$
Assignment Phase at the BS
while $\mathbf{X}$ is not feasible **do**
  if receive request from UL user i **then**
    if $b_{ij} - \hat{p}_j \geq \epsilon$ **then** # Bid accepted
      Update prices according to Equation (A.16)
      Report M2 to the previous assigned user i_j and the updated prices
      Update $i_j = i$ and report M1 to UL user i
      Update assignment $\mathbf{X}$ and if feasible, report M3 to UL user i_j
    else
      Report M2 and the updated prices $\hat{p}_j$ to UL user i
    end if
  else
    Remain assigned to the UL user on iteration t
  end if
end while
Output: $\mathbf{X}, \mathbf{p}^u, \mathbf{p}^d$
Report M4 to the UL and DL user their respective pairs and powers

The BS runs Algorithm 2 and initially needs to acquire or estimate all channel gains from UL and DL users, which can be done using reference signals similar to those standardized by 3rd Generation Partnership Project (3GPP) [104]. Next, the BS evaluates the optimal power allocation $\mathbf{p}^u, \mathbf{p}^d$ for all possible pairs based on the solution of problem (A.9) (see lines 1-2 on Algorithm 2). Then, the assignment benefits c_{ij} are evaluated and the corresponding row of each UL user i is sent (see line 3 on Algorithm 2). The value of ϵ is fixed and sent on line 3 of Algorithm 2. The selected UL users i_j for all DL users j is initially empty, and the set of possible UL users that a DL user may associate is defined as the set of UL users $\mathcal{I}$ (see line 4 on Algorithm 2). The prices $\hat{p}_j$ and the assignment matrix $\mathbf{X}$ are initialized with zero (see line 5).

The assignment phase at the BS continues until the assignment matrix $\mathbf{X}$ is not feasible (see line 6). If the BS receives request from UL user i and the bid is accepted, the prices are updated based on the new bid and the BS reports M2 to the previously assigned user i_j with the updated prices (see lines 6-10 on Algorithm 2). Then, the BS updates the assigned user, reports message M1 to UL user i and update the assignment $\mathbf{X}$ (see lines 11-12 on Algorithm 2). If the new assignment is feasible, the BS reports M3 to all UL users.

However, if the bid proposed by UL user i is not accepted, then the BS reports M2 to UE i with the updated prices (see line 14 on Algorithm 2). Notice that while the BS does not received requests, the assignment does not change (see line 17). Once a feasible assignment is found and message M3 is sent, the algorithm has as outputs the matrix assignment $\mathbf{X}$ and the power vectors $\mathbf{p}^u$ and $\mathbf{p}^d$. With the assignment and the power vectors, message M4 is sent to UL and DL users with their respective pairs and powers.

Therefore, by using Algorithms 1 and 2, we solve in a distributed manner problem (A.10), but it is important to know how many iterations the algorithms execute until a partial assignment is found, and how far this assignment is from the optimal solution. In order to address all these questions, in Section A.4.4 we show that the algorithms terminate within a bounded number of iterations and that the assignment given at the end is within $I\epsilon$ of being optimal.

A.4.4 Complexity and Optimality

In this subsection we derive a bound on the number of iterations of our proposed distributed auction algorithms in Theorem 2. Moreover, in Theorem 3 we show that the given assignment solution by Algorithms 1 and 2 is within $I\epsilon$ of being optimal.

Theorem 2. Consider I UL users and J DL users in a TNFD network. The distributed auction algorithms 1 and 2 terminate within a finite number of iterations bounded by $IJ^2\lceil\Delta/\epsilon\rceil$, where $\Delta = \max\limits_{\forall i,j} c_{ij} - \min\limits_{\forall i,j} c_{ij}$.

Proof. The proof of this theorem is along the lines of Xu et al. [105, Chapter 5], where we do not provide the complete proof herein due to the lack of space. ■

Theorem 2 shows that our algorithms terminate in a finite number of iterations bounded by $IJ^2\lceil\Delta/\epsilon\rceil$. However, we still need to know how far the solution is from the optimal assignment. In Theorem 3 we show that the feasible assignment at the end of the distributed auction is within $I\epsilon$ of being optimal, and if the benefits c_{ij} are integer and $\epsilon < 1/J$, the solution is optimal.

Theorem 3. Consider problem (A.10). The distributed auction algorithm described by Algorithms 1 and 2 terminate with a feasible assignment that is within $I\epsilon$ of being optimal. This feasible assignment is optimal if $c_{ij}, \forall i, j$ is integer and $\epsilon < 1/J$.

Proof. Once more, the proof of this theorem is along the lines of [105, Chapter 5]. ■

Based on Theorems 2 and 3, the distributed auction solution proposed by Algorithms 1 and 2 terminate within $IJ^2\lceil\Delta/\epsilon\rceil$ iterations, and in addition the feasible assignment at the end of the algorithms is within $I\epsilon$ of being optimal. Notice that in practice the benefits c_{ij} are seldom integer, implying that our solution to problem (A.10) is near optimal. Moreover, since the primal problem (A.2) is MINLP, the duality gap between the primal and dual solution is not zero, i.e., we should also take into account the duality gap on top of the gap between the distributed auction and the optimal assignment for problem (A.10). However, as we show in Section A.5, the gap between the exhaustive primal solution and the distributed auction is small.

A.5 Numerical Results and Discussion

In this section we consider a single cell system operating in the urban micro environment [106]. The maximum number of frequency channels is $F = 25$ that corresponds to the number of available frequency channel blocks in the a 5 MHz long term evolution (LTE) system [106]. The total number of served UE varies between $I + J = 8...50$, where we assume that $I = J$. We set the weights α_i^u and α_j^d based on a *path loss compensation rule*, where $\alpha_i^u = G_{ib}^{-1}$ and $\alpha_j^d = G_{bj}^{-1}$. The parameters of this system are set according to Table A.1.

To evaluate the performance of the distributed auction in this environment, we use the RUdimentary Network Emulator (RUNE) as a basic platform for system simulations and extended it to FD cellular networks. The RUNE FD simulation tool allows to generate the environment of Table A.1 and perform Monte Carlo simulations using either an exhaustive search algorithm to solve problem (A.2) or the distributed auction.

Initially, we compare the optimality gap between the exhaustive search solution of the primal problem (A.2), named herein as E-OPT, the optimal solution of the dual problem (A.8) using the power allocation based on the corner points and the centralized Hungarian algorithm for the assignment, named herein as C-HUN, and finally the solution of the dual problem (A.8) with the optimal power allocation but now with the distributed auction solution for the assignment, named herein as D-AUC. In the following, we compare how the distributed auction solution performs in comparison with a HD system, named herein as HD, and also a basic FD solution with random assignment and equal power allocation (EPA) for UL and DL users, named herein as R-EPA. Notice that since in HD systems two different time slots are required to serve all the UL and DL users, which implies that the sum spectral efficiency is divided by two.

In Figure A.2 we show the sum spectral efficiency between E-OPT, C-HUN and the proposed D-AUC as a measure of the optimality gap. We assume a small system with reduced number of users, 4 UL and DL users, and frequency channels, where we increase its number from 4 to 8. Moreover, we consider a SI cancelling level of β $=-100\,$dB.

Table A.1: *Simulation parameters*

Parameter	**Value**
Cell radius	$100\,\mathrm{m}$
Number of UL UEs $[I = J]$	$[4\ 5\ 6\ 25]$
Monte Carlo iterations	400
Carrier frequency	$2.5\,\mathrm{GHz}$
System bandwidth	$5\,\mathrm{MHz}$
Number of freq. channels $[F]$	$[4\ 5\ 6\ 25]$
LOS path-loss model	$34.96 + 22.7\log_{10}(d)$
NLOS path-loss model	$33.36 + 38.35\log_{10}(d)$
Shadowing st. dev. LOS and NLOS	$3\,\mathrm{dB}$ and $4\,\mathrm{dB}$
Thermal noise power $[\sigma^2]$	$-116.4\,\mathrm{dBm/channel}$
SI cancelling level $[\beta]$	$[-70\ \ -100 - 110]$ dB
Max power $[P^u_{\text{max}}] = [P^d_{\text{max}}]$	$24\,\mathrm{dBm}$
Minimum SINR $[\gamma^u_{\text{th}} = \gamma^d_{\text{th}}]$	[0] dB
Step size $[\epsilon]$	0.1

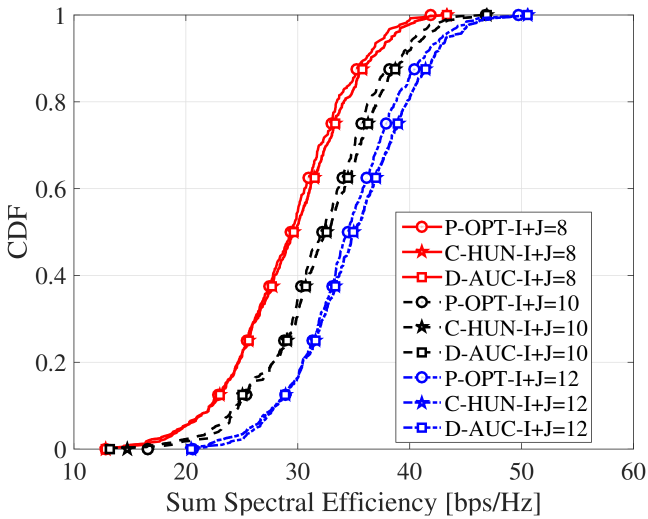

Figure A.2: *CDF of the optimality gap of the sum spectral efficiency for different users' load. We notice that the optimality gap between the proposed D-AUC and the exhaustive search solutions of the primal and dual, E-OPT and C-HUN, is low, which suggests that we can use a distributed solution based on the dual and still be close to the centralized optimal solution of the primal.*

We notice that the differences between the exhaustive search solutions, either E-OPT OR C-HUN, to the D-AUC is negligible, where in some cases the D-AUC achieves a higher performance than E-OPT due to lack of computational power to find the best powers. Figure A.2 clearly shows that the optimality gap is low for the distributed auction when compared to the centralized dual solution (C-HUN) and also to the primal solution (E-

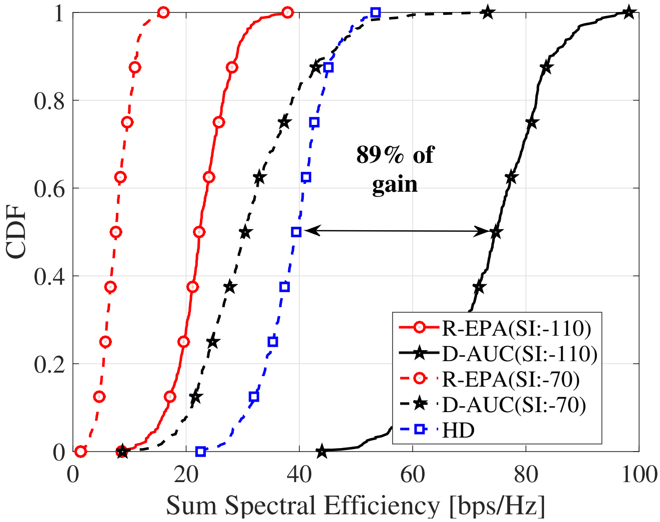

Figure A.3: *CDF of the sum spectral efficiency among all users for different SI cancelling levels. We notice that with $\beta = -110\,\mathrm{dB}$ the UE-to-UE interference is the limiting factor, where D-AUC mitigates this new interference and outperforms the HD mode and the R-EPA. When $\beta = -70\,\mathrm{dB}$, the SI is the limiting factor, where the mitigation of the UE-to-UE interference is not enough to bring gains to FD cellular networks.*

OPT). Therefore, we can use a distributed solution to solve the primal problem (A.2) and still achieve a solution close to the centralized optimal solution.

Figure A.3 shows the sum spectral efficiency between the current HD system, a naive FD implementation named R-EPA, and the proposed distributed solution D-AUC. We assume a small system fully loaded with 25 UL, DL users, and frequency channels, where we analyse the impact of the solutions for different SI cancelling levels of $-110\,\mathrm{dB}$ and $-70\,\mathrm{dB}$, i.e., $\beta = -110\,\mathrm{dB}$ and $-70\,\mathrm{dB}$. Notice that with a SI cancelling level of $-110\,\mathrm{dB}$ we achieve $89\,\%$ relative gain in the spectral efficiency at the 50th percentile, which is close to the expected doubling of FD networks. Moreover, the naive R-EPA performs approximately $43\,\%$ worse than the HD mode, which shows that despite the high SI cancelling level, we do not have any gain of using FD networks. This behaviour shows that we should also optimize the UL-DL pairing and the power allocation of the UL user and of the BS. When the SI level is $-70\,\mathrm{dB}$, HD outperforms the D-AUC and R-EPA with a relative gain of approximately $23\,\%$ and $81\,\%$ at the 50th percentile, respectively. This means that with low SI cancelling levels the D-AUC algorithm is not able to overcome the high self-interference, although the difference to HD is not high. As for the R-EPA, notice that its performance is even worse than before, which once more indicates that we should not use naive implementation of user pairing and power allocation on FD cellular networks. Overall, we notice that when the SI cancelling level is high, the UE-to-UE interference is the limiting factor, where our proposed D-AUC outperforms a naive FD implementation that disregards this interference. When the SI cancelling level is low, then the SI is the limiting factor, where optimizing the UE-to-UE interference is not enough to bring gains to FD cellular networks.

A.6 Conclusion

In this paper we considered the joint problem of user pairing and power allocation in FD cellular networks. Specifically, our objective was to maximize the weighted sum spectral efficiency of the users, where we can tune the weights to sum maximization or path loss compensation. This problem was posed as a mixed integer nonlinear optimization, which is hard to solve directly, thus we resorted to Lagrangian duality and developed a closed-form solution for the assignment and optimal power allocation. Since we were interested in a distributed solution between the BS and the users, we proposed a novel distributed auction solution to solve the assignment problem, whereas the power allocation was solved in a centralized manner. We showed that the distributed auction converges and that it has a guaranteed performance compared to the dual. The numerical results showed that our distributed solution drastically improved the sum spectral efficiency in a path loss compensation modelling, i.e. of the users with low spectral efficiency, when compared to current HD modes when the SI cancelling level is high. Furthermore, we noticed that with a high SI cancelling level, the impact of the UE-to-UE interference is severe and needs to be properly managed. Conversely, when the SI cancelling level is low, a proper management of the UE-to-UE interference is not enough to bring gains to FD cellular networks. Studying the case of asymmetric assignment, that is the case of unequal number of UL and DL users and the impact of the number of iterations and processing delays in the auction algorithm are left for future works.

Paper B

Fast-Lipschitz Power Control and User-Frequency Assignment in Full-Duplex Cellular Networks

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Fast-Lipschitz Power Control and User-Frequency Assignment in Full-Duplex Cellular Networks

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Abstract

In cellular networks, the three-node full-duplex transmission mode has the potential to increase spectral efficiency without requiring full-duplex capability of users. Consequently, three-node full-duplex in cellular networks must deal with self-interference and user-to-user interference, which can be managed by power control and user-frequency assignment techniques. This paper investigates the problem of maximizing the sum spectral efficiency by jointly determining the transmit powers in a distributed fashion, and assigning users to frequency channels. The problem is formulated as a mixed-integer nonlinear problem, which is shown to be non-deterministic polynomial-time hard. We investigate a close-to-optimal solution approach by dividing the joint problem into a power control problem and an assignment problem. The power control problem is solved by Fast-Lipschitz optimization, while a greedy solution with guaranteed performance is developed for the assignment problem. Numerical results indicate that compared with the half-duplex mode, both spectral and energy efficiencies of the system are increased by the proposed algorithm. Moreover, results show that the power control and assignment solutions have important, but opposite roles in scenarios with low or high self-interference cancellation. When the self-interference cancellation is high, user-frequency assignment is more important than power control, while power control is essential at low self-interference cancellation.

B.1 Introduction

In order to meet the need for explosive data volumes and data rates, wireless network operators seek to enhance the spectral efficiency in lower-frequency bands [1], and to exploit higher-frequency bands such as the millimiter waves. The research and standardization communities are currently studying physical layer technologies, including massive MIMO systems, spectrum sharing in mmWave networks, new waveforms, non-orthogonal multiple access technologies, and full-duplex communications [5, 6].

In-band full-duplex (FD) transceivers are expected to improve the attainable spectral efficiency of traditional wireless networks operating with half-duplex (HD) transceivers by a factor of two [6]. Due to recent advancements in mitigating the inherent self-interference (SI) by means of passive suppression, analog and/or digital cancellation, in-band FD technology is quickly approaching the phase of commercial deployments in low-power wireless networks [19].

Although in-band FD promises to instantly double the data capacity of existing technology, its deployment in wireless local area and cellular networks is challenging due to the large number of legacy devices and wireless access points. A viable introduction

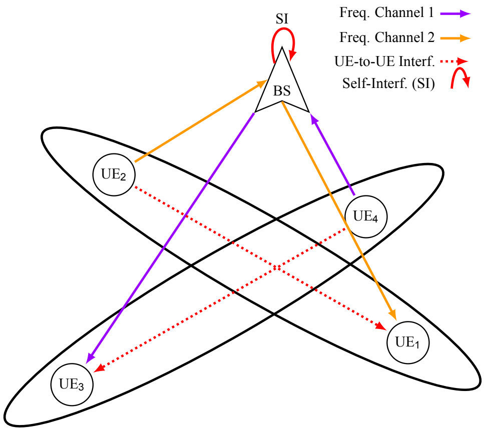

Figure B.1: *A cellular network employing three node FD with two UEs pairs. The base station selects pairs of UEs, represented by the ellipses, and jointly schedules them for FD transmission in the UL and DL. To mitigate UE-to-UE interference (red dotted line), it is advantageous to assign DL/UL users to for FD transmission in the same frequency that are far apart, such as* UE_1*-*UE_2 *and* UE_3*-*UE_4*.*

of FD technology in cellular networks is offered by three-node full-duplex (TNFD) deployments, in which only the wireless access points or base stations (BSs), typically equipped with multiple antennas, implement FD transceivers to support the simultaneous downlink (DL) and uplink (UL) communication with two distinct user equipments (UEs) on the same frequency channel [12]. In TNFD networks, the inherently present UE-to-UE interference may become the performance bottleneck, especially as the capability of FD transceivers to suppress SI improves.

To understand TNFD operations, consider the TNFD network with two UEs pairs in Figure B.1. Unlike when using HD transmissions, a TNFD network must deal with both SI and UE-to-UE interference, indicated by the red dotted lines between UE_1-UE_2 and UE_3-UE_4. The UE-to-UE interference power depends on the UEs locations and propagation environments, as well as the UE transmission powers. Coordination mechanisms are required in order to mitigate the negative effects of interference on the spectral efficiency of the system [10]. The two most important mechanisms are UE pair-frequency assignment and power control [10, 12, 90, 107]; together, these determine which UEs transmit simultaneously on the frequency channels and with which powers the UE and the BS will transmit. Therefore, it is crucial to understand the trade-offs between UL and DL performance of TNFD systems in the design of efficient and fair medium access control protocols, and also coordination mechanisms that help to realize the FD potential even for legacy UEs.

In this paper, we focus on the problem of joint power control and user-frequency channel assignment, assuming TNFD transmissions and a frequency selective wireless environment. Specifically, we investigate the fundamental problem of sum spectral efficiency maximization in a single-cell system. We formulate this problem as a mixed integer nonlinear optimization, which we call the *joint assignment and spectral efficiency maximization* (JASEM) problem. The decision variables are arranged in the assignment matrix, each element of which is a binary variable that indicates the association of an UL user to a DL user and a frequency channel. We show that the JASEM problem is a non-deterministic polynomial-time (NP)-hard problem, implying that no optimal solution in polynomial time can be obtained.

To find a close to optimal solution to JASEM, we decompose it into two parts that correspond to the power control problem and user-frequency channel assignment problem, respectively. It turns out that the assignment problem is also NP-hard. To solve it, we propose a greedy algorithm that has a guaranteed performance with respect to the optimal solution. To solve the power control problem, we develop a novel algorithm that is especially suited for TNFD networks. Our proposed power control algorithm is distributed and sets the signal-to-interference-plus-noise ratio (SINR) targets at each receiver such that the achieved sum rate is close to optimal. The sum rate maximization problem is non-convex, so we propose using the Fast-Lipschitz (FL) optimization [92] to solve it in a distributed and fast manner. A key component of the proposed power control algorithm is that it transforms the problem of finding the transmit power levels into a sequence of feasibility checking sub-problems, where the sum of the spectral efficiency targets is optimized while maintaining the feasibility of the power vector. The second element of the proposed algorithm consists of setting the transmit powers that minimize the sum power consumption and achieve the SINR targets. An important characteristic of the power control algorithm is that it operates in a distributed manner, taking advantage of the TNFD setup, where each transmit-receive pair sets its own transmit power levels based on locally available information typically transmitted by the legacy BSs ir order to facilitate mobility. Although we investigate the single-cell case, the proposed power control algorithm can also be used in multi-cell systems, provided that inter-cell interference measurements are available.

The analytical and numerical results show that the optimality gap between an exhaustive solution of JASEM and the proposed joint solution is small. Our proposed solution outperforms existing methods in the literature, such as random assignment without power control, and the use of HD mode in interference-limited scenarios. With the proposed distributed power control solution, we obtain gains in terms of energy saving, in addition to the spectral efficiency gains in interference-limited scenarios. Our overall finding is that smart assignment solutions are important when the scenario is interference-limited, whereas smart power control solutions are more important in SI-limited scenarios.

The remainder of the paper is organized as follows. Section B.2 discusses relevant and closely related works. Section B.3 presents the system model and main parameters, followed by the problem formulation and the notation used throughout the paper. Section B.4 analyses the power control problem. Using matrix analysis, we arrive at a sequence of feasibility checking sub-problems to maximize the sum spectral efficiency. Since the problem is not convex, we use the framework of FL optimization in Section B.5 to derive a distributed solution to approximate the optimal SINR targets in a distributed manner, and use these targets to obtain the transmit powers, also in a distributed manner. In Section B.6, we study the user-frequency channel assignment problem, for which we propose an approximate greedy solution. We show its guaranteed performance and, for benchmarking purposes, also propose a natural alternative solution to the assignment problem using the Hungarian algorithm for bipartite matching. Section B.7 presents numerical results and compares the performance of the proposed joint solution with alternative assignment and power allocation schemes in UE-to-UE interference limited and SI-limited scenarios.

B.2 Related Works

The impact of FD radios on cellular systems has been analysed recently in [10, 28, 78], which provide valuable insights into the design aspects and performance of such systems in terms of rate and energy performance. However, the problem of distributed joint power and user-frequency channel allocation in TNFD networks has not been addressed in these works.

Power allocation and user assignment have been analysed in [29, 85, 86]. In [86], the authors have used heuristics to assign UL and DL users to frequency channels in an ultra-dense network. In a similar manner, in [29] the authors have addressed the assignment and power allocation for small-cell networks, where the heuristic solutions of the assignment and power allocation algorithm are evaluated for scenarios with Rayleigh fading. The authors in [85] have analysed user-subcarrier assignment via matching theory. However, none of the works mentioned above have investigated a distributed power control and user-frequency channel assignment. We are interested in distributed solutions to offload the operational burden imposed by a large number of diverse devices on the BS.

Other papers have developed distributed algorithms that are applicable in TNFD networks [89, 90, 107]. The authors of [89] have tackled the problem of the UE-to-UE interference from an information theoretic perspective, without relating to resource allocation and power control. In [90], the authors have proposed a distributed power control for general wireless networks using approximation techniques. The proposed framework have not taken into account the specific aspects of TNFD in cellular networks, and thus the resulting performance are suboptimal. We have studied a weighted sum spectral efficiency maximization problem in our previous work [107]. In that paper, we have designed an auction theory based distributed algorithm to the assignment problem without taking into account frequency-selective fading, and assuming centralized power allocation. Notice that frequency selective channel introduces major technical challenges compared to [107] due to its variation between channels for UL and DL UEs, specially for distributed solution mechanisms that require information exchange between users.

A typical and natural objective of many physical layer procedures designed for TNFD cellular networks is to maximize the sum spectral efficiency [32, 79, 87, 88]. The authors in [79] have considered a joint subcarrier and power allocation problem, without taking into account the UE-to-UE interference. Similarly, the authors of [87] have taken into account the UE-to-UE interference, which has led to a formulation of the assignment problem as a 3D-matching problem. The authors have used a centralized water-filling solution to sub-optimally solve the power allocation problem. The work reported in [32] have considered the application of TNFD transmission mode in a cognitive femto-cell scenario with bidirectional transmissions from UEs, and have developed sum-rate optimal resource allocation and power control algorithms. The authors in [88] have considered scheduling and power allocation in multi-cell systems, and have proposed a distributed solution for the user selection and used geometric programming to solve the power allocation problem. However, these works have not considered a distributed solution approach for the power control problem and have not addressed the optimal user-frequency assignment problem.

Fast-Lipschitz (FL) optimization is a recently proposed framework to solve distributed optimization problems over networks by using fixed point iterations [92, 98, 99]. The main characteristic of a FL problem is that the optimal solution is found using fixed point iterations over the constraints, which are known for the fast convergence [96]. In addition, the structure of FL optimization does not rely on convexity or standard interference functions. Due to this fast and general solution approach, FL optimization has been used to solve general power control problems to minimize the sum power with different quality of service constraints [99]. However, FL optimization has not been used in TNFD networks, and the fundamental problem of sum spectral efficiency maximization has not been addressed before.

In the light of this survey of related literature, the main contributions of this paper are as follows:

- The problem formulation of JASEM and the proposed joint distributed power control using FL optimization and greedy assignment solution are new. The joint formulation to maximize the sum spectral efficiency is a natural objective in TNFD networks, and appears as a complex optimization for which no known methods are available.

- Due to the complexity of JASEM, we split the solution into two parts: power control problem and user-frequency assignment. For power control, we use the FL optimization framework to develop a distributed SINR setting close to the optimal and a power control solution that minimizes the sum power in the system. We propose a greedy solution to approximate the NP-hard problem of assigning UL and DL users to frequency channels.

- We evaluate the proposed joint solution by a realistic system simulator and gain insights that help design TNFD cellular networks operating in scenarios with both high and low SI capabilities.

B.3 System Model and Problem Formulation

B.3.1 System Model

We consider a single-cell cellular system in which the BS is FD capable, whereas the UEs served by the BS are only HD capable, as illustrated by Figure B.1. In Figure B.1, the BS is subject to SI and the UEs transmitting in the UL (UE_2 and UE_4) cause UE-to-UE interference to co-scheduled UEs receiving in the DL, that is to UE_1 and UE_3 respectively.

The number of UEs in the UL and DL are denoted by I and J, respectively, which are constrained by the total number of frequency channels in the system F, i.e., $I \leq F$ and $J \leq F$. Accordingly, the sets of UL and DL users are denoted by $\mathcal{I} = \{1, \ldots, I\}$ and $\mathcal{J} = \{1, \ldots, J\}$ respectively, and the set of frequency channels is denoted by $\mathcal{F} = \{1, \ldots, F\}$.

We assume a frequency selective environment, such that the composite channel gains depend on the frequency and consist of small- and large-scale fading. Let G_{ibf} denote the channel gain between transmitter UE i and the BS on frequency channel f, G_{bjf} denote the channel gain between the BS and the receiving UE j on frequency channel f, and G_{ijf} denote the interfering channel gain between the UL transmitter UE i and the DL receiver UE j on frequency channel f. To take into account the residual SI power that leaks to the receiver, we define β as the SI cancellation coefficient, such that the SI power at the receiver of the BS is βP_j^d when the transmit power is P_j^d.

The vector of transmit powers in the UL for all the UEs is denoted by $\mathbf{p^u} = [P_1^u \ldots P_I^u]^T$, whereas the vector of DL transmit powers by the BS is denoted by $\mathbf{p^d} = [P_1^d \ldots P_J^d]^T$. As illustrated in Figure B.1, the UE-to-UE interference depends heavily on the geometry of the co-scheduled UL and DL users, i.e., the *pairing* of UL and DL users, which also depends on the frequency channels assigned to UL and DL users. Therefore, UE pairing and frequency channel allocation are key functions of the system. Accordingly, we define the assignment matrix, $\mathbf{X} \in \{0, 1\}^{I \times J \times F}$, such that

$$x_{ijf} = \begin{cases} 1, & \text{if UE}_i \text{ is paired with UE}_j \text{ on frequency } f, \\ 0, & \text{otherwise.} \end{cases}$$

The SINR at the BS of transmitting user i and the SINR at the receiving user j of the BS, both assigned to frequency channel f, are given by

$$\gamma_{if}^u = \frac{P_i^u G_{ibf}}{\sigma^2 + \sum_{j=1}^J x_{ijf} P_j^d \beta}, \ \gamma_{jf}^d = \frac{P_j^d G_{bjf}}{\sigma^2 + \sum_{i=1}^I x_{ijf} P_i^u G_{ijf}}, \tag{B.1}$$

respectively, where x_{ijf} in the denominator of γ_{if}^u accounts for the SI at the BS, whereas x_{ijf} in the denominator of γ_{jf}^d accounts for the UE-to-UE interference caused by UE$_i$ to UE$_j$ on frequency f, and σ^2 is the noise power.

Thus, the achievable spectral efficiency by each user is given by the Shannon equation (in bits/s/Hz) for the UL and DL as

$$C_i^u = \sum_{f=1}^F C_{if}^u = \sum_{f=1}^F \log_2(1 + \gamma_{if}^u), \tag{B.2a}$$

$$C_j^d = \sum_{f=1}^F C_{jf}^d = \sum_{f=1}^F \log_2(1 + \gamma_{jf}^d), \tag{B.2b}$$

where the sums are taken along the frequency dimension because at most one SINR is non-zero for every UL and DL user.

B.3.2 Problem Formulation

Our goal is to devise the pairing and assignment of UEs in the UL and DL to frequency channels, that maximize the sum spectral efficiency over all users. Specifically, we formulate the joint assignment and spectral efficiency maximization (JASEM) problem as

$$\begin{aligned}
\underset{\mathbf{X},\mathbf{p}^u,\mathbf{p}^d}{\text{maximize}} \quad & \sum_{i=1}^{I} C_i^u + \sum_{j=1}^{J} C_j^d && \text{(B.3a)}\\
\text{subject to} \quad & P_i^u \le P_{\max}^u,\ \forall i, && \text{(B.3b)}\\
& P_j^d \le P_{\max}^d,\ \forall j, && \text{(B.3c)}\\
& \sum_{j=1}^{J}\sum_{f=1}^{F} x_{ijf} \le 1,\ \forall i, && \text{(B.3d)}\\
& \sum_{i=1}^{I}\sum_{f=1}^{F} x_{ijf} \le 1,\ \forall j, && \text{(B.3e)}\\
& \sum_{i=1}^{I}\sum_{j=1}^{J} x_{ijf} \le 1,\ \forall f, && \text{(B.3f)}\\
& x_{ijf} \in \{0,1\},\ \forall i,j,f. && \text{(B.3g)}
\end{aligned}$$

The main optimization variables are $\mathbf{p}^u$, $\mathbf{p}^d$ and $\mathbf{X}$. Constraints (B.3b) and (B.3c) limit the transmit powers per-user and per-channel DL power constraint, whereas constraints (B.3d)-(B.3f) assure that at most one UE in the DL can share the frequency resource with a UE in the UL and vice-versa. Note that if $I \neq J$, some users cannot be paired and will transmit in HD mode if there are frequency channels available.

Problem (B.3) belongs to the category of mixed integer nonlinear programming (MINLP), which is known for its high complexity and computational intractability. In addition, problem (B.3) belongs to the category of 3-D nonlinear assignment problems, where the problem with linear objective function is known to be NP-hard [108, Section 10.2]. Therefore, to arrive at a close to optimal solution to problem (B.3), we will split it into two parts corresponding to power control and frequency channel assignment problems. For power control, we propose to use Fast-Lipschitz optimization [92, 98] to develop a distributed SINR target setting solution. With the SINR targets from Fast-Lipschitz optimization , we find the power vectors that minimize the sum power consumption in a distributed fashion. As for the frequency channel assignment, it is known to be an axial 3-dimensional assignment problem (3-DAP) [108, Section 10.2], and it is NP-Hard, which motivates us to propose a greedy solution with guaranteed performance. We use the proposed greedy solution for the frequency assignment along with the distributed power control from Fast-Lipschitz optimization. We remark that the problem formulation (B.3) is original, and the solution approach we develop in the following is original as well.

B.3.3 Notation

Vectors and matrices are denoted by bold lower and upper case letters, respectively. We denote by $\mathbf{I}$ the identity matrix, and by $\mathbf{0}$ a vector or matrix where all elements are zero.

The gradient of a function $\mathbf{f}(\mathbf{x})$ is defined as the transpose of the Jacobian matrix, i.e., $[\nabla \mathbf{f}(\mathbf{x})]_{ij} = \partial f_j(\mathbf{x})/\partial x_i$, whereas $\nabla_i \mathbf{f}(\mathbf{x})$ denotes the ith row of $\nabla \mathbf{f}(\mathbf{x})$. Note that $\nabla \mathbf{f}(\mathbf{x})^k = (\nabla \mathbf{f}(\mathbf{x}))^k$, which is not to be confused with the kth derivative. The spectral radius is denoted $\rho(\cdot)$. Vector norms are denoted $|||\cdot|||$ and matrix norms are denoted $\|\cdot\|$. Unless specified $|||\cdot|||$ and $\|\cdot\|$ denote arbitrary norms. We denote by $|||\mathbf{A}|||_\infty = \max_i \sum_j |A_{ij}|$ the norm induced by the ℓ_∞ vector norm, where $\|\mathbf{x}\|_{\ell_\infty} = \max_i |x_i|$. These matrix norm definitions are coherent with [109]. All inequalities in this paper are intended *element-wise*, i.e., $\mathbf{A} \geq \mathbf{B}$ means $A_{ij} \geq B_{ij}$ for all i, j.

B.4 Power Control Analysis for JASEM

To develop a solution close to the optimal solution of problem (B.3), we split the problem into two parts corresponding to power control and frequency channel assignment problems. We first concentrate on the power allocation problem, and assume the assignment is already performed, whereas in Section B.6 we provide an approximate solution and a benchmark solution to the assignment problem. To this end, let us assume N pairs of UL and DL users, where $N = I = J$, and that a specific pair $n \triangleq (k, l)$ contains one UL and DL UEs sharing a frequency channel. In this case, the sum spectral efficiency maximization is formulated as:

$$\underset{\mathbf{p}^u, \mathbf{p}^d}{\text{maximize}} \quad \sum_{i=1}^{I} C_i^u + \sum_{j=1}^{J} C_j^d, \tag{B.4a}$$

$$\text{subject to} \quad \mathbf{p}^u, \mathbf{p}^d \in \mathcal{P}, \tag{B.4b}$$

where $\mathcal{P}$ is the set in which the power allocation constraints (B.3b)-(B.3c) are fulfilled. From results of [101], we know that the optimal transmit power allocation will have either P_i^u or P_j^d equal to zero or, $P_{\max}^u$ or $P_{\max}^d$, given that i and j share a frequency channel and form a pair. However, future cellular networks are expected to move from a fully centralized to a more distributed architecture [8], offloading the computation and orchestration burden imposed by a large number of diverse devices on the BS. With this objective in mind, we wish a distributed power control solution such that the UL and DL users can autonomously set their transmit powers and decrease the burden on the BS.

First, we rewrite problem (B.4) in a shortened form by defining $\mathbf{p} = [\mathbf{p}^u \, \mathbf{p}^d]^T$ and $K \triangleq I + J$. Similarly, we define the vector $\mathbf{G}_b = [\sigma^2/G_{1b}^u \ldots \sigma^2/G_{Ib}^u \ \sigma^2/G_{b1}^d \ldots \sigma^2/G_{bJ}^d]^T$ as the noise power divided by the interesting channel gains; and the matrix containing the interfering channel gains $\mathbf{F}$ of size $K \times K$ whose terms are defined as:

$$F_{lk} = \begin{cases} 0, & \text{if } x_{kl} = 0, \\ \beta / G^u_{kb}, & \text{if } k \leq I \text{ and } I \leq l \leq K, \\ G_{lk} / G^d_{bk}, & \text{if } I \leq k \leq K \text{ and } l \leq I. \end{cases} \tag{B.5}$$

Note that the index $k \leq I$ indicates a UL user, while $I \leq k \leq K$ indicates a DL user. We can now reformulate the SINR in Eq. (B.1) for the kth user sharing the frequency channel with the lth user in pair $n = (k, l)$ as

$$\gamma_k(\mathbf{p}) = \frac{P_k}{G_{bk} + (\mathbf{F}\mathbf{p})_k} = \frac{P_k}{G_{bk} + P_l F_{lk}}, \tag{B.6}$$

where P_k is the transmit power of user k, G_{bk} is the kth element of the column vector $\mathbf{G}_b$, and the expression $(\mathbf{F}\mathbf{p})_k$ stands for the kth element of the column vector $\mathbf{F}\mathbf{p}$, which results in a single term in the denominator of (B.6), representing the interference caused by user l to user k. Using this representation, we write problem (B.4) in a joint formulation of UL and DL users as:

$$\underset{\mathbf{p}}{\text{maximize}} \quad \sum_{k=1}^{K} \alpha_k \log_2(1 + \gamma_k(\mathbf{p})) \tag{B.7a}$$

$$\text{subject to} \quad P_k \leq P^{(k)}_{\text{max}}, \forall k, \tag{B.7b}$$

where $P^{(k)}_{\text{max}}$ is equal to P^u_{max} or P^d_{max} if k is a UL or DL user, respectively.

B.4.1 Problem Transformation

As a first step to solving problem (B.7), we consider the standard equivalent hypograph [110, Sec. 3.1.7] form of problem (B.7), in which one new variable and one new constraint are included:

$$\underset{\mathbf{p},\mathbf{t}}{\text{maximize}} \quad \sum_{k=1}^{K} t_k \tag{B.8a}$$

$$\text{subject to} \quad t_k \leq \alpha_k \log(1 + \gamma_k(\mathbf{p})), \forall k, \tag{B.8b}$$

$$P_k \leq P^{(k)}_{\text{max}}, \forall k, \tag{B.8c}$$

where the new variables $t_k > 0$ can be interpreted as spectral efficiency targets. We set the weights α_k as $1/\log(2)$ so that in Eq. (B.8b) we have the natural logarithm. Expanding the constraint (B.8b), we get

$$\begin{aligned}
\alpha_k \log(1+\gamma_k(\mathbf{p})) &\geq t_k, \\
\frac{P_k}{G_{bk}+P_l F_{lk}} &\geq (\exp(t_k/\alpha_k)-1), \\
P_k &\geq (\exp(t_k/\alpha_k)-1)(G_{bk}+P_l F_{lk}), \\
\mathbf{p} &\geq \mathbf{T}(\mathbf{G}_b+\mathbf{F}\mathbf{p}), \\
(\mathbf{I}_K-\mathbf{T}\mathbf{F})\mathbf{p} &\geq \mathbf{T}\mathbf{G}_b, \qquad \text{(B.9)}
\end{aligned}$$

where $\mathbf{T}$ denotes a diagonal matrix of size $K \times K$ with entries $(\exp(t_k/\alpha_k)-1)$. Notice that each diagonal entry $(\exp(t_k/\alpha_k)-1)$ of matrix $\mathbf{T}$ can be related to an adaptive SINR target γ_k^{tgt}, i.e., $\gamma_k^{\text{tgt}} = \exp(t_k/\alpha_k)-1$. In the next section, we will transform problem (B.8) into a sequence of *feasibility checking sub-problems*, by means of which the sum of the spectral efficiency targets can be optimized while maintaining the feasibility of the power vector with respect to constraints (B.8c) and (B.9).

The feasibility of the power vector and its relation to the SINR targets are established by the following useful lemma.

Lemma 1. Consider problem (B.8). The diagonal matrix $\mathbf{T}$ of adaptive SINR targets is feasible for problem (B.8) if and only if

$$\rho(\mathbf{T}\mathbf{F}) < 1. \qquad \text{(B.10)}$$

Proof. From inequality (B.9), we notice that $\mathbf{T}\mathbf{F} > \mathbf{0}$ and has a special structure, with non-negative entries only in the position of the interfering user. Thus, from the Perron-Frobenius Theorem [111], there exists a non-negative right eigenvector $\mathbf{p}$ of $\mathbf{T}\mathbf{F}$. Furthermore, we also know that $(\mathbf{I}-\mathbf{T}\mathbf{F})^{-1}$ is non-negative, which implies that the power vector that fulfils inequality (B.9) at equality is uniquely evaluated as

$$\mathbf{p} = (\mathbf{I}-\mathbf{T}\mathbf{F})^{-1}\mathbf{T}\mathbf{G}_b. \qquad \text{(B.11)}$$

Therefore, we can use eq. (B.11) to find the optimal powers for the given SINR targets. ■

From the SINR Eq. (B.6), it is clear that there is exactly one interfering user per frequency channel, which allows to decouple the UL and DL UEs sharing the same frequency channel in problem (B.8). Specifically, let us assume that for pair $n=(k,l)$ we have the matrices $\mathbf{F}^{(n)}$ and $\mathbf{T}^{(n)}$ composed of the entries related to the kth and lth users, respectively. Thus, the matrices $\mathbf{F}^{(n)}$, $\mathbf{T}^{(n)}$ and $\mathbf{T}^{(n)}\mathbf{F}^{(n)}$ can be shown to have the form of

$$\mathbf{F}^{(n)} = \begin{bmatrix} 0 & F_{lk} \\ F_{kl} & 0 \end{bmatrix}, \mathbf{T}^{(n)} = \begin{bmatrix} \gamma_k^{\text{tgt}} & 0 \\ 0 & \gamma_l^{\text{tgt}} \end{bmatrix}, \mathbf{T}^{(n)}\mathbf{F}^{(n)} = \begin{bmatrix} 0 & \gamma_k^{\text{tgt}} F_{lk} \\ \gamma_l^{\text{tgt}} F_{kl} & 0 \end{bmatrix}.$$

Therefore, given the simple structure of the $\mathbf{T}^{(n)}\mathbf{F}^{(n)}$, we can evaluate its eigenvalues as

$$\lambda^{(n)} = \pm\sqrt{\gamma_k^{\text{tgt}}\gamma_l^{\text{tgt}} F_{kl} F_{lk}}. \qquad \text{(B.12)}$$

In addition to Lemma 1, it will be useful to formulate an upper bound on the product of SINR targets

$$\gamma_k^{\text{tgt}}\gamma_l^{\text{tgt}} < \frac{1}{F_{kl}F_{lk}} = \frac{G_{kb}^u G_{bl}^d}{\beta G_{lk}}, \tag{B.13a}$$

$$\gamma_k^{\text{tgt}}\gamma_l^{\text{tgt}} < \frac{G_{kb}^u G_{bl}^d}{\beta G_{lk}}. \tag{B.13b}$$

Rewriting inequality (B.13b) as a function of the initial spectral efficiency targets t_k and t_l, we have the following inequality:

$$\left(\exp\left(\frac{t_k}{\alpha_k}\right)-1\right)\left(\exp\left(\frac{t_l}{\alpha_l}\right)-1\right) < \frac{G_{kb}^u G_{bl}^d}{\beta G_{lk}},$$

$$\exp\left(\frac{t_k}{\alpha_k}+\frac{t_l}{\alpha_l}\right) - \exp\left(\frac{t_k}{\alpha_k}\right) - \exp\left(\frac{t_l}{\alpha_l}\right) < \frac{G_{kb}^u G_{bl}^d}{\beta G_{lk}} - 1. \tag{B.14}$$

In order to transform the strict inequality in (B.14) into a nonstrict inequality, we introduce a predefined parameter $\epsilon < 1$ such that

$$\exp\left(\frac{t_k}{\alpha_k}+\frac{t_l}{\alpha_l}\right) - \exp\left(\frac{t_k}{\alpha_k}\right) - \exp\left(\frac{t_l}{\alpha_l}\right) \leq \epsilon\Big(\frac{G_{kb}^u G_{bl}^d}{\beta G_{lk}} - 1\Big). \tag{B.15}$$

With inequality (B.15) we can assure that the spectral efficiency targets fulfil Lemma 1 if (B.15) holds. Therefore, we can use inequality (B.15) to rewrite problem (B.8) as a function of the spectral efficiency targets t_k and t_l as:

$$\begin{aligned} \underset{t_k, t_l}{\text{maximize}} \quad & t_k + t_l && \text{(B.16a)} \\ \text{subject to} \quad & \text{(B.15)}, \\ & t_k, t_l \geq 0. && \text{(B.16b)} \end{aligned}$$

Notice that inequality (B.15) is not convex. Thus, problem (B.16) is not concave and there is no guarantee that a local optimal value is also globally optimal. To solve this problem in a distributed fashion, we resort to FL optimization [92].

B.5 Fast-Lipschitz SINR Target Updates and Distributed Power Control

Since problem (B.16) is not convex and we are interested in a distributed solution, we use FL optimization to solve the SINR setting problem. The main advantages of this framework is that the objective function or constraints are not required to be convex, and the optimal solution is found using fixed-point iterations, which are known for fast convergence. For clarity, the framework of FL optimization is summarized in Appendix B.9.1. Using this framework, we reformulate problem (B.16) using the relaxation of the FL form for problems with fewer constraints than variables, which is given in [98, Section V.B]. Consider a partitioned variable $\mathbf{x} = [t_k \;\; t_l]^T \in \mathcal{X} \subset \mathbb{R}^2$ and the following optimization problem:

$$\underset{\mathbf{x}}{\text{maximize}} \quad f_0(\mathbf{x}) \tag{B.17a}$$
$$\text{subject to} \quad t_k \leq f_{t_k}(\mathbf{x}), \tag{B.17b}$$
$$\mathbf{x} \in \mathcal{X}, \tag{B.17c}$$

where the functions $f_0(\mathbf{x})$, $f_{t_k}(\mathbf{x})$ and the set $\mathcal{X}$ are defined as

$$f_0(\mathbf{x}) = t_k + t_l, \tag{B.18a}$$
$$f_{t_k}(\mathbf{x}) = t_k - \gamma h(\mathbf{x}), \tag{B.18b}$$
$$\mathcal{X} = \left\{ \mathbf{x} : \begin{cases} t_k \in \mathcal{X}_k = \{t_k : a_k \leq t_k \leq b_k\} \\ t_l \in \mathcal{X}_l = \{t_l : a_l \leq t_l \leq b_l\} \end{cases} \right\},$$

where for clarity and for the sake of easy of presentation in the sequel, we introduce the functions $h(\mathbf{x}),\ u(\mathbf{x}), v(\mathbf{x})$ as:

$$h(\mathbf{x}) \triangleq \exp\left(\frac{t_k}{\alpha_k}\right)\left(\exp\left(\frac{t_l}{\alpha_l}\right) - 1\right) - \exp\left(\frac{t_l}{\alpha_l}\right) + \left(1 - \frac{G^u_{kb} G^d_{bl}}{\beta G_{lk}} + \epsilon\right), \tag{B.19a}$$
$$u(\mathbf{x}) \triangleq \frac{1}{\alpha_k} \exp\left(\frac{t_k}{\alpha_k}\right)\left(\exp\left(\frac{t_l}{\alpha_l}\right) - 1\right), \tag{B.19b}$$
$$v(\mathbf{x}) \triangleq \frac{1}{\alpha_l} \exp\left(\frac{t_l}{\alpha_l}\right)\left(\exp\left(\frac{t_k}{\alpha_k}\right) - 1\right). \tag{B.19c}$$

Note that γ in Eq. (B.18b) is the positive parameter used in [92, Section II.B] to transform constraint (B.15) into function $f_{t_k}(\mathbf{x})$, and that the upper bounds b_k, b_l can be understood as the maximum achievable spectral efficiency without interference. With this reformulation, we now state the following lemma to establish that problem (B.17) is Fast-Lipschitz.

Lemma 2. Consider optimization problem (B.17). If $v(\mathbf{x}) < u(\mathbf{x})$, and the parameter γ is constrained as

$$\frac{1}{2u(\mathbf{x})} < \gamma < \frac{1}{u(\mathbf{x})}, \tag{B.20}$$

then problem (B.17) is Fast-Lipschitz.

Proof. See Appendix B.9.2. ■

If the conditions of Lemma 2 are fulfilled, then we can find t_k using fixed point iterations. Using the proof in Appendix B.9.2, we fix t_l and find the optimal t_k from fixed point iterations $t_k^{(n+1)} = f_{t_k}(\mathbf{x})^{(n)}$. Thus, we need to keep updating t_l in order to obtain the solution that maximizes problem (B.17). Section B.5.1 shows a decentralized algorithm that uses the FL solution to the power control problem and can be executed by the UL and DL UEs in a distributed fashion.

Algorithm 3 FL Optimization to find t_k

1: Input: $G^u_{kb},\ G^d_{bl},\ G_{lk},\ \beta,\ \alpha_k,\ \alpha_l, t_k^{(0)}, a_k, b_k, t_l, \delta_p, \delta_a$
2: Evaluate $u(\mathbf{x}),\ v(\mathbf{x}),\ h(\mathbf{x})$ from Eqs. (B.19)
3: Set $\gamma^{(0)} = \frac{3}{4u(\mathbf{x})}$ and evaluate $f_{t_k}(\mathbf{x})^{(0)}$
4: Set $n = 0$
5: **while** $\left|t_k^{(n)} - t_k^{(n-1)}\right| < \delta_p$ **do**
6: $\quad n \leftarrow n + 1$
7: $\quad$ Update $t_k^{(n)} = \max\left\{\min\left\{f_{t_k}(\mathbf{x})^{(n-1)}, b_k\right\}, a_k\right\}$
8: $\quad$ Update $u(\mathbf{x}),\ v(\mathbf{x}),\ h(\mathbf{x})$ from Eqs. (B.19)
9: $\quad$ Set $\gamma^{(n)} = \frac{3}{4u(\mathbf{x})}$ and evaluate $f_{t_k}(\mathbf{x})^{(n)}$
10: $\quad$ **if** $\left|t_k^{(n)} - t_k^{(n-1)}\right| = \left|t_k^{(n-1)} - t_k^{(n-2)}\right|$ and $|h(\mathbf{x})| > \delta_a$ **then**
11: $\qquad$ Stop the algorithm and set $\zeta = 1$
12: $\quad$ **end if**
13: **end while**
14: **Output**: $t_k,\ \zeta$

B.5.1 Distributed SINR Target Updates using Fast-Lipschitz

Algorithm 3 comprises the steps that are necessary to find the optimal t_k, t_l of the FL optimization. The input variables of Algorithm 3 are the channel gains $G^u_{kb},\ G^d_{bl},\ G_{lk}$, and the SI cancellation term β. In practice, the BS would acquire or estimate all channel gains from UL and DL users, which can be done performing measurements on reference signals similar to those standardized by 3rd Generation Partnership Project (3GPP) [104]. The algorithm also needs the initial values for t_k, denoted as $t_k^{(0)}$, the upper and lower limits of t_k, denoted by a_k and b_k, the initial point for t_l, and some precision targets for convergence denoted by δ_p, δ_a. The precision targets can be predefined and broadcast by the BS, whereas the lower bounds are zero and the upper bounds can be sent via measurements to each user based on the channel gains and signal-to-noise ratio (SNR). The initial value for $t_k^{(0)}$ is decided based on the t_l given as input.

Initially, $u(\mathbf{x}),\ v(\mathbf{x}),\ h(\mathbf{x})$ are evaluated. Next, γ is set as the midpoint of the interval defined in Eq. (B.20) and the function $f_{t_k}(\mathbf{x})$ according to Eq. (B.18b) is evaluated (lines 2 and 3). Then, the algorithm updates t_k using the fixed point iterations and ensuring that $a_k \le t_k^{(n)} \le b_k$ (line 7). In the next step, $u(\mathbf{x}),\ v(\mathbf{x}),\ h(\mathbf{x}),\ \gamma$ and $f_{t_k}(\mathbf{x})$ are updated. The algorithm iterates until $\left|t_k^{(n)} - t_k^{(n-1)}\right| < \delta_p$ (line 5), which implies that the result δ_p is close to the fixed point.

As another stopping criterion, if $t_k^{(n)}$ has reached the upper/lower bound twice, the algorithm stops (line 10). This happens when the first stopping criterion cannot be fulfilled while keeping $t_k^{(n)}$ within the predefined bounds. We define this situation as an infeasibility. Note that the algorithm also enforces $|h(\mathbf{x})| > \delta_a$, so that the bound can be reached if $|h(\mathbf{x})|$ is small enough (from Eq. (B.15) it should be zero). Accordingly, $\zeta = 1$ is set to highlight that the initial value given to t_l does not lead to a feasible t_k. As outputs, the algorithm has the variable t_k and the infeasibility check ζ. From now on, we will refer to

Algorithm 4 Golden Search to find t_l

1: Input: $G^u_{kb},\ G^d_{bl},\ G_{lk},\ \beta,\ \alpha_k,\ \alpha_l, a_k, b_k, a_l, b_l$
2: Define $\phi \triangleq (-1+\sqrt{5})/2$
3: Evaluate $t_{l_1} = a_l\phi + (1-\phi)b_l$ and $t_{l_2} = (1-\phi)a_l + \phi b_l$
4: Evaluate $t_k|_{t_{l_1}}$ and $t_k|_{t_{l_2}}$ from Algorithm 3
5: **while** $t_{l_2} - t_{l_1} < \delta_g$ **do**
6:   **if** $(1-\zeta|_{t_{l_1}})(t_k|_{t_{l_1}} + t_{l_1}) \geq (1-\zeta|_{t_{l_2}})(t_k|_{t_{l_2}} + t_{l_2})$ **then**
7:     Update intervals: $b_l \leftarrow t_{l_2}, t_{l_2} \leftarrow t_{l_1}$
8:     Update t_k and t_l: $t_k \leftarrow t_k|_{t_{l_1}}$ and $t_l \leftarrow t_{l_1}$
9:     Update the evaluated value for $t_k|_{t_{l_2}}$: $t_k|_{t_{l_2}} \leftarrow t_k|_{t_{l_1}}$
10:     Update the new point to search: $t_{l_1} = a_l\phi + (1-\phi)b_l$
11:     Evaluate $t_k|_{t_{l_1}}$ from the new t_{l_1} using Algorithm 3
12:   **else**
13:     Update intervals: $a_l \leftarrow t_{l_1}, t_{l_1} \leftarrow t_{l_2}$
14:     Update t_k and t_l: $t_k \leftarrow t_k|_{t_{l_2}}$ and $t_l \leftarrow t_{l_2}$
15:     Update the evaluated value for $t_k|_{t_{l_1}}$: $t_k|_{t_{l_1}} \leftarrow t_k|_{t_{l_2}}$
16:     Update the new point to search: $t_{l_2} = a_l\phi + (1-\phi)b_l$
17:     Evaluate $t_k|_{t_{l_2}}$ from the new t_{l_2} using Algorithm 3
18:   **end if**
19: **end while**
20: **Output**: $t_l,\ t_k$

the outputs as $t_k|_{t_l}$ and $\zeta|_{t_l}$ to highlight that the outputs were evaluated with the input t_l.

Next, a unidimensional search algorithm is used to update t_l. Notice that the objective function has only one minimum, because problem (B.17) is Fast-Lipschitz and has a unique Pareto optimal solution [92]. To find this unique minimum, we propose to use the Golden Section search [112], because it does not rely on finding derivatives and requires only one new computation at every iteration as opposed to the widely used bisection method (see Algorithm 4).

In this algorithm the golden ratio ϕ is used as the constant reduction factor of the interval. Subsequently, the two points $t_{l_1},\ t_{l_2}$ are defined followed by evaluating t_k using the FL optimization in Algorithm 3 (lines 3-4). Then, the algorithm checks which part of the interval gives the higher sum $t_k + t_l$, if it is the lower part of the interval containing t_{l_1} (see line 6), namely $[a_l, t_{l_2}]$, or the upper part containing t_{l_2} (see line 12), namely $[t_{l_1}, b_l]$. Notice that the feasibility of the solution is also checked, on which the algorithm considers ζ in the sum.

Depending on the interval (lower or upper), the algorithm updates the upper/lower bound and the respective values of t_k (see lines 7-9 and 13-15). With this, we update the current value of t_k and t_l (see lines 8-14). In the following, t_{l_1} (t_{l_2} in the respective upper interval) is updated and the algorithm finds t_k for the new given value of t_l (see lines 10-11 and 16-17). The algorithm stops once the desired precision δ_g is achieved.

An important aspect of the Golden Search solution in Algorithm 4 is splitting the interval such that the solution used for t_k remains feasible. To this end, the algorithm must verify that the solution is feasible by considering ζ in the sum. However, we need to incorporate into the algorithm that when the infeasibility happens, it removes the branch of t_l that leads to infeasibility. To this end, the following Lemma 3 will be useful (see line 6).

Lemma 3. Let $t_l,\ t_k$ be given by Algorithm 4. Suppose that such t_l and t_k make Algorithm 3 infeasible. Then, the Golden Search solution cuts the branch that resulted in t_l and leads to the feasibility direction, which is the other branch of the search.

Proof. See Appendix B.9.3. ∎

Therefore, we can now solve at optimality the SINR setting problem (B.16). In the following subsection, we use the SINR targets obtained as the solution of the FL optimization to evaluate the power vector in a distributed manner.

B.5.2 Distributed Power Control

Using the optimal SINR targets obtained as the solution to problem (B.16), we can now develop a distributed power control algorithm based on the definition of standard interference functions [113], which for clarity is recalled below.

Definition 4. A function $\mathcal{I}(\mathbf{p}) : \mathbb{R}^n \to \mathbb{R}$ is a standard interference function if for all vector $\mathbf{p} \geq 0$ the following properties are satisfied:

- Positivity: $\mathcal{I}(\mathbf{p}) > 0$ for all $\mathbf{p}$;
- Monotonicity: If $\mathbf{p} \geq \mathbf{p}'$, then $\mathcal{I}(\mathbf{p}) > \mathcal{I}(\mathbf{p}')$;
- Scalability: For all scalar $\alpha > 1$, $\alpha\mathcal{I}(\mathbf{p}) > \mathcal{I}(\alpha\mathbf{p})$.

Notice that Eq. (B.9) can be written as an inequality based on an appropriate interference function $\mathcal{I}^W(\mathbf{p})$, as follows:

$$P_k \geq \mathcal{I}_k^W(\mathbf{p}) = \gamma_k^{\text{tgt}}(G_{bk} + F_{lk}P_l), \tag{B.21}$$

$$P_l \geq \mathcal{I}_l^W(\mathbf{p}) = \gamma_l^{\text{tgt}}(G_{bl} + F_{kl}P_k). \tag{B.22}$$

Using this form, it is straightforward to determine the power control update iterations that minimize the sum power and achieve the predefined SINR targets. The following lemma shows that with given feasible SINR targets, the transmit powers for the pair of UL-DL users sharing a frequency channel can be determined in a distributed manner while minimizing the sum power.

Lemma 4. Consider a feasible set of SINR targets for users k and l sharing a frequency channel. The power vector that minimizes the sum transmit power of the two users while attaining the SINR targets is found in a distributed manner as:

$$P_k(n+1) = \min\left\{P_{\max}^{(k)}, \mathcal{I}_k^W(\mathbf{p}(n))\right\}, \tag{B.23a}$$

$$P_l(n+1) = \min\left\{P_{\max}^{(l)}, \mathcal{I}_l^W(\mathbf{p}(n))\right\}, \tag{B.23b}$$

where n denotes the iteration counter.

Proof. From [113], it can be proved that the power vector found from Eqs. (B.23) denotes a sequence of monotone decreasing feasible power vectors that converge to a unique fixed point $\mathbf{p}^\star$ [113, Lemma 1], which minimizes the sum power that attain the SINR targets. If the SINR targets lead to a power higher than the maximum in (B.23), the proposed algorithm projects this power onto the set $\mathcal{P}$ defined by the $P_k \in [0, P_{\max}]$. ■

From the results of [101] we know that either user k or l or both transmit at maximum power. Therefore, in practice, $P_k(0) = P_{\max}^{(k)}$ and $P_l(0) = P_{\max}^{(l)}$ is typically a good starting point.

B.6 Assignment Solutions for JASEM

With the distributed solution to the powers, we are now interested in a solution for the frequency channel assignment. In this section we assume a set of fixed powers – such as the maximum power allocation ($P_{\max}^u$ and $P_{\max}^d$) – for both UL and DL transmissions. Recall that the optimal powers will have either P_i^u or P_j^d equal to zero, or $P_{\max}^u$ or $P_{\max}^d$, or both with $P_{\max}^u$ and $P_{\max}^d$, which motivates us to consider both users transmitting at maximum power. Since we cannot provide the optimal powers without knowing the assignment, we reasonably assume fixed powers and later optimize the powers using what we developed in sections B.4-B.5. With this assumption, we have the following combinatorial problem in binary variables $\mathbf{X}$:

$$\begin{aligned}
\underset{\mathbf{X}}{\text{maximize}} \quad & \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{f=1}^{F} s_{ijf}x_{ijf} && \text{(B.24a)}\\
\text{subject to} \quad & \sum_{j=1}^{J}\sum_{f=1}^{F} x_{ijf} = 1,\ \forall i, && \text{(B.24b)}\\
& \sum_{i=1}^{I}\sum_{f=1}^{F} x_{ijf} = 1,\ \forall j, && \text{(B.24c)}\\
& \sum_{i=1}^{I}\sum_{j=1}^{J} x_{ijf} = 1,\ \forall f, && \text{(B.24d)}\\
& x_{ijf} \in \{0,1\},\ \forall i,j,f, && \text{(B.24e)}
\end{aligned}$$

where the matrix $\mathbf{S} = [s_{ijf}] \in \mathbb{R}^{I\times J\times F}$ represents the sum of UL and DL spectral efficiencies on frequency resource f, i.e., $s_{ijf} = C_{if}^u + C_{jf}^d$. Furthermore, equalities (B.24b)-(B.24d) follow from the inequality terms (B.3d)-(B.3f), where it is now required to assign a frequency channel to every UL and DL user.

The assignment problem (B.24) has $(n!)^2$ possible assignments and is a well known NP-hard problem named axial 3-DAP [108, Section 10.2]. Thus, no optimal solution in polynomial time can be obtained, unless P $=$ NP. Nevertheless, in the following subsections we propose two solutions: a greedy solution with guaranteed performance,

Algorithm 5 Greedy Solution for the Axial 3-DAP

1: Input: $G_{ibf},\ G_{bjf},\ G_{ijf},\ \beta,\ P^u_{\max},\ P^d_{\max}$
2: Evaluate $s_{ijf} = C^u_{if} + C^d_{jf}$ for every i, j, f
3: $\mathcal{I}_a = \mathcal{J}_a = \mathcal{F}_a = \emptyset \leftarrow$ Sets of assigned users and resources are initially empty
4: **while** $\mid \mathcal{I}_a \mid \neq I$ **and** $\mid \mathcal{J}_a \mid \neq J$ **do**
5: $\quad s_{\max} = \max s_{ijf}, \forall i, j, f \leftarrow$ Sort in descending order s_{ijf}
6: $\quad (i^\star, j^\star, f^\star) = \arg\max s_{\max} \leftarrow$ Select the pair and frequency with maximum sum spectral efficiency
7: $\quad x_{i^\star j^\star f^\star} = 1 \leftarrow$ Assign the frequency channel $f^\star$ to pair $(i^\star, j^\star)$
8: $\quad \mathcal{I}_a = \mathcal{I}_a \cup \{i^\star\},\ \mathcal{J}_a = \mathcal{J}_a \cup \{j^\star\},\ \mathcal{F}_a = \mathcal{F}_a \cup \{f^\star\} \leftarrow$ Update the set of assigned resources
9: $\quad$ Remove the already assigned users and frequency $(i^\star, j^\star, f^\star)$ from s_{ijf}
10: **end while**
11: **Output**: $\mathbf{X}$

based on our previous work [30] (Section B.6.1), and another solution based on a modification of the Hungarian algorithm, especially adopted to the axial 3-DAP problem (Section B.6.2). We provide two solutions to highlight to compare our approximate solution to axial 3-DAP, the greedy solution, with an heuristic that splits the 3-DAP into two 2-D assignment problems.

B.6.1 Greedy Solution for the Axial 3-DAP

Differently from other works that reduce the assignment problem (B.24) to two dimensions [32], we aim at jointly assigning UE pairs to frequencies. A related work developed some other heuristics applicable to problem (B.24) but it relies on meta-heuristics using graph theory, and some simplifications of the cost s_{ijf} [108, Section 10.2.5]. This approach cannot be used in our case, because it either requires unacceptable simplifications or is computationally too complex. In our previous work [30], we proposed a greedy solution to jointly solve an axial 3-DAP and a power allocation problem aiming at maximizing the minimum spectral efficiency of a full-duplex cellular system. To solve the assignment problem, we use the approach from our previous work [30]. The axial 3-DAP can be understood as the maximization of a function over a constraint set that is the intersection of 3 matroids [108], which allows us to have performance guarantees in terms of sum spectral efficiency. More precisely, let us consider $\mathbf{X}^*$ and $\mathbf{X}_G$ as the optimal and the greedy solution found by our greedy algorithm, and let the objective function, the sum spectral efficiency, be $c(\mathbf{X}^*)$ and $c(\mathbf{X}_G)$, respectively. From Hausmann et al. [114, Corollary 4], any greedy solution aiming to solve the maximization or minimization of a real-valued weight function over a constraint set defined as the intersection of k matroids yields a performance guarantee of at least $1/k$, i.e., $c(\mathbf{X}_G)/c(\mathbf{X}^*) \geq 1/k$. Therefore, our proposed greedy solution has a performance guarantee of at least $1/3$. Actually, as the numerical results will show, the optimality gap is lower than the guaranteed.

Algorithm 5 comprises the steps of the greedy solution to solve the axial 3-DAP in (B.24). Initially, the s_{ijf} for all pairs and frequency channels are evaluated. Next, we define the sets that will contain the assigned users in UL, DL, and the frequency channels (see lines 2-3). Subsequently, the algorithm sorts the matrix $\mathbf{S}$ in descending order and

Algorithm 6 Solution of JASEM problem (B.3)

1: Solve the assignment problem using the greedy solution in Algorithm 5
2: Find in a distributed manner t_k and t_l using Algorithm 3-4
3: Find in a distributed manner $\mathbf{p}_u$ and $\mathbf{p}_d$ from t_k and t_l
4: **Output**: $\mathbf{X}, \mathbf{p}_u, \mathbf{p}_d$

selects the UE pair and frequency channel that achieves the highest s_{ijf} (see lines 5-7). With this, the assignment matrix $\mathbf{X}$ and the sets of UL and DL users, as well as the set of frequency channels already used are updated (see lines 7-8). Finally, the assigned users and frequency channels from the matrix $\mathbf{S}$ are removed. The loop is continued until all users have been assigned to a frequency channel (see line 9). Notice that the computationally demanding effort is sorting the matrix $\mathbf{S}$, which has worst-case complexity $O(n \log(n))$, where n is the dimension of the vector to be sorted. In our case, the matrix $\mathbf{S}$ is of dimension $I \times J \times F$, and since $I = F = J$, the worst-case complexity is $O(F^3 \log F)$.

B.6.2 Solution Approach Based on A Modified Hungarian Algorithm

Since problem (B.24) is NP-hard, we propose to develop an approximate solution based on a transformation of the axial 3-DAP into two 2-D assignment problems: one between the UL users and the frequency channels, and another between the DL users and the frequency channels. The idea here is to first assign the UL users to frequency channels such that the sum spectral efficiency for all UL users is maximized. We then use the information of the already assigned UL users to pair the DL users to frequency channels such that the sum spectral efficiency of all DL users is maximized. Both assignments can be solved optimally through the Hungarian algorithm [103, Section 3.2]. We propose this algorithm in order to benchmark against our proposed greedy solution, and also to show that splitting 3-DAP into 2-D assignment problems does not, in fact, lead to a good performance in terms of sum spectral efficiency.

B.6.3 Summary

The solution of the initial JASEM problem (B.3) can be summarized as follows in Algorithm 6. The algorithm initially solves the assignment problem of UL and DL users to frequency channels using the greedy solution in Algorithm 5. With the assignment, it uses a distributed solution provided in Algorithm 4 to find the close to optimal SINRs of UL and DL users. Finally, the algorithm uses the distributed power control iterations of (B.23) to set the power vectors of UL and DL users.

The assignment solution in Algorithm 5 provides an approximation to the assignment problem (B.24), while Algorithm 4 provides a close to optimal solution to the power control problem (B.16). Thus, by using both algorithms we are providing a suboptimal solution to the JASEM problem (B.3). However, as the numerical results in Section B.7 show, the proposed solution method provides a close to optimal solution to JASEM.

Table B.2: *Simulation parameters*

Parameter	Value
Cell radius	$100\,\mathrm{m}$
Number of UL UEs $[I = J]$	[4 5 25]
Monte Carlo iterations	400
Carrier frequency	$2.5\,\mathrm{GHz}$
System bandwidth	$5\,\mathrm{MHz}$
Number of freq. channels $[F]$	[4 5 25]
LOS path-loss model	$34.96 + 22.7\log_{10}(d)$
NLOS path-loss model	$33.36 + 38.35\log_{10}(d)$
LOS probability	$\min(18/d, 1)(1-\exp(-d/36))+$ $\exp(-d/36)$
Shadowing st. dev. LOS	$3\,\mathrm{dB}$
Shadowing st. dev. NLOS	$4\,\mathrm{dB}$
Thermal noise power $[\sigma^2]$	$-116.4\,\mathrm{dBm}$/channel
Average user speed	$3\,\mathrm{km/h}$
User antenna height	$1.5\,\mathrm{m}$
BS antenna height	$10\,\mathrm{m}$
SI cancelling level $[\beta]$	$[-70\ \ -110]$ dB
UE max power $[P^u_{\max}]$	$24\,\mathrm{dBm}$
BS max power $[P^d_{\max}]$	$24\,\mathrm{dBm}$

B.7 Numerical Results and Discussion

We consider a single cell system operating in an urban micro environment assuming 2.5 GHz carrier frequency and a system bandwidth of 5 MHz [106, 115, 116]. For this bandwidth in an long term evolution (LTE) system [106], the maximum number of frequency channels is $F = 25$, which corresponds to the number of available frequency channel blocks. The total number of served UE varies between $I + J = 8...50$, where we assume that after UE pairing the number of UE transmitting in UL (I) is equal to the number of UE receiving in DL (J). The parameters of this system are set according to Table B.2.

To evaluate the performance of the proposed assignment and power control solution in this environment, we use the RUdimentary Network Emulator (RUNE) as a basic platform for system simulations [117] and extend it to FD cellular networks. With the RUNE FD simulation tool, the environment specified in Table B.2 can be readily generated. Below we report the results obtained by Monte Carlo simulations using either an exhaustive search algorithm to solve problem (B.3) or the proposed solution for assignment and power control.

Section B.7.1 analyses the optimality gap between the initially proposed sum rate maximization problem in (B.3), named P-OPT, our greedy solution for the axial 3-DAP with FL optimization for the power control, named G-FLIP, and the Hungarian assignment solution with FL optimization for the power control, termed H-FLIP.

In the following we compare the performance of these algorithms in two distinct scenarios: interference limited scenario and SI limited scenario. In Section B.7.2, we assume the scenario is interference limited, i.e, the SI cancellation level β is good (low β) such that the UE-to-UE interference is the limiting factor. Conversely, in Section B.7.3 we consider an SI limited scenario, i.e., the SI cancellation is poor (high β) such that the performance limiting factor is the SI.

Sections B.7.2-B.7.3 compare the performance of G-FLIP with four other algorithms:

1. Assignment according to the greedy solution but with equal power allocation (EPA), named G-EPA;
2. Random assignment with EPA, named R-EPA;
3. Random assignment but with the proposed FL power control, named R-FLIP;
4. Half-Duplex, named HD.

Notice that in the HD solution we need to assign UL and DL users to frequency channels, for which we use the Hungarian assignment, which in this situation is optimal.

B.7.1 Analysis of Optimality Gap

To evaluate the optimality gap between the proposed G-FLIP, H-FLIP and P-OPT, we evaluate the objective function of problem (B.3). Recall from Section B.3.2 that P-OPT is NP-hard, thus we solve it by exhaustive search for scenarios with a small number of users and frequency channels. Specifically, we consider a small system with reduced number of UL and DL users and frequency channels. Moreover, we consider a SI cancelling level of $-110\,\mathrm{dB}$, i.e., with $\beta = -110\,\mathrm{dB}$.

In Figure B.2 we show the convergence of the fixed-point iterations $t_k^{(n+1)} = f_{t_k}(\mathbf{x})^{(n)}$ for a pair of UL-DL users. As expected, the solution converges fast, in approximately 12 iterations, and with an accuracy of 10^{-6}.

With this, we can now evaluate the performance of the proposed G-FLIP and H-FLIP with respect to P-OPT. Figure B.3 shows the cumulative distribution function (CDF) of the sum spectral efficiency for all users, which is the objective function of problem (B.3). As expected, P-OPT yields the highest sum spectral efficiency, followed by G-FLIP and H-FLIP. Recall that P-OPT solves the NP-hard problem (B.3), and an optimality gap is expected. In addition, H-FLIP results the lowest sum spectral efficiency, which shows that greedily assigning all UL and DL users to frequency channels is better than doing the assignment in separated phases for UL and DL users.

To understand the relative optimality gap between the algorithms, Figure B.4 shows the relative optimality gap between P-OPT and G-FLIP and H-FLIP. As the number of users increases, the optimality gap between P-OPT and G-FLIP increases from approximately $6.5\,\%$ to $9\,\%$ at the 50-th percentile, i.e., the gap remains small as the number of users

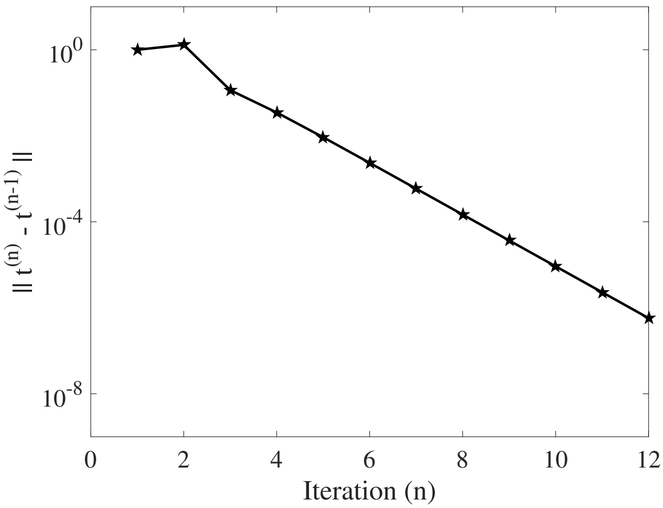

Figure B.2: *Convergence of the FL power control algorithm 3. Notice the solution converges in approximately 12 iterations with an accuracy of* 10^{-6}.

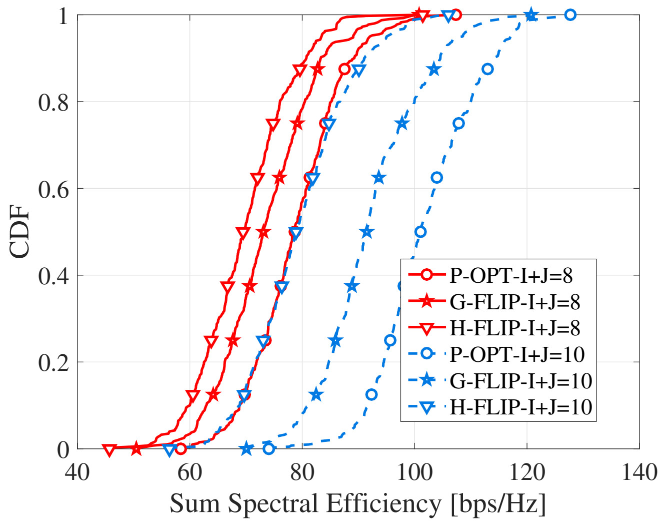

Figure B.3: *CDF of the sum spectral efficiency with reduced number of users. The proposed G-FLIP achieves a performance close to the optimal P-OPT and a better than H-FLIP. Notice that H-FLIP has the lowest sum spectral efficiency regardless of the number of users.*

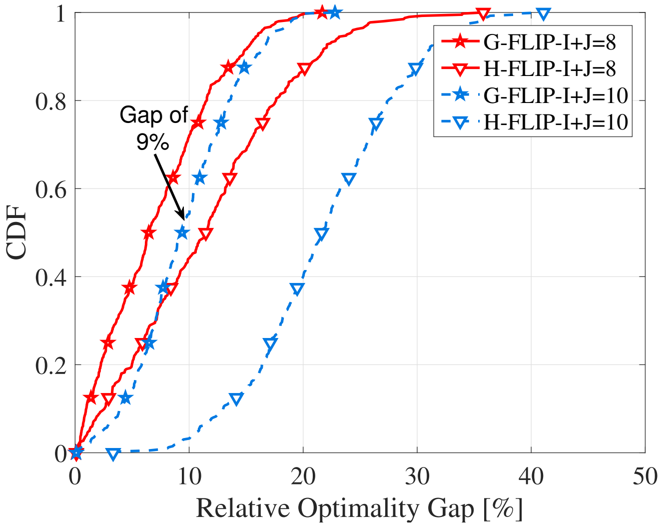

Figure B.4: *CDF of the relative optimality gap between P-OPT and the proposed G-FLIP and H-FLIP. The relative gap slowly increases with the number of users for G-FLIP. Conversely, for H-FLIP the relative gap almost doubles when increasing the number of users.*

increases in the system. However, for H-FLIP, the relative gap increases more when the number of users increases, it grows from approximately $11.5\,\%$ to $21.5\,\%$. This result suggests that G-FLIP is more suited to systems with a large number of users than H-FLIP.

B.7.2 Analysis for Interference-limited Regime

In this subsection we study the performance of G-FLIP and compare it with G-EPA, R-EPA, R-FLIP and HD in terms of sum spectral efficiency and total power consumption. Here we assume a very high SI cancellation level of $-110\,\mathrm{dB}$, which represents an interference-limited situation. Our objective is to evaluate the impact of frequency channel allocation and power control on the sum spectral efficiency and system-wide power consumption and, ultimately, to demonstrate the gains of the proposed algorithms.

Figure B.5 shows the CDF of the sum spectral efficiency. The proposed G-FLIP algorithm has a gain of approximately $16\,\%$ at the 50-th percentile with respect to HD systems, and recall that this HD solution has the optimal assignment that could be hardly used in a real system. Notice that R-FLIP performs close to HD, with a relative difference of approximately $4\,\%$ at the 50-th percentile. R-EPA achieves the lowest sum spectral efficiency among all algorithms, and with the usage of a smart power control such as the FL, the performance improves in approximately $30.6\,\%$. This is shown as a power control (PC) gain in Figure B.5. By comparing G-FLIP and G-EPA, notice that the PC relative gain is small, approximately $1.6\,\%$. The large gains come from the usage of the

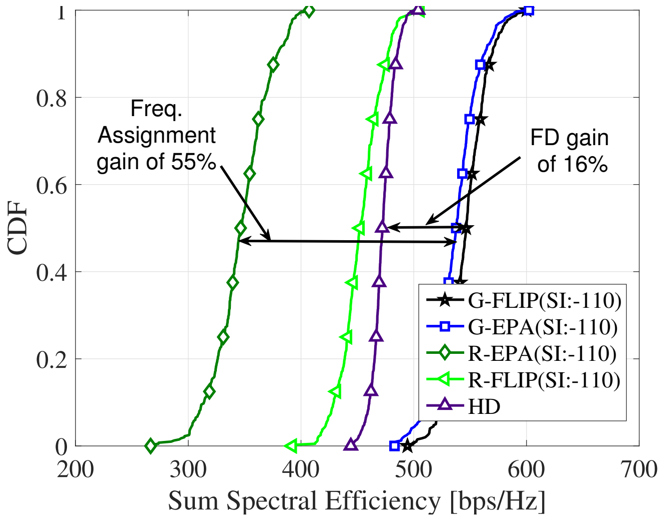

Figure B.5: *CDF of the sum spectral efficiency with* $\beta = -110\,\mathrm{dB}$*, i.e., the system is limited by the UE-to-UE interference. We notice that G-FLIP has a relative gain of approximately* $16\,\%$ *with respect to HD systems. In addition, most of the gains can be achieved by a smart frequency assignment rather than a smart power control.*

greedy frequency assignment, as can be observed by comparing G-EPA with R-EPA, the relative gain is approximately $55\,\%$. Therefore, for interference-limited scenarios, most of the gains can be achieved by a smart frequency assignment, and the power control has little impact on the sum spectral efficiency.

Figure B.6 shows the CDF of the total transmit power. Notice that G-EPA, R-EPA and HD are represented by a straight line, which is a result of using a fixed power allocation at maximum value. By comparing G-FLIP with any algorithm using EPA, we can observe a relative energy saving of approximately $48\,\%$. In addition, R-FLIP has a lower power consumption than G-FLIP. This is because the greedy assignment is able to transmit with a higher power on better (i.e. higher gain) channels.

The high energy saving gains with FL power control are expected. With the SINR targets obtained by FL optimization, the distributed power control algorithm by [113] can be used to set the powers that reach the target and minimize the sum power. Recall that due to the strict inequality in Eq. (B.14), we are close to (but do not necessarily reach) the optimal SINRs that maximizes the sum spectral efficiency. Thus, our solution with FL power control realizes a system whose spectral efficiency is close to optimal but with low power consumption.

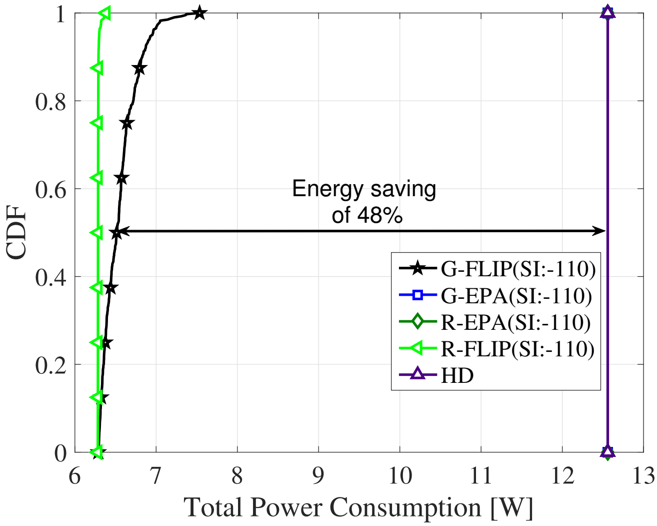

Figure B.6: *CDF of the total power consumption with a system limited by the UE-to-UE interference. We notice that using FL power control solution we have approximately* $48\,\%$ *of energy saving gains.*

B.7.3 Analysis for SI-limited Regime

In an opposite manner, in subsection B.7.3 we analyse a system limited by the SI, on which the SI cancelling level is $-70\,\mathrm{dB}$. Similar to subsection B.7.2, the performance of the proposed algorithm G-FLIP is compared to G-EPA, R-EPA, R-FLIP and HD.

Figure B.7 shows the sum spectral efficiency, and as expected, the performance has degraded for all algorithms with full-duplex communications. The proposed G-FLIP achieves the best performance among the algorithms with full-duplex, and the relative difference to HD is small, approximately $5\,\%$ at the 50-th percentile. Notice that the PC gain between G-FLIP and G-EPA is higher, approximately $28\,\%$ at the 50-th percentile. Differently from Figure B.5, now most of the gains come from power control, and this is shown by the relative difference between R-EPA and R-FLIP, with a relative gain of approximately $170.5\,\%$ at the 50-th percentile. In addition, R-FLIP outperforms G-EPA, which shows that a smart power control outperforms a smart frequency assignment in the SI-limited regime. Therefore, power control, instead of the frequency assignment, has the important role for the SI-limited regime.

Figure B.8 shows the total power consumption, and as before, we have a high energy saving when using FL power control. For G-FLIP, an energy saving gain of approximately $42\,\%$ in the 50-th percentile can be observed, which is smaller than the gain in Figure B.6. The reason for this result is that the BS transmits at a higher power for many users to compensate for the high SI.

Overall, when the scenario is limited by the SI the power control is more important

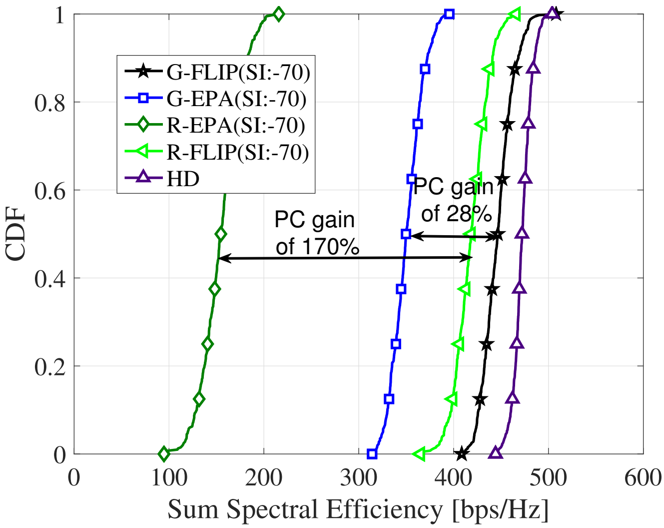

Figure B.7: *CDF of the sum spectral efficiency with* $\beta = -70\,\mathrm{dB}$*, i.e., the system is limited by the SI. We notice a performance degradation for full-duplex communications, but the relative difference between G-FLIP and HD is only* $5\,\%$*. Now, most of the gains can be achieved by a smart power control instead of a smart frequency assignment algorithm.*

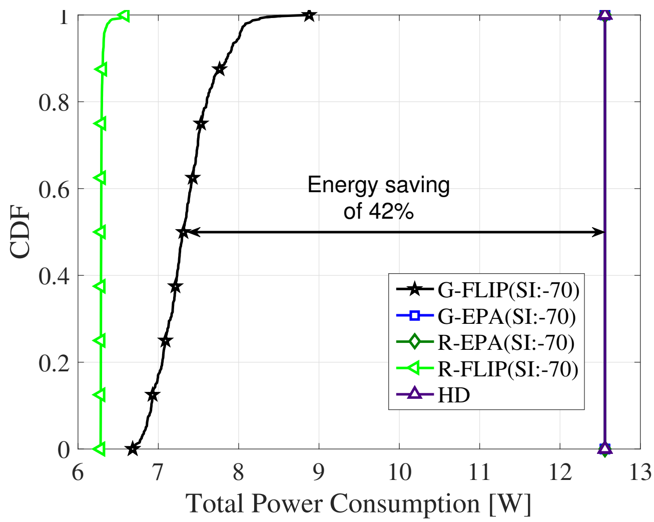

Figure B.8: *CDF of the total power consumption when the system is limited by the SI. The proposed solution G-FLIP provides a relative energy saving of approximately* $42\,\%$ *with respect to HD and all other algorithms transmitting with EPA.*

than the frequency assignment, and with the proposed G-FLIP we have a small relative difference of $5\,\%$ to HD but transmitting at lower power, with an energy gain of approximately $42\,\%$.

B.8 Conclusion

In this paper we considered the joint assignment and spectral efficiency maximization problem in FD cellular networks. Our objective was to maximize the sum spectral efficiency in a distributed manner while optimizing the assignment of users to frequency channels. This problem was posed as a mixed integer nonlinear optimization, called JASEM, which was shown to be NP-hard. To solve JASEM, we split it into two subproblems, power control and frequency channel assignment. For the power control, we proposed a novel distributed SINR setting solution based on Fast-Lipschitz optimization. The user-frequency assignment problem was also NP-hard, and we proposed a greedy solution with guaranteed performance in terms of sum spectral efficiency. Using such an optimization, we provided a distributed SINR setting power control algorithm that can be used with our greedy solution to provide a solution close to optimality for the initial JASEM problem.

The numerical results showed that our solution approach improved the spectral efficiency of the users with in an interference-limited regime, achieved higher spectral efficiency than HD transmissions, and showed large energy saving gains. The results also indicated that a smart assignment solution achieves more gains in interference-limited regime than smart power control solution, whereas this situation is reversed in SI-limited regime.

In future works, we intend to analyse the impact of assignment and power control in multi-cell FD systems in terms of sum spectral efficiency and fairness.

B.9 Appendix

B.9.1 Fast-Lipschitz Optimization

For the sake of self containment and the need for introducing a preliminary result, we now give a brief formal definition of FL problems. For a thorough discussion of FL properties we refer the reader to [92, 98].

Definition 5. A problem is said to be on *FL form* if it can be written as

$$\begin{aligned} \underset{\mathbf{x}}{\text{maximize}} \quad & \mathbf{f}_0(\mathbf{x}) \\ \text{subject to} \quad & x_i \leq f_i(\mathbf{x}) \quad \forall i \in \mathcal{A} \\ & x_i = f_i(\mathbf{x}) \quad \forall i \in \mathcal{B}, \end{aligned} \tag{B.25}$$

where

- $\mathbf{f}_0(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}^m$ is a differentiable scalar ($m = 1$) or vector valued ($m \geq 2$) function.

- $\mathcal{A}$ and $\mathcal{B}$ are complementary subsets of $\{1, \ldots, n\}$.
- For all i, f_i : $\mathbf{f}_0(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}$ is a differentiable function.

From the individual constraint functions we form the vector valued function $\mathbf{f} : \mathbb{R}^n \rightarrow \mathbb{R}^n$ as $\mathbf{f}(\mathbf{x}) = \begin{bmatrix} f_1(\mathbf{x}) & \cdots & f_n(\mathbf{x}) \end{bmatrix}^T$.

Remark 1. We restrict our attention to a *bounding box* $\mathcal{D} = \{\mathbf{x} \in \mathbb{R}^n \,|\, \mathbf{a} \leq \mathbf{x} \leq \mathbf{b}\}$. We assume $\mathcal{D}$ contains all candidates for optimality and that $\mathbf{f}$ maps $\mathcal{D}$ into $\mathcal{D}$, $\mathbf{f} : \mathcal{D} \rightarrow \mathcal{D}$. This box arises naturally in practice, since any real-world decision variable, such as transmitting power, must be bounded.

Definition 6 (Pareto Optimal)**.** A vector $\mathbf{x}^*$ is called a Pareto optimal point if there is no $\mathbf{x} \in \mathcal{D}$ such that $\mathbf{f}_0(\mathbf{x}) > \mathbf{f}_0(\mathbf{x}^*)$, i.e., if $\mathbf{f}_0(\mathbf{x}^*)$ is a maximal element of the set $\mathcal{D}$ with respect to the natural partial ordering defined by the cone $\mathbb{R}^m_+$ [118].

In practice, a Pareto optimal solution is a vector for which is impossible to improve one component without decreasing another component. The Pareto optimal solutions are derived by converting a vector optimization problem into a scalar one via scalarization of the objective function [118].

Definition 7. A problem is said to be *Fast-Lipschitz* when it can be written on FL form and admits a unique Pareto optimal solution $\mathbf{x}^*$, defined as the unique solution to the system of equations

$$\mathbf{x}^* = \mathbf{f}(\mathbf{x}^*).$$

A problem written on FL form is not automatically Fast-Lipschitz. There are several qualifying conditions that, when fulfilled, guarantee that problem (B.25) is Fast-Lipschitz (see Table B.3). Note that only at least one condition in Table B.3 needs to be satisfied. The special condition (**GQC**) is implied by (**Q**$_3$) and thus (**GQC**) is more general in the sense that if (**Q**$_3$) holds, then (**GQC**) holds. However, this implies also that (**GQC**) can be more conservative than (**Q**$_3$).

Note that we only show the (**Q**$_3$) in this paper. There are already six known qualifying conditions in FL optimization [98]. We did not report the other conditions in this paper because they are not used here and therefore are not included in this appendix.

B.9.2 Proof of Lemma 2

In the formulation of problem (B.17), there are no constraints for variable t_l. Nevertheless, we can enforce $t_l \in \mathcal{X}_l$ twice and get the equivalent problem

$$\begin{aligned} \underset{\mathbf{x}}{\text{maximize}} \quad & f_0(\mathbf{x}) && \text{(B.26a)} \\ \text{subject to} \quad & t_k \leq f_{t_k}(\mathbf{x}), && \text{(B.26b)} \\ & t_l \leq f_{t_l}(\mathbf{x}) = b_l, && \text{(B.26c)} \\ & \mathbf{x} \in \mathcal{X}. && \text{(B.26d)} \end{aligned}$$

Table B.3: *Fast-Lipschitz qualifying conditions from [98].* $\boldsymbol{Q}_3$ *implies the general condition (**GQC**), but ($\boldsymbol{Q}_3$) is much easier to use from an analytical and computational point of view compared to (**GQC**).*

General Qualifying Condition	
GQC	**(GQC.a)** $\nabla \mathbf{f}_0(\mathbf{x}) \geq \mathbf{0}$ with non-zero rows **(GQC.b)** $\lvert\lvert\lvert \nabla \mathbf{f}(\mathbf{x}) \rvert\rvert\rvert < 1$ There exists a $k \in \{1, 2, \dots\} \cup \infty$ such that **(GQC.c)** when $k < \infty$, then $\nabla \mathbf{f}(\mathbf{x})^k \geq \mathbf{0}$ **(GQC.d)** when $k > 1$, then $\left\lvert\left\lvert\left\lvert \sum_{l=1}^{k-1} \nabla \mathbf{f}(\mathbf{x})^l \right\rvert\right\rvert\right\rvert_\infty < \mathbf{q}(\mathbf{x})$, where $\mathbf{q}(\mathbf{x}) \triangleq \min_j \dfrac{\min_i [\nabla \mathbf{f}_0(\mathbf{x})]_{ij}}{\max_i [\nabla \mathbf{f}_0(\mathbf{x})]_{ij}}$
Qualifying Condition 3	
Q₃	**($\mathbf{Q}_3$.a)** $\nabla \mathbf{f}_0(\mathbf{x}) > \mathbf{0}$ **($\mathbf{Q}_3$.b)** $\lvert\lvert\lvert \nabla \mathbf{f}(\mathbf{x}) \rvert\rvert\rvert_\infty < \dfrac{\mathbf{q}(\mathbf{x})}{1 + \mathbf{q}(\mathbf{x})}$

This problem is Fast-Lipschitz if

$$\nabla \mathbf{f}(\mathbf{x}) = \begin{bmatrix} \nabla_{t_k} f_{t_k}(\mathbf{x}) & \nabla_{t_k} f_{t_l}(\mathbf{x}) \\ \nabla_{t_l} f_{t_k}(\mathbf{x}) & \nabla_{t_l} f_{t_l}(\mathbf{x}) \end{bmatrix} \text{ and } \nabla f_0(\mathbf{x}) = \begin{bmatrix} \nabla_{t_k} f_0(\mathbf{x}) \\ \nabla_{t_l} f_0(\mathbf{x}) \end{bmatrix}$$

fulfils the condition (**GQC**) of Table B.3. Since $f_{t_l}(\mathbf{x}) = b_l$ is constant, and $f_0(\mathbf{x}) = t_k + t_l$, $\nabla \mathbf{f}(\mathbf{x})$ and $\nabla f_0(\mathbf{x})$ simplify to

$$\nabla \mathbf{f}(\mathbf{x}) = \begin{bmatrix} \nabla_{t_k} f_{t_k}(\mathbf{x}) & 0 \\ \nabla_{t_l} f_{t_k}(\mathbf{x}) & 0 \end{bmatrix} \text{ and } \nabla f_0(\mathbf{x}) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Following the steps in [98], we can consider problem (B.17) with a fixed $t_l \in \mathcal{X}_l$ and with t_k as the only variable, and then rewrite as

$$\underset{\mathbf{x}}{\text{maximize}} \quad f_{0|t_l}(t_k) \tag{B.27a}$$

$$\text{subject to} \quad t_k \leq f_{t_k|t_l}(t_k), \tag{B.27b}$$

$$t_k \in \mathcal{X}_k. \tag{B.27c}$$

For clarity, we refer to problem (B.27) as *subproblem* (B.27). We use Proposition 20 in [98] to prove that problem (B.17) is Fast-Lipschitz. To this end, we need to prove the following:

(a) Subproblem (B.27) fulfils the (**GQC**) for all $t_l \in \mathcal{X}_l$;

(b.iii) It holds that for all $\mathbf{x} \in \mathcal{X}$

$$\frac{\lvert\lvert\lvert \nabla_{t_l} f_{t_k}(\mathbf{x}) \rvert\rvert\rvert_\infty}{1 - \lvert\lvert\lvert \nabla_{t_k} f_{t_k}(\mathbf{x}) \rvert\rvert\rvert_\infty} < \frac{\delta_{t_l}(\mathbf{x})}{\Delta_{t_k}(\mathbf{x})}, \tag{B.28}$$

where $\delta_{t_l}(\mathbf{x}) = \nabla_{t_l} f_0(\mathbf{x})$ and $\Delta_{t_k}(\mathbf{x}) = \nabla_{t_k} f_0(\mathbf{x})$.

To prove (a) we need to check (**GQC**), and since the qualifying condition ($\mathbf{Q}_3$) implies (**GQC**), we use ($\mathbf{Q}_3$) to subproblem (B.27). This is equivalent to check the following two sub-conditions:

$$(\mathbf{Q}_3\textbf{.a})\;\; \nabla_{t_k} f_{0|t_l}(t_k) > 0;$$

$$(\mathbf{Q}_3\textbf{.b})\;\; \left|\left|\left|\nabla_{t_k} f_{t_k|t_l}(t_k)\right|\right|\right|_\infty < \tfrac{q(t_k)}{1+q(t_k)}.$$

The gradients $\nabla_{t_k} f_{0|t_l}(t_k)$, $\nabla_{t_k} f_{t_k|t_l}(t_k)$, and the function $q(t_k)$ are given by

$$\begin{aligned} \nabla_{t_k} f_{0|t_l}(t_k) &= 1, \\ \nabla_{t_k} f_{t_k|t_l}(t_k) &= 1 - \gamma u(\mathbf{x}), \\ q(t_k) &= 1. \end{aligned}$$

Since $\nabla_{t_k} f_{t_k|t_l}(t_k)$ is scalar, we have the following for ($\mathbf{Q}_3$**.b**):

$$\left|\left|\left|\nabla_{t_k} f_{t_k|t_l}(t_k)\right|\right|\right|_\infty = \left|1 - \gamma u(\mathbf{x})\right|.$$

Notice that ($\mathbf{Q}_3$**.a**) is fulfilled because $\nabla_{t_k} f_{0|t_l}(t_k) = 1 > 0$. Let us now analyse ($\mathbf{Q}_3$**.b**):

$$|1 - \gamma u(\mathbf{x})| < \frac{1}{2}. \tag{B.30}$$

Note that we can have $(1 - \gamma u(\mathbf{x})) > 0$, which implies the following bound on γ:

$$\frac{1}{2u(\mathbf{x})} < \gamma < \frac{1}{u(\mathbf{x})}. \tag{B.31}$$

As for the case when $(1 - \gamma u(\mathbf{x})) < 0$, we have the bound on γ as:

$$\frac{1}{u(\mathbf{x})} < \gamma < \frac{3}{2u(\mathbf{x})}. \tag{B.32}$$

To prove (b.iii), we need to first derive the gradients $\nabla_{t_l} f_{t_k}(\mathbf{x})$, $\nabla_{t_k} f_{t_k}(\mathbf{x})$, and also $\delta_{t_l}(\mathbf{x})$ and $\Delta_{t_k}(\mathbf{x})$, which are given by

$$\nabla_{t_l} f_{t_k}(\mathbf{x}) = -\gamma v(\mathbf{x}), \tag{B.33a}$$

$$\nabla_{t_k} f_{t_k}(\mathbf{x}) = 1 - \gamma u(\mathbf{x}), \tag{B.33b}$$

$$\delta_{t_l}(\mathbf{x}) = 1, \tag{B.33c}$$

$$\Delta_{t_k}(\mathbf{x}) = 1. \tag{B.33d}$$

Since $\nabla_{t_l} f_{t_k}(\mathbf{x})$ and $\nabla_{t_k} f_{t_k}(\mathbf{x})$ are scalars, we have the following for (b.iii):

$$\begin{aligned} |||\nabla_{t_l} f_{t_k}(\mathbf{x})|||_\infty &= \left|-\gamma v(\mathbf{x})\right|, \\ |||\nabla_{t_k} f_{t_k}(\mathbf{x})|||_\infty &= \left|1 - \gamma u(\mathbf{x})\right|. \end{aligned}$$

Let us now analyse inequality (B.28):

$$\frac{\gamma v(\mathbf{x})}{1-|1-\gamma u(\mathbf{x})|} < 1, \tag{B.35a}$$

$$|1-\gamma u(\mathbf{x})| < 1-\gamma v(\mathbf{x}), \tag{B.35b}$$

which implies the following inequalities:

- $1-\gamma u(\mathbf{x})>0$ implies that $1-\gamma u(\mathbf{x})<1-\gamma v(\mathbf{x})$, and then

$$\gamma\Big(v(\mathbf{x})-u(\mathbf{x})\Big) < 0. \tag{B.36}$$

  Since $\gamma > 0$, inequality (B.36) is fulfilled if

$$v(\mathbf{x}) < u(\mathbf{x}), \text{ and if} \tag{B.37}$$

$$\gamma < \frac{1}{u(\mathbf{x})}. \tag{B.38}$$

- $1-\gamma u(\mathbf{x})<0$ implies that $-1+\gamma u(\mathbf{x})<1-\gamma v(\mathbf{x})$, and then

$$\gamma\Big(v(\mathbf{x})+u(\mathbf{x})\Big) < 2,$$

$$\frac{1}{u(\mathbf{x})} < \gamma < \frac{2}{v(\mathbf{x})+u(\mathbf{x})}.$$

Note that we also require that $v(\mathbf{x}) < u(\mathbf{x})$ when $1-\gamma u(\mathbf{x}) < 0$, because otherwise we would have $2/(v(\mathbf{x})+u(\mathbf{x}))<1/(u(\mathbf{x}))$, which violates the bound on γ defined above.

Based on the requirements from (a) and (b.iii), we have inequalities on γ that allow us to use fixed point iterations to solve problem (B.17). To rely only on $u(\mathbf{x})$, we choose the case when $1-\gamma u(\mathbf{x})>0$, which results in the following requirement:

$$\frac{1}{2u(\mathbf{x})} < \gamma < \frac{1}{u(\mathbf{x})},$$

and $v(\mathbf{x}) < u(\mathbf{x})$. If for such conditions on t_k and t_l there is no γ that satisfies inequality (B.20), then problem (B.17) is not Fast-Lipschitz.

B.9.3 Proof of Lemma 3

From Algorithm 3, the infeasibility of t_k occurs when the upper or lower bound for t_k is reached and $|h(\mathbf{x})|$ is not small enough (see line 10). From the expression of $f_{t_k}(\mathbf{x})$ in Eq. (B.18b), we can take its second derivatives with respect to t_k and t_l:

$$\frac{\partial^2 f_{t_k}(\mathbf{x})}{\partial t_k^2} = -\frac{\gamma}{\alpha_k}u(\mathbf{x}), \tag{B.40a}$$

$$\frac{\partial^2 f_{t_k}(\mathbf{x})}{\partial t_l^2} = \frac{\gamma}{\alpha_l}v(\mathbf{x}), \tag{B.40b}$$

$$\frac{\partial^2 f_{t_k}(\mathbf{x})}{\partial t_k \partial t_l} = -\frac{\gamma}{\alpha_l\alpha_k}\exp\left(\frac{t_k}{\alpha_k}\right)\exp\left(\frac{t_l}{\alpha_l}\right). \tag{B.40c}$$

If t_k hits the upper bound and $|h(\mathbf{x})|$ is not small enough, we should increase t_l, because it increases $f_{t_k}(\mathbf{x})$ since the function is monotonically non-decreasing in t_l from Eq. (B.40b). Then, this will cause a decrease in t_k, because the function is monotonically non-increasing in t_k from Eq. (B.40a). Now, if t_k hits the lower bound and $|h(\mathbf{x})|$ is not small enough, the behaviour is the opposite, and we should decrease t_l, which also decreases $f_{t_k}(\mathbf{x})$ and later increases t_k.

Therefore, if for a given t_l we face later an infeasibility on t_k, we should cut the branch we used and go for the other direction in the search.

Paper C

Spectral Efficient and Fair User Pairing for Full-Duplex Communication in Cellular Networks

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Spectral Efficient and Fair User Pairing for Full-Duplex Communication in Cellular Networks

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Abstract

A promising new transmission mode in cellular networks is the three-node full-duplex mode, which involves a base station with full-duplex capability and two half-duplex user transmissions on the same frequency channel for uplink and downlink. The three-node full-duplex mode can increase spectral efficiency, especially in the low transmit power regime, without requiring full-duplex capability at user devices. However, when a large set of users is scheduled in this mode, self-interference at the base station and user-to-user interference can substantially hinder the potential gains of full-duplex communications. This paper investigates the problem of grouping users to pairs and assigning frequency channels to each pair in a spectral efficient and fair manner. Specifically, the joint problem of user uplink/downlink frequency channel pairing and power allocation is formulated as a mixed integer nonlinear problem that is solved by a novel joint fairness assignment maximization algorithm. Realistic system level simulations indicate that the spectral efficiency of the users having the lowest spectral efficiency is increased by the proposed algorithm, while a high ratio of connected users in different loads and self-interference levels is maintained.

C.1 Introduction

Traditional cellular networks operate in half-duplex (HD) transmission mode, in which a user equipment (UE) or the base station (BS) either transmits or receives on any given frequency channel. However, the ever increasing demand to support the transmission of unprecedented data quantities has led the research community to investigate new wireless transmission technologies. Recently, in-band full-duplex (FD) has been proposed as a key enabling technology to drastically increase the spectral efficiency of conventional wireless transmission modes. In-band FD has the potential to double the spectral efficiency of traditional wireless systems operating in HD, such as time division duplex or frequency division duplex modes [16, 78]. Recent advances in antenna design, interference cancellation algorithms, self-interference (SI) suppression techniques and prototyping of FD transceivers have meant that, FD is becoming a realistic technology component of advanced wireless – including cellular – systems, especially in the low transmit power regime [17, 18].

FD transmission modes include bidirectional full-duplex and three-node full-duplex (TNFD) modes [12]. In bidirectional full-duplex, two FD-capable nodes (either a UE or the BS) transmit and receive on the same time-frequency resource. In contrast, TNFD involves three nodes, but only one of them needs to have FD capability. The FD-capable

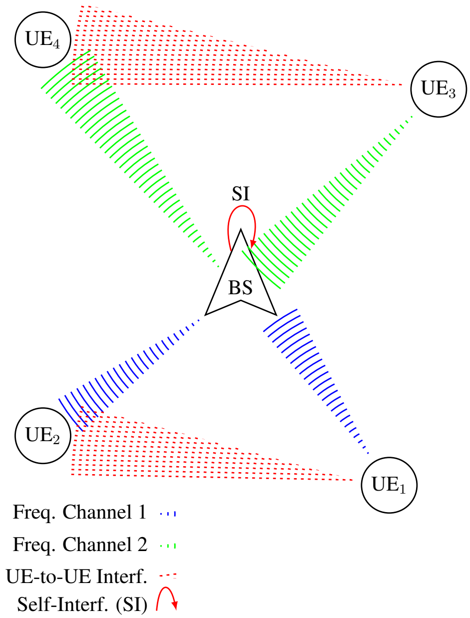


Figure C.1: *A full duplex cellular network employing TNFD with two UEs pairs. The BS selects pairs of UE (pairing) and jointly schedules them for TNFD transmission by allocating frequency channels in the UL and DL. As the figure illustrates, apart from SI, TNFD experiences the new UE-to-UE interference that might limit the efficiency of FD communications.*

node transmits to its receiver node while receiving from another transmitter node on the same frequency channel. In cellular networks, the BS is FD-capable and transmits in the downlink (DL) and receives in the uplink (UL) from the HD UEs (Figure C.1). From Figure C.1, it is clear that FD operation in a cellular environment experiences new types of interference, aside from the inherently present SI.

Because of the level of UE-to-UE interference depends on the UE locations and their transmission powers, coordination mechanisms are needed to mitigate the negative effect of the interference on the spectral efficiency of the system [10]. A key element of such mechanisms is UE *pairing* and frequency channel selection that determines which UEs should be scheduled for simultaneous UL and DL transmissions on specific frequency channels. As pointed out in [12], the simultaneous bidirectional communication capabilities of the nodes in bidirectional full-duplex and TNFD networks mean that the fairness among nodes may degrade by a factor of two compared with HD communications. Despite that fairness is widely recognized as important and has been well investigated in more traditional wireless communication context [91], to the best of our knowledge, max-min fairness has not been addressed in the full-duplex literature for TNFD. Hence, it is crucial to design efficient and fair medium access control protocols and physical layer procedures capable of supporting adequate pairing mechanisms that arise in full-duplex communications.

In this paper we focus on the problem of joint frequency channel selection and transmit power allocation assuming TNFD transmissions and a frequency selective wireless environment. Specifically, we investigate schemes that maximize the spectral efficiency of the user with the lowest achieved spectral efficiency. We formulate this problem as a mixed integer nonlinear optimization, which we call the *joint assignment and fairness maximization* (JAFM) problem, where the decision variables are the assignment matrices of UL and DL, and where each element of the matrix is a binary variable indicating the association of the user to a frequency channel. We show that the JAFM problem is a non-deterministic polynomial-time (NP)-hard problem, implying that no optimal solution in polynomial time can be obtained. We derive a closed-form approximate solution by resorting to Lagrangian duality theory, and prove the optimal power allocation for the dual problem. However, the closed-form assignment cannot be solved efficiently, because this assignment problem is also NP-hard, but now with respect to the assignment variables. Therefore, we propose a greedy solution to this assignment problem and evaluate the optimal transmit powers for all UEs based on the results given by the dual solution. We also show that the duality gap between the greedy solution and the primal is bounded and diminishes as the number of frequency channels increases.

The remainder of the paper is organized as follows. Section C.2 discusses some relevant and closely related works to our problem. Section C.3 presents the system model and main parameters, followed by the problem formulation. In Section C.4, we study the impact of the problem constraints based on the JAFM problem formulation, and show that the SINR and power constraints create an admissible area on which UL and DL users can share a frequency channel. In Section C.5 we analyse the proposed JAFM problem and derive a closed-form approximate solution for the assignment and show the optimal power allocation based on the Lagrangian dual problem. In Section C.6, we propose a greedy approximation to the primal JAFM problem based on the dual problem of Section C.5. Section C.7 presents numerical results and compares the performance of the proposed solution with different assignments and power allocation, and the impact of the users' load and SI cancelling on the fairness of FD cellular networks. The results show that our proposed solution outperforms existing methods from the literature, such as random assignment without power control, and guarantees connection to more users, and maintains a higher level of fairness. Moreover, we show that our joint assignment and power allocation solution should not be split into separated procedures, because the performance achieved by such splitting schemes is similar to a random assignment with equal power allocation.

C.2 Related Works

The impact of FD radios on the design of cellular systems has been analysed only relatively recently, in [10, 28, 78]. These works provide valuable insights into the design and performance of FD cellular systems in terms of rate and energy performance. However, the problem of fair and efficient joint power and channel allocation has not been addressed.

Transmit power optimization for FD wireless networks is the topic of [119] and [120]. A dynamic power allocation scheme that maximizes the sum-rate of FD bidirectional transmissions is developed in [119], while an optimal power control scheme for FD decode-and-forward relaying systems is proposed in [120]. However, the results of these works are not directly applicable to FD networks operating in TNFD mode, since the joint channel and power allocation problem under fairness constraints has not been considered therein.

Most of the recent works consider the joint subcarrier and power allocation problem [79] and the joint duplex mode selection, channel allocation, and power control problem [32] in FD networks. The cellular network model in [79] is applicable to FD mobile nodes rather than to networks operating in TNFD mode. The work reported in [32] considers the case of TNFD transmission mode in a cognitive femto-cell context with bidirectional transmissions from UEs and develops sum-rate optimal resource allocation and power control algorithms. Our objective differs from that of [32] in that we aim at maximizing the transmission rate of the low rate users and thereby taking into account the fairness of the FD system. We do not consider the proposed algorithm for rate maximization in our performance evaluation, because in [32] heterogeneous networks with bidirectional transmission from UEs were considered, which substantially changes the problem.

The aspects of fairness and quality of service (QoS) are addressed by [12, 121], and [80]. Specifically, a QoS-driven power allocation approach for bidirectional FD links is proposed by [121] and a heterogeneous statistical QoS provisioning framework is developed in [80]. Both of those papers focus on the bidirectional FD link case without considering the implications of TNFD transmissions. In contrast, [12] discusses both the bidirectional and the TNFD modes and emphasizes the importance of fairness. However, [12] does not provide a power control and channel allocation scheme that is developed with such objectives in mind.

Several deliverables of the DUPLO project are related to our work [9]. These deliverables provide methodology and numerical results on the performance of cellular and ad hoc networks employing FD radios combined with power control, channel allocation and other physical and medium access control layer algorithms. However, a fairness-optimizing joint channel and power allocation has not been investigated in that project.

In the light of this survey of related literature, we summarize our main contributions as follows:

- The problem formulation of JAFM and our proposed solution is new. This problem is particularly important in practical systems employing TNFD mode, where its need was first pointed out in [12].

- We propose a necessary condition for a pair of UL and DL users to share a frequency channel based on the SI cancelling level of the system, which helps us to define the admissible pairs for the assignment problem.

- Due to the intractability of JAFM, we propose a greedy joint assignment and fairness maximization algorithm (JAFMA) that is based on Lagrangian duality theory, where we show that the duality gap between JAFMA and JAFM is bounded and diminishes as the number of channels in the system increases.

- We evaluate JAFMA by a realistic system simulator and argue that the obtained numerical results and the insights help design FD cellular networks.

C.3 System Model and Problem Formulation

C.3.1 System Model

We consider a single-cell cellular system in which only the BS is FD capable, whereas the UEs operate in time division duplex mode, as illustrated in Figure C.1. In the example scenario of Figure C.1, the BS is subject to SI and the UEs in the UL (UE_1 and UE_3) cause UE-to-UE interference to UE_2 and UE_4 in DL. Table C.4 shows the main sets, constants and variables used throughout the paper. We assume that within a large number of connected users in the cellular network, a smaller subset of these users are selected by an appropriate scheduler for data transmission in the UL and reception in the DL. The number of UEs in the UL and DL is denoted by I and J, respectively. Their sum is constrained by the total number of frequency channels in the system F, i.e., $I \leq F$ and $J \leq F$. The set of UL and DL users is denoted by $\mathcal{I} = \{1, \ldots, I\}$ and $\mathcal{J} = \{1, \ldots, J\}$, respectively, and the set of resources by $\mathcal{F} = \{1, \ldots, F\}$. In this paper we assume that at most one channel is used by a UE.

Our system model includes a frequency selective environment, such that the composite channel gains depend on the frequency. We assume that the path gains consist of small- and large-scale fading. Specifically, let G_{ibf} denote the path gain between transmitter i in the UL and the BS on frequency channel f, whereas G_{bjf} denotes the path gain between BS and the receiving UE j in the DL on frequency channel f, and G_{ijf} denotes the interfering path gain between transmitter i in the UL and the receiver j in the DL on frequency channel f. The vector of transmit powers in the UL is denoted by $\mathbf{p^u} = [P_1^u \ldots P_I^u]$, whereas the vector of the DL transmit powers on the DL frequency channels is denoted by $\mathbf{p^d} = [P_1^d \ldots P_J^d]$.

To take into account the residual value of the SI power that leaks to the receiver, we define β as the SI cancellation coefficient, such that the SI power at the receiver of the BS is βP_j^d when the transmit power is P_j^d. For example, at a transmit power of $20\,\mathrm{dBm}$ $(100\,\mathrm{mW})$, and assuming a noise floor around $-90\,\mathrm{dBm}$, the transmit SI has to be cancelled by $\beta = -110\,\mathrm{dB}$ to reduce it to the same level as the noise floor [16, 18].

As illustrated in Figure C.1, the UE-to-UE interference depends on the geometry of the co-scheduled UL and DL users. For example, assuming a greater path loss between UE_1 and UE_4 than between UE_1 and UE_2, it may be advantageous to pair UE_1 and UE_4 for simultaneous UL/DL transmission on the same frequency channel f. Therefore, the UE pairing along with the channel allocation are key functions of the system. Accordingly, we define two assignment matrices, $\mathbf{X}^u \in \{0, 1\}^{I \times F}$ for the UL and $\mathbf{X}^d \in \{0, 1\}^{J \times F}$ for the

Table C.4: *Definition of sets, constants and variables*

Sets	
$\mathcal{F}$	Set of frequency channels
$\mathcal{I}$	Set of UL users
$\mathcal{J}$	Set of DL users
Constants	
β	SI cancelling term
σ^2	Thermal noise power on frequency channel f
γ_{th}^u	Minimum SINR required for UL users
γ_{th}^d	Minimum SINR required for DL users
G_{bjf}	Path gain between BS and DL user j on frequency f
G_{ibf}	Path gain between UL user i and BS on frequency f
$P_{\max}^u$	UL maximum transmit power
$P_{\max}^d$	DL maximum transmit power
Variables	
P_i^u	Transmit power of UL user i
P_j^d	Transmit power of DL user j
x_{if}^u	Assignment of UL user i on frequency f
x_{jf}^d	Assignment of DL user j on frequency f

DL, such that

$$x_{if}^u = \begin{cases} 1, & \text{if UE}_i \text{ transmits in frequency } f \\ 0, & \text{otherwise,} \end{cases}$$

and

$$x_{jf}^d = \begin{cases} 1, & \text{if UE}_j \text{ receives in frequency } f \\ 0, & \text{otherwise.} \end{cases}$$

The signal-to-interference-plus-noise ratio (SINR) at the BS due to transmitting user i in the UL on frequency channel f is

$$\gamma_{if}^u = \frac{x_{if}^u P_i^u G_{ibf}}{\sigma^2 + \sum_{j=1}^J x_{jf}^d \beta P_j^d}, \tag{C.1}$$

where x_{if}^u accounts for usage of frequency channel f by UL user i, and the sum in the denominator for the possible SI created by the simultaneous transmission by the BS on frequency channel f. Similarly, the SINR at the receiving user j due to the BS on frequency channel f is given by

$$\gamma_{jf}^d = \frac{x_{if}^d P_j^d G_{bjf}}{\sigma^2 + \sum_{i=1}^I x_{if}^u P_i^u G_{ijf}}, \tag{C.2}$$

where the x_{jf}^d accounts for usage of frequency channel f by the BS and the sum in the denominator for the intra-cell interference between UE_i and UE_j on frequency channel f.

Thus, the spectral efficiency for each user is given by the Shannon Eq. (in bits/s/Hz) for the UL and DL as

$$C_i^u = \sum\nolimits_{f=1}^{F} C_{if}^u = \sum\nolimits_{f=1}^{F} \log_2(1+\gamma_{if}^u), \tag{C.3a}$$

$$C_j^d = \sum\nolimits_{f=1}^{F} C_{jf}^d = \sum\nolimits_{f=1}^{F} \log_2(1+\gamma_{jf}^d), \tag{C.3b}$$

where the sum over the frequency dimension is because at most one SINR is non-zero along the frequency dimension.

C.3.2 Problem Formulation

Our goal is to jointly consider the problem of frequency assignment to UEs in the UL and DL (*pairing*), while maximizing the minimum spectral efficiency of all users (*max-min spectral efficiency*), thus increasing the fairness of the system. Specifically, the problem is formulated as a joint assignment and fairness maximization (JAFM) problem as follows

$$\underset{\mathbf{X}^u,\mathbf{X}^d,\mathbf{p}^u,\mathbf{p}^d}{\text{maximize}} \quad \min_{\forall i,j}\{C_i^u, C_j^d\} \tag{C.4a}$$

$$\text{subject to} \quad \sum_{f=1}^{F} \gamma_{if}^u \geq \gamma_{\text{th}}^u,\ \forall i, \tag{C.4b}$$

$$\sum_{f=1}^{F} \gamma_{jf}^d \geq \gamma_{\text{th}}^d,\ \forall j, \tag{C.4c}$$

$$P_i^u \leq P_{\text{max}}^u,\ \forall i, \tag{C.4d}$$

$$P_j^d \leq P_{\text{max}}^d,\ \forall j, \tag{C.4e}$$

$$\sum_{i=1}^{I} x_{if}^u \leq 1,\ \forall f, \tag{C.4f}$$

$$\sum_{f=1}^{F} x_{if}^u \leq 1,\ \forall i, \tag{C.4g}$$

$$\sum_{j=1}^{J} x_{jf}^d \leq 1,\ \forall f, \tag{C.4h}$$

$$\sum_{f=1}^{F} x_{jf}^d \leq 1,\ \forall j, \tag{C.4i}$$

$$x_{if}^u,\ x_{jf}^d \in \{0,1\},\ \forall i,j,f. \tag{C.4j}$$

The optimization variables of JAFM are $\mathbf{p}^u$, $\mathbf{p}^d$, $\mathbf{X}^u$ $\mathbf{X}^d$. Constraints (C.4b) and (C.4c) ensure a minimum SINR to be achieved in the DL and UL, respectively. Constraints (C.4d) and (C.4e) limit the transmitting power and constraints (C.4f)-(C.4i) assure that only one UE in the UL and DL can use frequency channel f and that any given frequency channel is used by at most one UE in the UL and DL. Let us further denote by $\mathcal{X}$ the set of matrices $\mathbf{X}^u$ and $\mathbf{X}^d$ that satisfy the constraints (C.4f)-(C.4j) and by $\mathcal{P}$ the set of vectors $\mathbf{p}^u$ and $\mathbf{p}^d$ that satisfy the constraints (C.4d)-(C.4e).

JAFM is a mixed integer nonlinear programming (MINLP) problem that belongs to the category of multi-level nonlinear bottleneck assignments, wherein the linear integer sub-problem is NP-hard [103]. Therefore, to analyse JAFM, we develop an approximate approach based on Lagrangian duality in Section C.5. Afterwards, by using the insights given by the dual problem, we propose a greedy approximation to problem (C.4) in Section C.6. Moreover, for simplicity we rename the primal problem (C.4) as P-JAFM.

Before analysing problem (C.4), we establish some preliminary results that will be instrumental in establishing an approximate solution to problem (C.4) in the subsequent sections.

C.4 Preliminary Results

In this section, we characterize the maximum number of users sharing the frequency channel, and the region of the transmit power for which users can share frequency channels. We use these two results in the following section to derive an approximate solution to problem (C.4).

TNFD implies that an UL and a DL transmission share a frequency channel f. The following lemma shows that in a TNFD system, complying with constraints (C.4f)-(C.4i), the number of users sharing a frequency channel f is bounded.

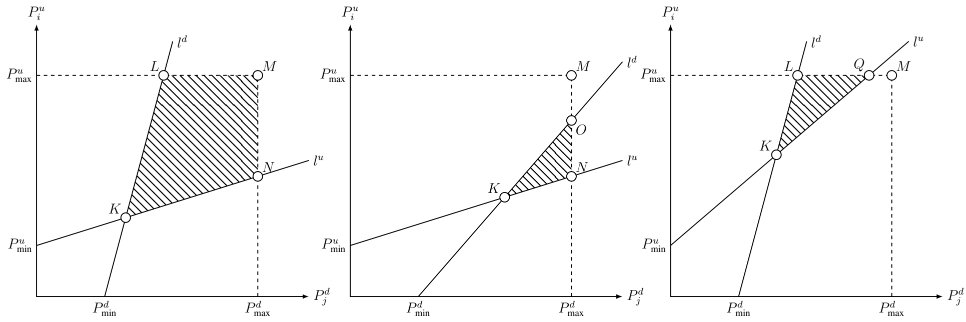
Lemma 5. Consider optimization Problem (C.4). Then, for every frequency channel f and feasible assignment matrices $\mathbf{X}^u$ and $\mathbf{X}^d$, the following inequality holds

$$\sum\nolimits_{i=1}^{I} x_{if}^u + \sum\nolimits_{j=1}^{J} x_{jf}^d \leq 2. \tag{C.5}$$

Proof. Since $I, J \leq F$, we have the inequality $I + J \leq 2F$. Moreover, if we sum constraints (C.4f) and (C.4h), we notice that $\sum_{i=1}^{I} x_{if}^u + \sum_{j=1}^{J} x_{jf}^d$ is at most 2. ■

Lemma 5 establishes that for feasible assignment matrices $\mathbf{X}^u$ and $\mathbf{X}^d$, there can be at most one pair of UL-DL users sharing a frequency channel f. We will use this result in the following subsection to derive an approximate closed-form solution to the assignment matrices of problem (C.4).

Now we turn our attention to characterize the admissible area of transmit powers for which a pair of UL and DL users can share a frequency channel f. In problem C.4, constraints (C.4b)-(C.4e) require a minimum SINR level for all users, implying that we need to identify the set of corresponding transmit powers within the constraint set $\mathcal{P}$. Based on the SINR and power constraints of Problem C.4, we establish Lemma 6 to determine whether a pair is admissible or not. We refer to the set of such powers as *admissible areas* (Figure C.2).



(a) *Area 1 given by* $\gamma^u_{iMf} > \gamma^u_{th}$ *and* $\gamma^d_{jMf} \geq \gamma^d_{th}$

(b) *Area 2 given by* $\gamma^u_{iMf} > \gamma^u_{th}$ *and* $\gamma^d_{jMf} < \gamma^d_{th}$

(c) *Area 3 given by* $\gamma^u_{iMf} \leq \gamma^u_{th}$ *and* $\gamma^d_{jMf} \geq \gamma^d_{th}$

Figure C.2: *The 3 admissible areas for a user i in the UL and a user j in the DL to share a frequency channel f that fulfil constraints* (C.4b)-(C.4e).

Figure C.2 shows three possible areas in terms of (P_j^d, P_i^u) where both the UL and DL users fulfil the minimum SINR requirement. In the figure, the lines l^u and l^d correspond to constraints (C.4b) and (C.4c) whereas the rectangular area drawn by the dashed line represents the constraint set $\mathcal{P}$. The points P^u_{min} and P^d_{min} fulfil constraints (C.4b) and (C.4c) with equality and HD transmission from both users, and can be calculated as

$$P^u_{\text{min}} = \frac{\gamma^u_{\text{th}}\sigma^2}{G_{ibf}}, \qquad P^d_{\text{min}} = \frac{\gamma^d_{\text{th}}\sigma^2}{G_{bjf}}. \tag{C.6}$$

The point K is the intersection point between the lines l^u and l^d, whereas points L, M, N, O and Q help to define the admissible (dashed) area.

Figure C.2a shows the first area, where $\gamma^u_{iMf} > \gamma^u_{\text{th}}$ and $\gamma^d_{jMf} \geq \gamma^d_{\text{th}}$, with corner points $L,\ M$ and N. At corner point L, the UL user is at the maximum power P^u_{max} while the DL user fulfils constraint (C.4c) with equality, whose coordinates are $(P^d_{jLf}, P^u_{\text{max}})$ where P^d_{jLf} is given by

$$P^d_{jLf} = \frac{\gamma^d_{\text{th}}(\sigma^2 + P^u_{\text{max}}G_{iLjLf})}{G_{bjLf}}. \tag{C.7}$$

Similarly, at corner point N the DL user is at maximum power P^d_{max} while the UL user fulfils constraint (C.4b) with equality, whose coordinates are $(P^d_{\text{max}}, P_{iNf})^u$ where P^u_{iNf} is given by

$$P^u_{iNf} = \frac{\gamma^u_{\text{th}}(\sigma^2 + P^d_{\text{max}}\beta)}{G_{iNbf}}. \tag{C.8}$$

The second area is shown in Figure C.2b, where $\gamma^u_{iMf} > \gamma^u_{\text{th}}$ and $\gamma^d_{jMf} < \gamma^d_{\text{th}}$, with corner points O and N. The corner point N is the same as in the first area, but at corner point O, the DL user is at maximum power P^d_{max} and also fulfils constraint (C.4c) with equality. Thus, the coordinates of point O are $(P^d_{\text{max}}, P^u_{iOf})$, where P^u_{iOf} is given by

$$P^u_{iOf} = \frac{P^d_{\text{max}} G_{bjOf} - \gamma^d_{\text{th}} \sigma^2}{G_{iOjOf} \gamma^d_{\text{th}}}. \tag{C.9}$$

Finally, Figure C.2c shows the third area, where $\gamma^u_{iMf} \leq \gamma^u_{\text{th}}$ and $\gamma^d_{jMf} \geq \gamma^d_{\text{th}}$, with corner points L and Q. The corner point L is the same as in the first area, but at corner point Q, the UL user is at maximum power P^u_{max} and fulfils constraint (C.4b) with equality. Similarly to corner point O, the coordinates of point Q are $(P^d_{jQf}, P^u_{\text{max}})$, where P^d_{jQf} is given by

$$P^d_{jQf} = \frac{P^u_{\text{max}} G_{iQbf} - \gamma^u_{\text{th}} \sigma^2}{\beta \gamma^u_{\text{th}}}. \tag{C.10}$$

From constraints (C.4b)-(C.4e), we can derive the following Lemma that will help us to determine if an admissible area exists for a given pair of UL and DL users. We say that a pair of users is an admissible pair if an associated admissible area exists.

Lemma 6. Consider optimization Problem (C.4). A pair of an UL user i and a DL user j sharing frequency channel f is an *admissible pair* if the residual SI term is such that $\beta \leq \beta^{\text{max}}_{ijf}$, where β^{max}_{ijf} is

$$\beta^{\text{max}}_{ijf} = \begin{cases} \dfrac{G_{bjf}(P^u_{\text{max}} G_{ibf} - \gamma^u_{\text{th}} \sigma^2)}{\gamma^u_{\text{th}} \gamma^d_{\text{th}} (P^u_{\text{max}} G_{ijf} + \sigma^2)}, \text{if } \gamma^d_{jMf} > \gamma^d_{\text{th}}, \\ \dfrac{P^d_{\text{max}} G_{bjf} G_{ibf} - \sigma^2 \gamma^d_{\text{th}} (G_{ijf} \gamma^u_{\text{th}} + G_{ibf})}{\gamma^u_{\text{th}} \gamma^d_{\text{th}} P^d_{\text{max}} G_{ijf}}, \\ \text{if } \gamma^d_{jMf} \leq \gamma^d_{\text{th}}. \end{cases} \tag{C.11}$$

Proof. See Appendix C.9.3. ■

Based on the previous result, a pair of UL and DL users is admissible if that pair fulfils Lemma 6. As we will see in the next section, once this is guaranteed, the optimal transmit powers lie in the admissible area to which the pair belongs.

C.5 Solution via Lagrange Dual Problem

In this section we analyse JAFM through duality theory, where we first derive an equivalent version of problem (C.4) in Section C.5.1, which will help us to mathematically treat the objective function (C.4a) as a linear function. In Section C.5.2 we form the partial Lagrangian function and derive a closed-form solution for the assignments $\mathbf{X}^u, \mathbf{X}^d$, and in Section C.5.3 we show that the optimal power allocation, given a pair of users reusing frequency channel f, is within the admissible areas shown in Section C.4. Finally, Section C.5.4 summarizes the insights given by the solution of the dual problem, which are important to develop the greedy approximation of problem (C.4) on Section C.6, named JAFMA.

C.5.1 Problem Transformation

As a first step of solving problem (C.4), we consider the standard equivalent hypograph [110, Sec. 3.1.7] form of problem (C.4), where one new variable and two more constraints are included:

$$\begin{aligned} \underset{\mathbf{X}^u,\mathbf{X}^d,\mathbf{p}^u,\mathbf{p}^d,t}{\text{maximize}} \quad & t && \text{(C.12a)} \\ \text{subject to} \quad & C_i^u \geq t, \forall i, && \text{(C.12b)} \\ & C_j^d \geq t, \forall j && \text{(C.12c)} \\ & \text{Constraints (C.4b)-(C.4j)} && \text{(C.12d)} \end{aligned}$$

where $t > 0$ is an additional variable with respect to (C.4). Notice that problem (C.12), similarly to problem (C.4a), is a MINLP. It is possible to relax its integer constraints, but the resulting problem is not convex as established by the following result.

Result 1. The continuous relaxation of problem (C.12), given by letting $\mathbf{X}^u \in [0,1]^{I\times F}$ and $\mathbf{X}^d \in [0,1]^{J\times F}$, is not convex.

Proof. See Appendix C.9.1. ■

It is important to know that the relaxed problem is not convex, because it shows that relaxing the integer constraint does not lead to a problem that can be solved by conventional solvers and methods, which motivates us to develop an alternative solution approach.

C.5.2 Solution for Assignment Matrices $\mathbf{X}^u, \mathbf{X}^d$

We can now form the *partial* Lagrangian function by considering constraints (C.12b)-(C.12d) of problem (C.12) and ignoring the integer constraints (C.4f)-(C.4j). To this end, we introduce Lagrange multipliers $\boldsymbol{\lambda}^u,\ \boldsymbol{\lambda}^d,\ \boldsymbol{\delta}^u,\ \boldsymbol{\delta}^d$, where the superscript u denotes the dimension of I and d of J, respectively. The partial Lagrangian is a function of the Lagrange multipliers and the optimization variables $\mathbf{X}^u, \mathbf{X}^d, \mathbf{p}^u, \mathbf{p}^d, t$ as follows:

$$\begin{aligned} L(\boldsymbol{\lambda}^{u,d}, \boldsymbol{\delta}^{u,d}, \mathbf{X}^{u,d}, \mathbf{p}^{u,d}, t) \triangleq t\left(\sum_{i=1}^{I} \lambda_i^u + \sum_{j=1}^{J} \lambda_j^d - 1\right) + \sum_{i=1}^{I}\left(\delta_i^u\left(\gamma_{\text{th}}^u - \sum_{f=1}^{F}\gamma_{if}^u\right) - \lambda_i^u C_i^u\right) \\ + \sum_{j=1}^{J}\left(\delta_j^d\left(\gamma_{\text{th}}^d - \sum_{f=1}^{F}\gamma_{jf}^d\right) - \lambda_j^d C_j^d\right), \end{aligned} \tag{C.13}$$

where by a slight abuse of notation we introduce $\boldsymbol{\lambda}^{u,d}$ to denote the two vectors $\boldsymbol{\lambda}^u$ and $\boldsymbol{\lambda}^d$, and we assume $\mathbf{X}^u, \mathbf{X}^d \in \mathcal{X}$, and $\mathbf{p}^u, \mathbf{p}^d \in \mathcal{P}$.

Let $g(\boldsymbol{\lambda}^{u,d}, \boldsymbol{\delta}^{u,d})$ denote the dual function obtained by minimizing the partial Lagrangian (C.13) with respect to the Lagrange multipliers and variable t. Thus, the dual function is

$$g(\boldsymbol{\lambda}^{u,d}, \boldsymbol{\delta}^{u,d}) = \inf_{t\in\mathbb{R}^+\ \mathbf{X}^{u,d}\in\mathcal{X}\ \mathbf{p}^{u,d}\in\mathcal{P}} L(\boldsymbol{\lambda}^{u,d}, \boldsymbol{\delta}^{u,d}, \mathbf{X}^{u,d}, \mathbf{p}^{u,d}, t) \tag{C.14a}$$

$$g(\boldsymbol{\lambda}^{u,d}, \boldsymbol{\delta}^{u,d}) = \begin{cases} \inf\limits_{\mathbf{X}^{u,d}\in\mathcal{X}\ \mathbf{p}^{u,d}\in\mathcal{P}} \Big[\sum\limits_{i} q_i^u(\mathbf{X}^{u,d}, \mathbf{p}^{u,d}) + \\ \sum\limits_{j} q_j^d(\mathbf{X}^{u,d}, \mathbf{p}^{u,d})\Big], \quad \text{if } \sum_i \lambda_i^u + \sum_j \lambda_j^d = 1 \\ -\infty, \qquad \text{otherwise,} \end{cases} \tag{C.14b}$$

where from equality (C.14b) follows that the linear function $t\big(\sum_{i=1}^{I}\lambda_i^u + \sum_{j=1}^{J}\lambda_j^d - 1\big)$ is lower bounded when it is identically zero, and $q_i^u(\mathbf{X}^{u,d}, \mathbf{p}^{u,d})$ and $q_j^d(\mathbf{X}^{u,d}, \mathbf{p}^{u,d})$ are

$$q_i^u(\mathbf{X}^{u,d}, \mathbf{p}^{u,d}) \triangleq \delta_i^u\Big(\gamma_{\text{th}}^u - \sum\nolimits_{f=1}^{F}\gamma_{if}^u\Big) - \lambda_i^u C_i^u, \tag{C.15a}$$

$$q_j^d(\mathbf{X}^{u,d}, \mathbf{p}^{u,d}) \triangleq \delta_j^d\Big(\gamma_{\text{th}}^d - \sum\nolimits_{f=1}^{F}\gamma_{jf}^d\Big) - \lambda_j^d C_j^d. \tag{C.15b}$$

Notice that we can find the infimum of $q_i^u(\mathbf{X}^{u,d}, \mathbf{p}^{u,d})$ as

$$\inf_{\substack{\mathbf{X}^{u,d}\in\mathcal{X}\\ \mathbf{p}^{u,d}\in\mathcal{P}}} q_i^u(\mathbf{X}^{u,d}, \mathbf{p}^{u,d}) = \delta_i^u\Big(\gamma_{\text{th}}^u - \gamma_{if^{(i)}}^{u,\max}\Big) - \lambda_i^u \log_2\Big(1 + \gamma_{if^{(i)}}^{u,\max}\Big), \tag{C.16}$$

where $f^{(i)} = \arg\max\limits_{f}\ \gamma_{if}^{u,\max}$ and $\gamma_{if}^{u,\max} = \max_{\mathbf{p^{u,d}}\in\mathcal{P}}\gamma_{if}^u$. We define $\gamma_{if}^{u,\max}$ as the maximum SINR that UL user i can achieve on frequency resource f, where the maximization is done over the power variables $\mathbf{p}^{u,d} \in \mathcal{P}$. Note that $\gamma_{if}^{u,\max}$ is a function of the given assignment matrices $\mathbf{X}^{u,d}$, and it should be evaluated for the DL user that is receiving in frequency resource f. Due to constraints (C.4f)-(C.4j) on the partial Lagrangian and using Lemma 5, the summations in (C.15) include a single frequency channel that maximizes the $\gamma_{if}^{u,\max}$ in (C.16). Then, the chosen $f^{(i)}$ is unique, because a UL user cannot be connected to more than one frequency channel. Based on this, we propose a solution for x_{if}^u and x_{jf}^d given by

$$x_{if^{(i)}}^{u\star} = \begin{cases} 1, & \text{if } f^{(i)} = \arg\max\limits_{f}\gamma_{if}^{u,\max} \\ 0, & \text{otherwise,} \end{cases} \tag{C.17a}$$

$$x_{jf^{(j)}}^{d\star} = \begin{cases} 1, & \text{if } f^{(j)} = \arg\max\limits_{f}\gamma_{jf}^{d,\max} \\ 0, & \text{otherwise.} \end{cases} \tag{C.17b}$$

Remark 2. Recall we need to find the dual function (C.14b) of problem (C.4), which includes the infimum of the partial Lagrangian function (C.14a). This requires finding the infimum of $q_i^u(\mathbf{X}^{u,d}, \mathbf{p}^{u,d})$ and $q_j^d(\mathbf{X}^{u,d}, \mathbf{p}^{u,d})$ as in (C.15), and (C.16). These infima are obtained at $x_{if}^{u\star}$ and $x_{jf}^{d\star}$, given by Eqs (C.17), because Eqs. (C.17) assign to UL and DL users frequency channels $f^{(i)}$and $f^{(j)}$ that maximizes $\gamma_{if}^{u,\max}$ and $\gamma_{jf}^{d,\max}$, thus minimizing $q_i^u(\mathbf{X}^{u,d}, \mathbf{p}^{u,d})$ and $q_j^d(\mathbf{X}^{u,d}, \mathbf{p}^{u,d})$.

Note that from Eqs. (C.17) and constraints (C.4f)-(C.4h), we can uniquely associate a user i in the UL with a frequency channel f and do the same with a user j in the DL. However, the solutions are still intertwined through the SINRs γ^u_{if} and γ^d_{jf}. This implies that if an UL user changes its allocation, this modification will impact the DL users, whose modifications are then reflected in the SINRs γ^u_{if} and γ^d_{jf}, which makes the problem of finding the assignment matrix complex. Due to the high complexity of this solution, we use the results from Section C.5.3 and Section C.5.4 to reformulate Eqs. (C.17) as an NP-Hard problem in Section C.6, where we propose a greedy approximation solution whose solution is the assignment matrices searched for in Eqs. (C.17).

C.5.3 Solution for $\mathbf{p}^u, \mathbf{p}^d$

Since the SINR from the UL is not separable from that of the DL, they must be analyzed jointly. We therefore need to find the powers that jointly minimize expression (C.14b) $g(\boldsymbol{\lambda}^{u,d}, \boldsymbol{\delta}^{u,d})$, thus we can formulate the subproblem of power setting as

$$\begin{aligned} \underset{\mathbf{p}^u, \mathbf{p}^d}{\text{minimize}} \quad & -\sum_{i=1}^{I} \left(\delta^u_i \gamma^u_{if(i)} + \lambda^u_i \log_2(1 + \gamma^u_{if(i)}) \right) \\ & -\sum_{j=1}^{J} \left(\delta^d_j \gamma^d_{jf(j)} + \lambda^d_j \log_2(1 + \gamma^d_{jf(j)}) \right) && \text{(C.18a)} \\ \text{subject to} \quad & \mathbf{p}^u, \mathbf{p}^d \in \mathcal{P}, && \text{(C.18b)} \end{aligned}$$

where it is assumed that all the frequencies $f(i)$ and $f(j)$ have been assigned without the terms related to γ^u_{th} and γ^d_{th}, since these terms are constant and do not affect the problem in (C.18).

The following lemma reveals an important property of the optimal transmit powers that will be useful in finding the solutions to (C.18).

Lemma 7. Consider optimization Problem C.18. The optimal transmit power pair $(P^{u\star}_i, P^{d\star}_j)$ of a user pair sharing the same frequency channel has at least one component equal to either P^u_{max} or P^d_{max}.

Proof. Suppose that there is a user i' or j' that do not share the same frequency channel. Then the interference term is zero and the SINR is equal to the signal-to-noise ratio (SNR) and the maximum power minimizes the objective sum of problem (C.18) for these users. Alternatively, suppose user i in the UL shares the frequency channel f with user j in the DL.

User i is only associated with user j and vice-versa, since these users use the same frequency channel, i.e., $f(i) = f(j)$. Therefore, the optimization for these two users can be taken independently of all the other users.

The objective function can be redefined as

$$O(P_i^u, P_j^d) \triangleq \frac{\delta_i^u P_i^u G_{ibf}}{\sigma^2 + P_j^d \beta} + \lambda_i^u \log_2 \left(1 + \frac{P_i^u G_{ibf}}{\sigma^2 + P_j^d \beta}\right) + \frac{\delta_j^d P_j^d G_{bjf}}{\sigma^2 + P_i^u G_{ijf}} +$$
$$\lambda_j^d \log_2 \left(1 + \frac{P_j^d G_{bjf}}{\sigma^2 + P_i^u G_{ijf}}\right) \tag{C.19}$$

Thus, we have that for $\alpha \in \mathbb{R}, \alpha > 1$ and $\mathbf{p}^u, \mathbf{p}^d \in \mathcal{P}$:

$$O(\alpha P_i^u, \alpha P_j^d) = \frac{\delta_i^u P_i^u G_{ibf}}{\frac{\sigma^2}{\alpha} + P_j^d \beta} + \lambda_i^u \log_2 \left(1 + \frac{P_i^u G_{ibf}}{\frac{\sigma^2}{\alpha} + P_j^d \beta}\right) + \frac{\delta_j^d P_j^d G_{bjf}}{\frac{\sigma^2}{\alpha} + P_i^u G_{ijf}}$$
$$+ \lambda_j^d \log_2 \left(1 + \frac{P_j^d G_{bjf}}{\frac{\sigma^2}{\alpha} + P_i^u G_{ijf}}\right)$$

Note that $O(\alpha P_i^u, \alpha P_j^d) > O(P_i^u, P_j^d)$. Therefore, similarly to [101], the optimal power pair $(P_i^{u\star}, P_j^{d\star})$ is bounded by the maximum power constraints $(P^u_{\text{max}}, P^d_{\text{max}})$, which concludes the proof. ■

From this lemma it follows that we must look for solutions into the two functions of the form $O(P_i^u, P^d_{\text{max}})$ and $O(P^u_{\text{max}}, P_j^d)$. Next, the following result establishes that the points that maximize $O(P_i^u, P_j^d)$ are in the *corner points* of the constraint set $\mathcal{P}$.

Lemma 8. The function $O(P_i^u, P^d_{\text{max}})$ given in Eq. (C.19) is convex for positive P_i^u and is maximized at the corner points of the constraint set $\mathcal{P}$.

Proof. See Appendix C.9.2. ■

Lemma 8 implies that to maximize $O(P_i^u, P_j^d)$ we must look in the corner points of the constraint set $\mathcal{P}$. Moreover, as shown in Section C.4, we should look in a subset of $\mathcal{P}$ in order to fulfil the SINR constraints (C.4b)-(C.4c), the admissible area of pair i and j. Based on the optimal transmit powers and the closed-form solution for the assignment, we will now analyse the Lagrange dual problem and use its implications to develop the greedy approximation of P-JAFM on Section C.6.

C.5.4 Insights from the Dual

Once the transmit powers are determined looking for the corner points in the admissible area of the UL-DL pairs as shown in Section C.4, we can formulate the Lagrange dual problem to problem (C.12) as

$$\underset{\boldsymbol{\lambda}^{u,d},\ \boldsymbol{\delta}^{u,d}}{\text{maximize}} \quad g(\boldsymbol{\lambda}^{u,d},\ \boldsymbol{\delta}^{u,d}) \tag{C.20a}$$

$$\text{subject to} \quad \sum\nolimits_{i=1}^{I} \lambda_i^u + \sum\nolimits_{j=1}^{J} \lambda_j^d = 1, \tag{C.20b}$$

$$\lambda_i^u,\ \lambda_j^d,\ \delta_i^u,\ \delta_j^d \geq 0, \forall i, j, \tag{C.20c}$$

where recall that $g(\boldsymbol{\lambda}^{u,d},\ \boldsymbol{\delta}^{u,d})$ is the solution of problem (C.14b). Notice that we can interpret the dual problem above as a weighted sum spectral efficiency maximization of both UL and DL users, where the weights are chosen based on a convex combination of both users' spectral efficiencies.

For every pair, it is necessary to decide which corner point within the admissible area will be used. Since the values of $\boldsymbol{\lambda}^{u,d}$ are tied through Eq. (C.17) and the SINRs, we need to check all possible combinations of the admissible areas of different pairs. However, by analyzing the structure of the dual function, we note that $\boldsymbol{\delta}^{u,d}$ are lower bounded by 0 and the terms $\delta_i^u\left(\gamma_{\text{th}}^u - \gamma_{if}^u\right)$ and $\delta_j^d\left(\gamma_{\text{th}}^d - \gamma_{jf}^d\right)$ are negative for a pair (i,j) associated with frequency f.

Since the dual maximizes $g(\boldsymbol{\lambda}^{u,d},\ \boldsymbol{\delta}^{u,d})$ and the terms above are negative, either the SINRs are equal to the minimum (γ_{th}^u or γ_{th}^d), or $\boldsymbol{\delta}^{u,d}$ is 0. Thus, the terms with $\boldsymbol{\delta}^{u,d}$ do not impact $g(\boldsymbol{\lambda}^{u,d},\ \boldsymbol{\delta}^{u,d})$. The dual maximizes the negative convex combination of the spectral efficiencies of UL and DL, which results in a value close to 1 for the $\lambda^{u,d}$ related to the user with minimum spectral efficiency in the system, implying that the optimal value of the Lagrange dual problem is close to the negative of the minimum spectral efficiency in the system. Moreover, recall that the closed-form solutions for $\mathbf{X}^{u,d}$ on (C.17) are still tied through the SINRs γ_{if}^u and γ_{jf}^d, meaning that the solution for the assignment is still complex.

C.6 Approximate Solution via Greedy Method

Given the results on the closed-form solution to the assignment problem (C.17) the optimal power allocation problem (C.18) from Section C.5, we can solve the dual problem by checking exhaustively which pair of UL and DL users jointly solve Eq. (C.17), where the power allocation for each pair is given by the solution of the admissible areas (see Section C.5.3). Furthermore, Section C.5.4 showed that the dual problem (C.20) maximizes the user with minimum spectral efficiency in the system. Therefore, we can solve the dual problem by selecting the pair of UL and DL users that maximize the spectral efficiency of the user with minimum spectral efficiency of the pair.

However, such exhaustive solution demands an extremely high number of iterations that depend on the number of frequency channels, which in practical cellular systems is essentially impossible due to the high number of frequency channels. Thus, in order to use the results from Section C.5 and solve the dual problem (C.20) in an efficient manner, we reformulate the dual problem (C.20) on Section C.6.1 to solve the assignment and propose a greedy approximation on Section C.6.2 that aims at maximizing the sum of the minimum spectral efficiency of the UL-DL pairs, named JAFMA. Finally, in Section C.6.3 we discuss the efficiency and complexity of JAFMA when compared to the P-JAFM (C.4) and dual (C.20) problems.

C.6.1 The Dual as a 3-Dimensional Assignment Problem

As seen in Lemma 6, the SI cancellation term β must fulfil $\beta \leq \beta_{ijf}^{\max}$ for every assigned pair of UL user i and DL user j on frequency channel f. Thus, an algorithm that aims to solve the JAFM problem needs to identify the UL-DL user *candidates* for every frequency f. Notice that due to the minimum SINR constraints (C.4b) and (C.4c) in JAFM, a given UL user i or DL user j may not have a pair with which it would form an admissible pair. On the other hand, once a pair fulfils Lemma 6, there is an admissible area where both users fulfil constraints (C.4b) and (C.4c).

From the reasoning in Section C.5.4 and using Lemma 8, it follows that we can select the corner point that maximizes the minimum spectral efficiency of the two users in the pair, and thereby maximizes the sum of the minimum spectral efficiencies over all pairs. Note that the power allocation problem (C.18) formulation comes from minimizing $q_i^u(\mathbf{X}^{u,d}, \mathbf{p}^{u,d}) + q_j^d(\mathbf{X}^{u,d}, \mathbf{p}^{u,d})$ for the admissible pair (i, j). Thus, we can use the observations on maximizing the sum of the minimum spectral efficiency to reformulate the solution for the assignment (C.17) as an axial 3-dimensional assignment problem (3-DAP) given by:

$$\underset{\mathbf{X}}{\text{maximize}} \quad \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{f=1}^{F} s_{ijf}x_{ijf} \tag{C.21a}$$

$$\text{subject to} \quad \sum_{j=1}^{J}\sum_{f=1}^{F} x_{ijf} = 1, \ \forall i, \tag{C.21b}$$

$$\sum_{i=1}^{I}\sum_{f=1}^{F} x_{ijf} = 1, \ \forall j, \tag{C.21c}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ijf} = 1, \ \forall f, \tag{C.21d}$$

$$x_{ijf} \in \{0, 1\}, \ \forall i, j, f. \tag{C.21e}$$

The matrix $\mathbf{S} = [s_{ijf}] \in \mathbb{R}^{I\times J\times F}$ represents the minimum spectral efficiency of the pair of users i and j on frequency channel f, which is explicitly defined as $s_{ijf} = \min\{C_{if}^u, C_{jf}^d\}$. We define $\mathbf{X} = [x_{ijf}] \in \{0, 1\}^{I\times J\times F}$ as the binary assignment matrix. Recall that C_{if}^u and C_{jf}^d depend on the powers, thus we select the power vector from the corner point (P_j^d, P_i^u) that maximizes s_{ijf}. The selection of the corner point that maximizes s_{ijf} is according to the solution of problem (C.18), which uses Lemma 8 to assure that the optimal solution is indeed one of the corner points.

Remark 3. From Remark 2 it follows that we can assign UL and DL users frequency channels $f(i)$ and $f(j)$ that maximize $\gamma_{if}^{u,\max}$ and $\gamma_{jf}^{d,\max}$ from the minimization of $q_i^u(\mathbf{X}^{u,d}, \mathbf{p}^{u,d})$ and $q_j^d(\mathbf{X}^{u,d}, \mathbf{p}^{u,d})$, and this is precisely the formulation of the axial 3-DAP (C.21). Therefore, the solutions to problem (C.21) are the assignment matrices searched for in equation (C.17) and together with the optimal power allocation, solves the dual problem (C.20). For clarity, we rename the axial 3-DAP (C.21) problem as the D-JAFM problem to indicate its connection with the dual problem.

The D-JAFM problem has $(n!)^2$ possible assignments and is a well known NP-hard problem [103, Section 8.2]. Therefore, by using the insights in the preceding section, we develop a greedy algorithm to solve D-JAFM and approximate the P-JAFM problem.

C.6.2 A Greedy Approximation Solution to the JAFM Problem

Differently from other works that reduce the assignment problem (C.21) to two-dimensions [32], we aim at jointly assigning the pairs to frequencies. We propose a greedy algorithm with reduced complexity to solve the D-JAFM problem, where using the power allocation from Section C.5.3 we approximate P-JAFM problem. Algorithm 7 shows the steps of the proposed greedy solution, called joint fairness assignment maximization algorithm (JAFMA). JAFMA evaluates $\beta_{ijf}^{\max}$ for every user and frequency (see line 5). If the assumptions of Lemma 6 are fulfilled for the pair (i, j) on frequency channel f, JAFMA evaluates the powers that maximize the minimum spectral efficiency s_{ijf} based on the admissible area for the pair (see Figure C.2 and line 7 of Algorithm 7).

Once we have the s_{ijf} for all the admissible pairs, JAFMA sorts the matrix $\mathbf{S}$ in descending order of the minimum spectral efficiency of users in the pair (see line 14). If the maximum value is not unique, JAFMA selects the pairs that attain this maximum and within these pairs, JAFMA selects the pair with maximum sum of spectral efficiency (see line 17). If the maximum is unique, select the pair and frequency that achieves that maximum (see line 19).

If there is a pair that has not been assigned to a frequency channel, JAFMA checks which user (either UL or DL) has higher spectral efficiency and assigns the frequency channel to it (see lines 22-23). As for the non-selected user, JAFMA accounts it for the disconnected users statistics (see line 24), which will help us to later define the ratio of connected users. At the end, JAFMA removes the assigned users and frequency channels from the matrix $\mathbf{S}$ and continues the loop over all users until all users have been assigned to a frequency channel (see lines 29-30).

When $I + J < 2F$, i.e., when there are more resources than pairs, JAFMA selects the pair with minimum spectral efficiency and *reassigns* the newly available resources to the users in the selected pair starting from the user with minimum spectral efficiency (see line 32). The users will select the frequency channel that gives the highest path gain within the available resources (see line 35 and 38). This loop will continue until all the initially available resources have been used. At the end, JAFMA selects the pairs that maximize the minimum spectral efficiency within each pair, which means that JAFMA maximizes the sum of the pairwise minimum spectral efficiencies of the system.

With JAFMA in hand, we can compare its performance with existing algorithms to assign and to allocate power that current system designers use. For the frequency assignment, the simplest solution is a random allocation of frequency channel to UL and DL users [122]. For the power allocation, the common solution is equal power allocation (EPA) with maximum power transmission for UL users and for the BS [100, 123]. In

Algorithm 7 Joint Fairness Assignment Maximization Algorithm (JAFMA)

Input: $G_{ibf},\ G_{bjf},\ G_{ijf},\ \beta,\ P^u_{\text{max}},\ P^d_{\text{max}}$
for $i = 1$ **to** I **do**
    for $j = 1$ **to** J **do**
        for $f = 1$ **to** F **do**
            Evaluate β^{max}_{ijf} according to Eq. (C.11)
            if $\beta \leq \beta^{\text{max}}_{ijf}$ (see Lemma 6) **then**
                Evaluate the corner point (P^d_j, P^u_i) that maximizes $s_{ijf} = \min\{C^u_{if}, C^d_{jf}\}$ (see Section C.4)
            end if
        end for
    end for
end for
$\mathcal{I}_a = \mathcal{J}_a = \mathcal{F}_a = \emptyset \leftarrow$ Sets of assigned users and resources are initially empty
while $\mid \mathcal{I}_a \mid \neq I$ **and** $\mid \mathcal{J}_a \mid \neq J$ **do**
    $s_{\text{max}} = \max s_{ijf}, \forall i, j, f \leftarrow$ Sort in descending order s_{ijf}
    if s_{max} is not unique **then**
        $\mathcal{S}_{\text{max}} \leftarrow$ Set of users with minimum spectral efficiency of the pair equals to s_{max}
        $(i^\star, j^\star, f^\star) = \arg\max_{\mathcal{S}_{\text{max}}} C^u_{if} + C^d_{jf} \leftarrow$ Select the pair and frequency with maximum sum of the spectral efficiency within the set of users $\mathcal{S}_{\text{max}}$
    else
        $(i^\star, j^\star, f^\star) = \arg\max s_{\text{max}} \leftarrow$ Select the pair and frequency with maximum minimum spectral efficiency
    end if
    if $s_{\text{max}} = 0 \leftarrow$ If this pair is not admissible **then**
        Assign the frequency channel to the user with maximum spectral efficiency using maximum power (either P^u_{max} or P^d_{max})
        Assign P^u_{max} or P^d_{max} for the chosen user
        Assign 1 in the assignment matrix (either $x^u_{i^\star f^\star}$ or $x^d_{j^\star f^\star}$) to the selected user and 0 to the non-selected
    else
        $x^u_{i^\star f^\star} = 1$ and $x^d_{j^\star f^\star} = 1$
        Evaluate $(P^d_{j^\star}, P^u_{i^\star})$ using the admissible areas and the SINRs and spectral efficiencies using Eqs. (C.1)-(C.3)
    end if
    $\mathcal{I}_a = \mathcal{I}_a \cup \{i^\star\},\ \mathcal{J}_a = \mathcal{J}_a \cup \{j^\star\},\ \mathcal{F}_a = \mathcal{F}_a \cup \{f^\star\} \leftarrow$ Update the set of assigned resources
    Remove the already assigned users and frequency $(i^\star, j^\star, f^\star)$ from s_{ijf}
end while
while $\mid \mathcal{F}_a \mid \neq F$ **do**
    $(i^\star, j^\star, f^\star) = \arg\min_{\mathcal{I}_a, \mathcal{J}_a, \mathcal{F}_a}\{C^u_{if}, C^d_{jf}\} \leftarrow$ Get the users and frequency with minimum spectral efficiency within the already assigned
    $\mathcal{F}_{av} = (\mathcal{F} \setminus \mathcal{F}_a) \cup \{f^\star\} \leftarrow$ New set of available resources
    Select the frequency channel $f^\dagger$ within $\mathcal{F}_{av}$ that gives the highest desired path gain for the user with minimum spectral efficiency selected (G_{ibf} if UL or G_{bjf} if DL)
    $\mathcal{F}_a = \mathcal{F}_a \cup \{f^\dagger\} \leftarrow$ Update the set of assigned resources
    $\mathcal{F}_{av} = (\mathcal{F}_{av} \setminus \{f^\dagger\}) \leftarrow$ Update the set of available resources
    Select the frequency channel $\hat{f}$ within $\mathcal{F}_{av}$ that gives the highest desired path gain now for the other user that was reusing frequency $f^\star$
    $\mathcal{F}_a = \mathcal{F}_a \cup \{\hat{f}\} \leftarrow$ Update the set of assigned resources
end while
Output: $\mathbf{X}^u, \mathbf{X}^d, \mathbf{p}^u, \mathbf{p}^d$

Section C.7 we will compare the performance of JAFMA with different variations of a random assignment and EPA.

C.6.3 Discussion

The complexity of JAFMA relies on sorting in descending order the values of minimum spectral efficiency of every pair on the frequency channel, which can be interpreted as sorting a matrix with dimensions $I \times J \times F$ (see the matrix element s_{ijf} on line 7). With the heap sorting algorithm, the worst-case solution has complexity of $O(n \log n)$ [124], where n is the size of the vector. Since the matrix to be sorted has size $I \times J \times F$, we represent it as a vector of size IJF, and assuming that $I = J = F$, the worst-case complexity of the greedy solution is $O(F^3 \log F)$. If compared with the exhaustive solution of D-JAFM that has worst-case complexity of $O(F!^2)$ and considering $F = 10$, JAFMA requires $O(10^3)$ while D-JAFM requires $O(10^{13})$. Although problem (C.21) has been studied before, to the best of our knowledge, our proposed greedy solution has never been proposed. Other heuristics were developed for this problem [108, Section 10.2.5], but they rely on metaheuristics, graph theory, and on some simplifications of the cost s_{ijf}. These other solutions cannot be used in our case, because they either require unacceptable simplifications or are computationally too complex.

Remark 4. To analyse the performance of the proposed JAFMA, let us assume that $\mathbf{X}^\star$ and $\mathbf{X}_G$ are the optimal solution of D-JAFM and the one given by JAFMA, respectively, and let the objective function (C.21a) be $c(\mathbf{X})$. In Hausmann et al. [114, Corollary 4], it is proved in a more general context of problem (C.21) (intersection of matroids) that any greedy solution, without specifying any particular solution, yields a performance guarantee of $1/3$, i.e., $c(\mathbf{X_G})/c(\mathbf{X}^\star) \geq 1/3$. Therefore, our greedy solution is an *approximated* solution of D-JAFM. Moreover, as we will see in the numerical results on Section C.7, the proposed JAFMA has a tighter performance than the guaranteed for any greedy algorithm.

Note that problem (C.21) aims at solving the closed-form solution of the assignment (C.17) taking into account the optimal power allocation (see Section C.5.3) and insights from the dual problem (see Section C.5.4), which shows that D-JAFM has its basis on the dual problem (C.20). Therefore, it is necessary to take into account the duality gap between D-JAFM and P-JAFM problems. Recall that the objective function of P-JAFM (C.4) is the minimum spectral efficiency of UL and DL users.

Based on the works of Weeraddana et al. [125], we show in Proposition 1 that the relative duality gap diminishes when the total number of resources in the system F increases. The concept of relative duality gap is extended to the relative optimality gap, where $d^\star$ can also be the solution given by JAFMA.

Proposition 1. Consider $p^\star$ as the optimal solution of problem P-JAFM in (C.4), and $d^\star$ as the optimal solution of problem D-JAFM in (C.21), and that the number of UL users, I, and DL users, J, do not increase with the frequency channels. The duality gap $(p^\star - d^\star)/p^\star$ diminishes as the number of frequency channels increases.

Proof. See Appendix C.9.4. ■

Table C.5: *Simulation parameters*

Parameter	Value
Cell radius	$100\,\mathrm{m}$
Number of UL UEs $[I = J]$	[4 19 25]
Monte Carlo iterations	400
Carrier frequency	$2.5\,\mathrm{GHz}$
System bandwidth	$5\,\mathrm{MHz}$
Number of freq. channels $[F]$	[25]
LOS path-loss model	$34.96 + 22.7\log_{10}(d)$
NLOS path-loss model	$33.36 + 38.35\log_{10}(d)$
LOS probability	$\min(18/d, 1)(1 - \exp(-d/36)) + \exp(-d/36)$
Shadowing st. dev. LOS	$3\,\mathrm{dB}$
Shadowing st. dev. NLOS	$4\,\mathrm{dB}$
Thermal noise power $[\sigma^2]$	$-116.4\,\mathrm{dBm}$/channel
Average user speed	$3\,\mathrm{km/h}$
User antenna height	$1.5\,\mathrm{m}$
BS antenna height	$10\,\mathrm{m}$
SI cancelling level $[\beta]$	$[-70\ -100]$ dB
UE max power $[P^u_{\text{max}}]$	$24\,\mathrm{dBm}$
BS max power $[P^d_{\text{max}}]$	$24\,\mathrm{dBm}$
Minimum SINR $[\gamma^u_{\text{th}} = \gamma^d_{\text{th}}]$	[0 5] dB

With the characterization of the duality gap and the performance loss between the proposed greedy algorithm and optimal solution of the dual, we recall here that the proposed algorithm JAFMA is an *approximated primal* solution to problem (C.4).

C.7 Numerical Results

In this section we consider a single cell system operating in the urban micro environment assuming 2.5 GHz carrier frequency and a system bandwidth of 5 MHz [106, 115, 116]. The maximum number of frequency channels is $F = 25$ that corresponds to the number of available frequency channel blocks in the frequency domain of a 5 MHz long term evolution (LTE) system [106]. The total number of served UE varies between $I + J = 8...50$, where we assume that after UE pairing the number of UE transmitting in UL (I) is equal to the number of UE receiving in DL (J). The parameters of this system are set according to Table C.5.

To evaluate the performance of JAFMA in this environment, we use the RUdimentary Network Emulator (RUNE) as a basic platform for system simulations [117] and extended it to FD cellular networks. The RUNE FD simulation tool allows to generate the environment of Table C.5 and perform Monte Carlo simulations using either an exhaustive search algorithm to solve problem (C.4) or JAFMA.

In order to understand and quantify the gap between the initially proposed JAFM in (C.4) (P-JAFM), the dual problem converted to axial 3-DAP in (C.21) (D-JAFM) and the greedy solution named JAFMA, we analyse the relative duality gap between the three algorithms in Section C.7.1. In Sections C.7.2 and C.7.3 we compare the performance of JAFMA for different users' loads and SI cancelling levels, respectively, with three other algorithms:

1. Random assignment with EPA, named R-EPA;
2. Assignment according to JAFMA but with EPA, named AF-EPA;
3. Random assignment with power allocation according to JAFMA, named R-FMA.

Recall that other algorithms previously proposed for rate maximization, e.g. [32], are not considered here because our scenario does not suit the solution proposed by them. Although the sum rate maximization and fairness trade-off is not analysed, we aim to solve an essential problem, which is how to achieve fairness and how fair can we be using full-duplex communications. For a careful analysis of the rate-fairness trade-off in general wireless networks, see for example [126].

C.7.1 Analysis of Optimality Gap

To evaluate the optimality gap between the proposed JAFMA, P-JAFM and D-JAFM, we evaluate the objective function of problem (C.4), i.e., the minimum spectral efficiency of UL and DL users.

Recall from Section C.3.2 and Section C.6.2 that the P-JAFM and D-JAFM are NP-hard, thus we solve them by exhaustive search for scenarios with a small number of users and frequency channels. Specifically, for P-JAFM and D-JAFM, we consider a small system with reduced number of users, 4 UL and DL users, and frequency channels, where we increase its number from 4 to 8. Moreover, we consider a SI cancelling level of $-100\,\mathrm{dB}$, i.e., with $\beta = -100\,\mathrm{dB}$ and $\gamma_{\text{th}}^{u} = \gamma_{\text{th}}^{d} = 0\,\mathrm{dB}$.

Figure C.3 shows the cumulative distribution function (CDF) of minimum spectral efficiency among all UL and DL users, which is the objective function of problem (C.4), and an important performance indicator of cellular networks. At full load, when all users need to use FD transmission, the relative gap between JAFMA and P-JAFM is approximately $69\,\%$ at the 50th percentile. Similarly, the gap between D-JAFM and P-JAFM is approximately $57\,\%$. Moreover, notice that in approximately $30\,\%$ of the Monte Carlo iterations at least one user was disconnected. However, the number of resources increases, both JAFMA and D-JAFM get closer to P-JAFM: approximately $1\,\%$ for JAFMA and $0.6\,\%$ for D-JAFM at the 50th percentile.

Figure C.4 shows the CDF of the relative optimality gap for the same configuration as that used in Figure C.3, and here we highlight how close JAFMA is to the optimal solution achieved by P-JAFM. At high load the relative optimality gap is approximately $67\,\%$ for JAFMA and $50\,\%$ for D-JAFM at the 50th percentile. When the number of channels is increased to 8, in $59\,\%$ of the cases the gap is approximately zero for JAFMA and at most $14\,\%$. These figures clearly show that the optimality gap diminishes with an increasing number of resources as proved in Appendix C.9.4.

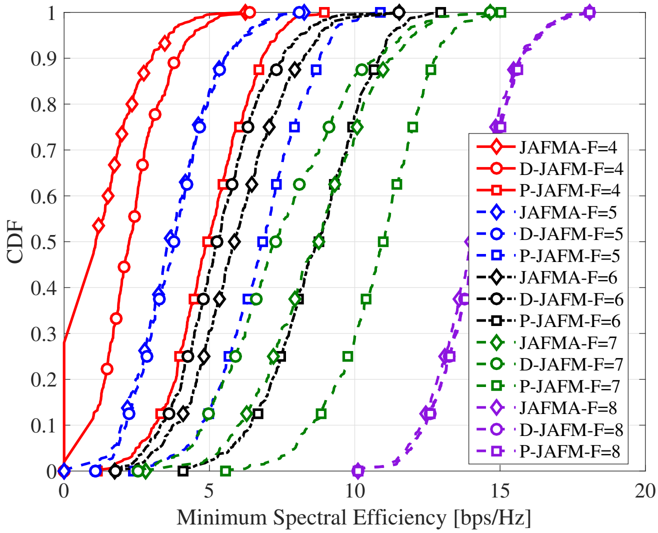


Figure C.3: *CDF of the minimum spectral efficiency among all users. We notice that as we increase the number of frequency channels in the system, the gap between JAFM and P-JAFM diminishes, where in the* 50*th percentile this relative gap is approximately* $1\,\%$.

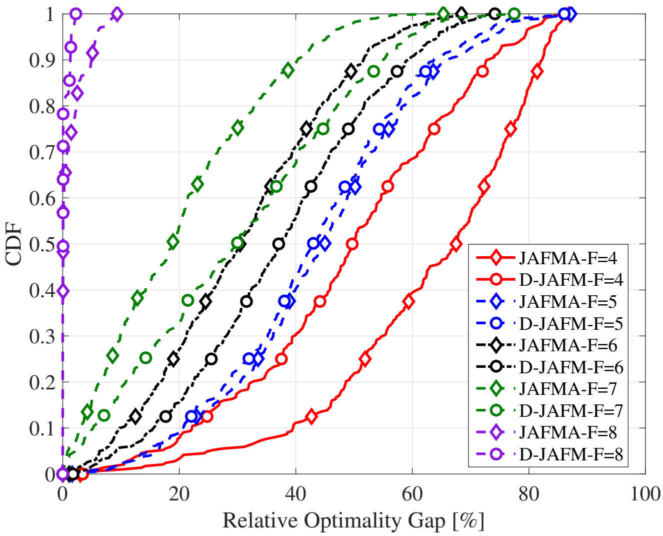


Figure C.4: *CDF of the relative optimality gap of JAFMA and D-JFMA with P-JFMA. We clearly see that the optimality gap diminishes when the number of frequency channels is increased, where in* $57\,\%$ *of the cases the gap is approximately zero for JAFMA.*

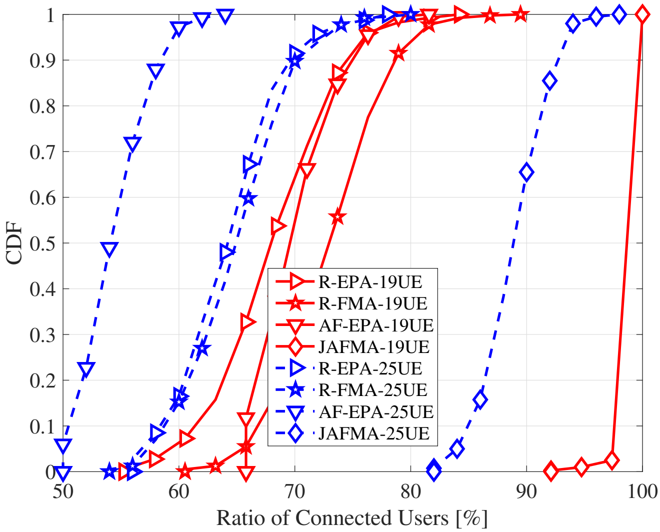


Figure C.5: *CDF of the ratio of connected users in the system for different users' load. Notice that JAFMA guarantees connection to at least* $92\,\%$ *in a system with 19 UL and DL users and* $82\,\%$ *in a system with 25 UL and DL users. Thus, JAFMA is able to maintain a high ratio of connected users although the system is completely loaded.*

C.7.2 Fairness Performance Analysis for Different Users' Loads

In this subsection we study the performance of JAFMA and compare it with R-EPA, AF-EPA and R-FMA in terms of the achieved ratio of connected users and the achieved fairness measured by the well known Jain's fairness index [91]. To characterize the fairness achieved by this system, we employ a modified versions of Jain's index that is weighed by the number of connected users [91]. In simple terms, the modified Jain's fairness index punishes the algorithms that disconnect users rather than considering the connected users only, which is given by

$$J_{mod} = \left(1 - \frac{N_d}{I+J}\right) \frac{\left(\sum_k^{I+J} C_k\right)^2}{(I+J)\sum_k^{I+J} C_k^2}, \tag{C.22}$$

where C_k is the spectral efficiency of user k (either UL or DL) and N_d is the number of disconnected users. Note that N_d represents how many users, within the ones that are scheduled to transmit, cannot communicate with the BS fulfilling a minimum SINR threshold γ_{th}.

We analyse how the fairness of the system changes with different users' load considering $\gamma_{\text{th}}^u = \gamma_{\text{th}}^d = 5\,\text{dB}$ and a SI cancelling level of $-70\,\text{dB}$, i.e., with $\beta =-70\,\text{dB}$. Recall that the SINR threshold γ_{th} also impacts the number of disconnected users. Due to this sensitivity, we consider a high minimum SINR threshold, which allows us to evaluate if full-duplex communications can guarantee such threshold.

Figure C.5 shows the CDF of the ratio of connected users, where $100\,\%$ means that all

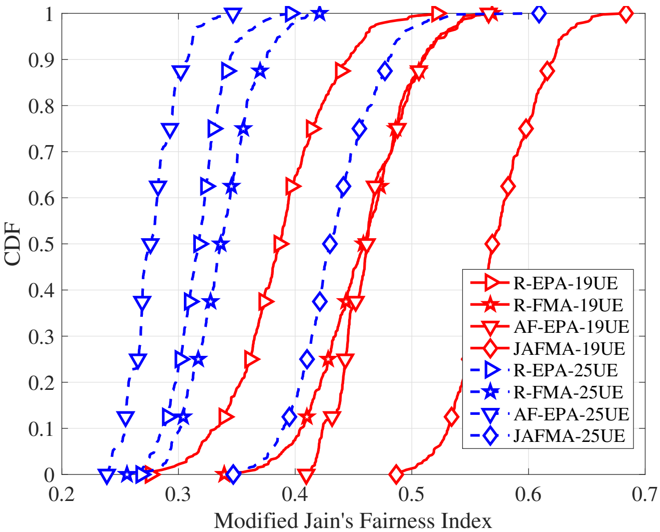


Figure C.6: *CDF of the modified Jain's fairness index among all UL and DL users for different users' load. We notice that as we increase the number of users JAFMA increases its relative difference to AF-EPA and R-FMA, with* 35 % *at the* 50*th percentile.*

users are connected and use a frequency channel. Notice that for 19 UL and DL users the percentage of connected users with JAFMA is at least 92 % (lowest ratio of connected users in the right axis), while for AF-EPA is at least 66 %, for R-FMA is at least 61 % for R-EPA is at least 55 %. Notice that R-FMA crosses AF-EPA, meaning that the power allocation for pairs randomly assigned can improve connection of the users, instead of assigning them in a proper manner and allocating maximum power. For 25 UL and DL users, the performance of all algorithms is degraded, but JAFMA still shows superior performance guaranteeing connection to at least 82 % of all users. In contrast, AF-EPA has at least 50 %, R-FMA has at least 54 % and R-EPA at least 56 % connected users. Notice that R-EPA has a higher connection ratio than AF-EPA, meaning that considering only the assignment of JAFMA should not be taken into consideration alone, but also with the power allocation. Moreover, R-FMA crosses R-EPA, but they are still close, showing that in a high load the gain of optimizing the power allocation for a pair randomly allocated is negligible. This result shows that JAFMA maintains a high ratio of connected users although the system is fully loaded and that the fairness assignments and power allocation of JAFMA should not be split and merged with other techniques.

Figure C.6 shows the CDF of modified Jain's fairness index among all UL and DL users. Notice that for 19 UL and DL users the relative difference is already clear between JAFMA, AF-EPA and R-FMA, approximately 23 % at the 50th percentile. The performance between AF-EPA and R-FMA is similar for at least 50 % of the cases (see above 50th percentile), thus regardless of how good the assignment or power allocation are, if they are taken into consideration alone they will have the same effect in the fairness.

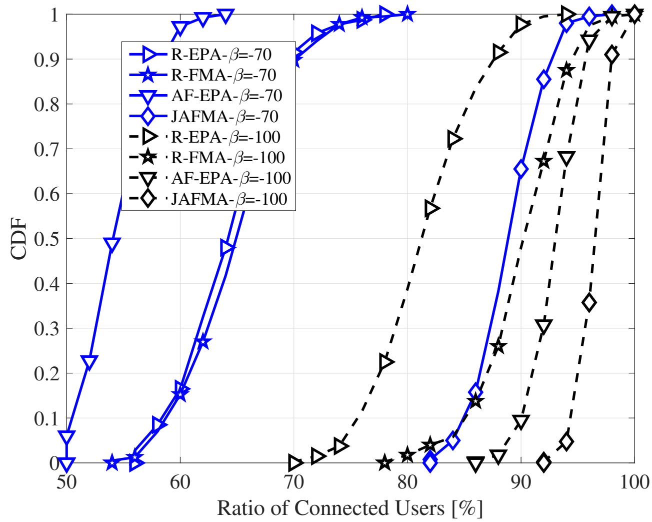


Figure C.7: *CDF of the ratio of connected users in the system for different values of β. Notice that JAFMA guarantees connection to at least $82\,\%$ in a system with high SI level, i.e., JAFMA guarantees a high connection ratio to users in system with severe SI.*

As expected, R-EPA has poor performance, not only because of the random assignment of frequency channels but also because of using EPA with maximum power, which increases the SI in the UL. As the load of the system increases to 25 UL and DL users, the relative difference between JAFMA and AF-EPA increases to $35\,\%$ at the 50th percentile. JAFMA achieves the highest modified Jain's fairness index for different users' load, meaning that not only the fairness of the system is maintained but also the ratio of connected users is improved. Again, we notice that considering only one feature of JAFMA, either fairness assignment or power allocation, does not perform much better than a simple random allocation with maximum power, thus JAFMA should be used as it is.

C.7.3 Fairness Performance Analysis for SI Cancelling Levels

It is important to understand the impact of SI on the performance of JAFMA To this end, we now assume a high user load with 25 UL and DL users and vary β as $-70\,\mathrm{dB}$ and $-100\,\mathrm{dB}$, i.e., from a system with severe SI level to a modest SI level.

Figure C.7 shows the CDF of the ratio of connected users for different SI cancelling levels, i.e., for different values of β. Notice that when β is $-100\,\mathrm{dB}$, JAFMA guarantees connection to at least $92\,\%$ of all users, while AF-EPA has at least $86\,\%$, R-FMA has at least $78\,\%$ and R-EPA guarantees at least $70\,\%$. The difference between AF-EPA and R-FMA is more clear now, which shows that with modest SI cancelling there is an improvement in optimizing only the assignment instead of only the power allocation. When β increases to $-70\,\mathrm{dB}$, all algorithms guarantee connection to fewer users than before. However, JAFMA

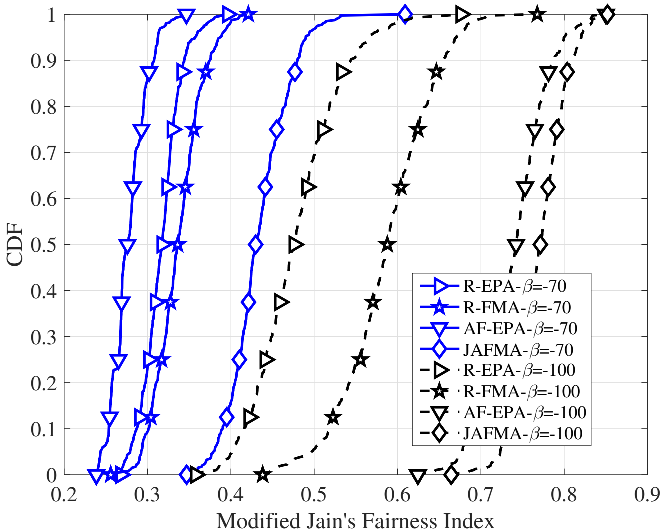


Figure C.8: *CDF of the modified Jain's index among all UL and DL users for different SI cancelling levels. We notice that as β increases, the relative difference between JAFMA and AF-EPA also increases.*

clearly outperforms the other algorithms, guaranteeing connection to at least $82\,\%$ of all users, while AF-EPA decreases to $50\,\%$, R-FMA decreases to $55\,\%$ and R-EPA to $56\,\%$. Again, we see that considering only the assignment or power allocation of JAFMA, the performance is worse or the same as using a random assignment, as the performance of AF-EPA and R-FMA show.

Figure C.8 shows the modified Jain's fairness index and we notice that for $\beta = -100\,\mathrm{dB}$ the relative difference between JAFMA and AF-EPA is approximately $4\,\%$ while between JAFMA and R-EPA is $62\,\%$ at the 50th percentile. Notice that as expected from Figure C.7, there is a relative gain of approximately $26\,\%$ of optimizing the assignment instead of the power allocation, as it is shown by the difference between AF-EPA and R-FMA at the 50th percentile. However, when we increase β to $-70\,\mathrm{dB}$, the relative difference between JAFMA and AF-EPA at the 50th percentile is approximately $56\,\%$ and higher than the relative difference between JAFMA and R-EPA, which is approximately $35\,\%$ and the relative difference between JAFMA and R-FMA (approximately $28\,\%$). Notice that as β increases, the relative difference between JAFMA with AF-EPA and R-FMA increase, where AF-EPA becomes worse than R-EPA and R-FMA becomes close to R-EPA. Therefore, JAFMA should be taken into account as a joint algorithm for different SI levels, where neither the assignment nor the power allocation should be split to create new techniques.

C.8 Conclusion

In this paper we considered the joint problem of user pairing and frequency channel selection in FD cellular networks. Specifically, our objective was to maximize the minimum spectral efficiency of the user with the lowest achieved spectral efficiency. This problem was posed as a mixed integer nonlinear optimization, called JAFM, which was shown to be NP-hard. We resorted to Lagrangian duality and developed a closed-form solution for the assignment and found the optimal power allocation. Since the closed-form solution could not be solved explicitly, we used duality theory, combined with a greedy solution algorithm, JAFMA, to approximate the primal problem. We derived the duality gap between JAFMA and the primal problem, and we showed that JAFMA has a guaranteed performance, which allowed us to denominate as a greedy approximation to the primal problem. The numerical results showed that JAFMA improved the spectral efficiency of the users with low spectral efficiency and achieved the highest ratio of connected users in a wide range of scenarios.

The results also indicated that the optimization of the assignment and power allocation should be solved jointly; otherwise, a random allocation with EPA achieves a similar performance. In future works, we intend to study distributed schemes to jointly assign UL and DL users to frequency channels, to inspect in details the rate-fairness trade-off, and to analyse the impact of multi-cell interference in the fairness of FD systems.

C.9 Appendix

C.9.1 Proof of Result 1

To check if the problem (C.12) is convex, we need to preliminary check that all the inequality constraints are convex, thus we start with constraint (C.12b). Suppose that user i in the UL is interfering with user j in the DL, where user i is transmitting only on frequency channel f, i.e., $x^u_{if} = 1$. Thus, constraint (C.12b) can be rewritten as

$$(2^t - 1)(\sigma^2 + \sum\nolimits_{j=1}^{J} x^d_{jf} P^d_j \beta) - P^u_i G_{ibf} \leq 0. \tag{C.23}$$

If the Hessian of inequality (C.23) with respect to variables t, x_{jf}, P^u_i, P^d_j is positive semi-definite, then problem (C.12) is convex. However, the second leading minor is not positive, which implies that the matrix is not positive semi-definite, i.e., constraint (C.23) is not convex. Since we proved that a simpler version of the relaxed problem is not convex, the general version cannot be convex either.

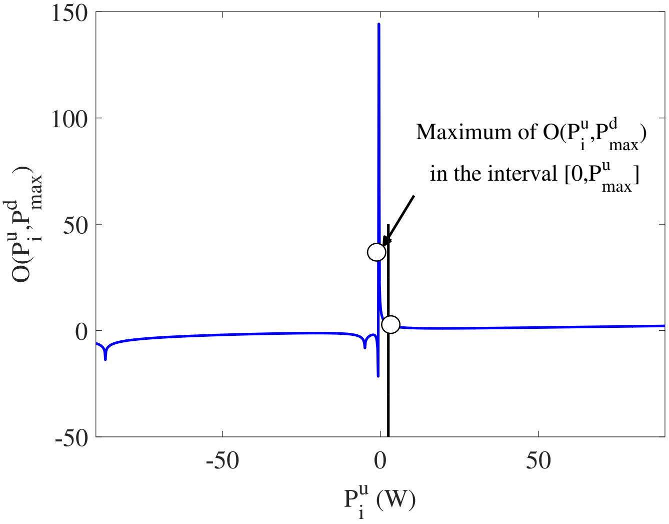


Figure C.9: *Illustrative plot of $O(P_i^u, P_{max}^d)$ that shows the possible maxima of the function and its transitions in the poles.*

C.9.2 Proof of Lemma 8

We take the first derivative of $O(P_i^u, P_{\max}^d)$ with respect to P_i^u, which is given by

$$\frac{\partial O(P_i^u, P_{\max}^d)}{\partial P_i^u} = \frac{\delta_i^u G_{ibf}}{\sigma^2 + P_{\max}^d \beta} - \frac{\delta_j^d P_{\max}^d G_{ijf} G_{bjf}}{(\sigma^2 + P_i^u G_{ijf})^2} + \frac{\lambda_i^u G_{ibf}}{\ln(2)(P_i^u G_{ibf} + P_{\max}^d \beta + \sigma^2)} - \frac{\lambda_j^d P_{\max}^d G_{ijf} G_{bjf}}{\ln(2)(P_i^u G_{ijf} + P_{\max}^d G_{bjf} + \sigma^2)(\sigma^2 + P_i^u G_{ijf})} \quad \text{(C.24)}$$

We notice from Eq. (C.24) that the derivative has three distinct poles all of which are located at negative powers, namely $P_{i,l}^u$ tends to ∞ for $l = 1, 2, 3$, since the function is continuous for positive powers, tends to infinity as P_i^u tends to infinity, and is given by the sum of a strictly decreasing function and a strictly increasing function, thus it must have a U shape for positive P_i^u. These are inflection points for the derivative. Therefore, the function must look like Figure C.9, and thus for $P_i^u > 0$ the function is convex and it is maximized in the corner points of the interval $[0, P_{\max}^u]$.

C.9.3 Proof of Lemma 6

In Figure C.2, we can find the coordinates of point K (P_{jK}, P_{iK}) by solving the system of equations

$$\frac{P_{iK}^u G_{ibf}}{\sigma^2 + P_{jK}^d \beta} = \gamma_{\text{th}}^u, \qquad \frac{P_{jK}^d G_{bjf}}{\sigma^2 + P_{iK}^u G_{ijf}} = \gamma_{\text{th}}^d, \quad \text{(C.25)}$$

which gives the following coordinates

$$P_{iK}^{u} = \frac{\gamma_{\text{th}}^{u}\sigma^2(G_{bjKf} + \beta\gamma_{\text{th}}^{d})}{G_{bjKf}G_{iKbf} - \beta G_{iKjKf}\gamma_{\text{th}}^{u}\gamma_{\text{th}}^{d}}, \tag{C.26a}$$

$$P_{jK}^{d} = \frac{\gamma_{\text{th}}^{d}\sigma^2(G_{iKjKf}\gamma_{\text{th}}^{u} + G_{iKbf})}{G_{bjKf}G_{iKbf} - \beta G_{iKjKf}\gamma_{\text{th}}^{u}\gamma_{\text{th}}^{d}}. \tag{C.26b}$$

However, the point K needs to be constrained to the boxed region defined by the power constraints, thus we get the following inequalities

$$0 < \frac{\gamma_{\text{th}}^{u}\sigma^2(G_{bjKf} + \beta\gamma_{\text{th}}^{d})}{G_{bjKf}G_{iKbf} - \beta G_{iKjKf}\gamma_{\text{th}}^{u}\gamma_{\text{th}}^{d}} \leq P_{\max}^{u}, \tag{C.27a}$$

$$0 < \frac{\gamma_{\text{th}}^{d}\sigma^2(G_{iKjKf}\gamma_{\text{th}}^{u} + G_{iKbf})}{G_{bjKf}G_{iKbf} - \beta G_{iKjKf}\gamma_{\text{th}}^{u}\gamma_{\text{th}}^{d}} \leq P_{\max}^{d}. \tag{C.27b}$$

Inspired by [102] and using inequalities (C.27), we can derive now an upper bound for the SI cancellation term for any two users i and j that share a frequency channel f and fulfil constraints (C.4b)-(C.4e), i.e., any user in the area with parallel lines defined by points above K and within the boxed area. The bound is $\beta \leq \beta_{ijf}^{\max}$, where $\beta_{ijf}^{\max}$ is given by

$$\beta_{ijf}^{\max} = \begin{cases} \beta_u \triangleq \dfrac{G_{bjf}(P_{\max}^{u}G_{ibf} - \gamma_{\text{th}}^{u}\sigma^2)}{\gamma_{\text{th}}^{u}\gamma_{\text{th}}^{d}(P_{\max}^{u}G_{ijf} + \sigma^2)}, \\ \beta_d \triangleq \dfrac{P_{\max}^{d}G_{bjf}G_{ibf} - \sigma^2\gamma_{\text{th}}^{d}(G_{ijf}\gamma_{\text{th}}^{u} + G_{ibf})}{\gamma_{\text{th}}^{u}\gamma_{\text{th}}^{d}P_{\max}^{d}G_{ijf}}. \end{cases} \tag{C.28}$$

Thus, $\beta \leq \beta_{ijf}^{\max} = \min\{\beta_u, \beta_d\}$. To understand when β_u and β_d are chosen, we see that $\beta_u > \beta_d$ when $\gamma_{jMf}^{d} \leq \gamma_{\text{th}}^{d}$, i.e., $\min\{\beta_u, \beta_d\} = \beta_d$. In the other case, $\beta_u \leq \beta_d$ when $\gamma_{jMf}^{d} > \gamma_{\text{th}}^{d}$, i.e., $\min\{\beta_u, \beta_d\} = \beta_u$. Therefore,

$$\beta_{ijf}^{\max} = \begin{cases} \beta_u, \text{if } \gamma_{jMf}^{d} > \gamma_{\text{th}}^{d}, \\ \beta_d, \text{if } \gamma_{jMf}^{d} \leq \gamma_{\text{th}}^{d}. \end{cases} \tag{C.29}$$

Therefore, a pair of UL and DL users is admissible on frequency channel f if the evaluated β_{ijf} fulfils inequality $\beta \leq \beta_{ijf}^{\max}$.

C.9.4 Proof of Proposition 1

Based on the work of Weeraddana et al. [125], the proof of the duality gap relies on three assumptions of an equivalent version of problem (C.12). Let us define it as

$$
\begin{aligned}
\text{minimize} \quad & -\sum\nolimits_{f=1}^{F} \min\{t_f^u, t_f^d\} && \text{(C.30a)}\\
\text{subject to} \quad & \sum\nolimits_{f=1}^{F} C_{if}^u \geq \sum\nolimits_{f=1}^{F} t_f^u, \forall i, && \text{(C.30b)}\\
& \sum\nolimits_{f=1}^{F} C_{jf}^d \geq \sum\nolimits_{f=1}^{F} t_f^d, \forall j && \text{(C.30c)}\\
& \text{Constraints (C.4b)-(C.4j)} && \text{(C.30d)}\\
& 0 \leq t_f^u \leq t_{\max}^u + C_{n_f^u f}^u, \forall f && \text{(C.30e)}\\
& 0 \leq t_f^d \leq t_{\max}^d + C_{n_f^d f}^d, \forall f && \text{(C.30f)}
\end{aligned}
$$

where the variables are $\mathbf{t}^{u,d}$, $\mathbf{X}^{u,d}$ and $\mathbf{p}^{u,d}$. We define $n_f^u = \arg\max_{i \in \mathcal{I}} C_{if}^u$ and $n_f^d = \arg\max_{j \in \mathcal{J}} C_{jf}^d$, with $t_{\max}^u, t_{\max}^d < \infty$ as upper bound of both optimal solution components $t_f^{u\star}$ and $t_f^{d\star}$ of problem (C.30). For example, we use $t_{\max}^u = \max_{i \in \mathcal{I}, f \in \mathcal{F}} C_{if}^u$ and $t_{\max}^d = \max_{j \in \mathcal{J}, f \in \mathcal{F}} C_{jf}^u$ throughout this article. Similarly to [125], we can show that problem (C.30) is equivalent to the original MINLP (C.12) and the optimal value $P^\star$ of (C.30) is equal to the optimal value $p^\star$ of MINLP (C.12), i.e., $P^\star = p^\star$.

From now on, we refer to problem (C.30) as the *modified MINLP* from (C.12), which is in a similar form of Proposition 4 from [125] (indices i and j have a different meaning in that work), where

1. $\mathcal{J} = \mathcal{F}$ and $J = F$;
2. $\mathbf{y}_f = (\mathbf{z}_f^u, \mathbf{z}_f^d, \mathbf{p}^u, \mathbf{p}^d, t_f^u, t_f^d) \in \mathbb{R}^y$, with $\mathbf{z}_f^u = (x_{if}^u)_{i \in \mathcal{I}}$, $\mathbf{z}_f^d = (x_{jf}^d)_{j \in \mathcal{J}}$ and $y = 2(I + J + 1)$;
3. $f_f(\mathbf{y}_f) = -\sum_{f=1}^{F} \min\{t_f^u, t_f^d\}$;
4. $\mathcal{Y}_f = \Big\{ \Big((x_{if}^u)_{i \in \mathcal{I}}, (x_{jf}^d)_{j \in \mathcal{J}}, (P_i^u)_{i \in \mathcal{I}}, (P_j^d)_{j \in \mathcal{J}}, t_f^u, t_f^d \Big)$
   $\Big| \sum_{i=1}^{I} x_{if}^u \leq 1, \sum_{j=1}^{J} x_{jf}^d \leq 1, \ \forall f, x_{if}^u, \ x_{jf}^d \in \{0, 1\},$
   $P_i^u \leq P_{\max}^u, \forall i, P_j^d \leq P_{\max}^d, \forall j, \ t_f^u \in \big[0, t_{\max}^u + C_{n_f^u f}^u\big]$
   $t_f^d \in \big[0, t_{\max}^d + C_{n_f^d f}^d\big] \Big\}$
5. $\mathbf{h}_f^1(\mathbf{y}_f) = \Big(t_f^u - C_{1f}^u, \ldots, t_f^u - C_{n_f^u f}^u, \ldots, t_f^u - C_{If}^u \Big) \in \mathbb{R}^{Q_1}$, with $Q_1 = I$,
   $\mathbf{h}_f^2(\mathbf{y}_f) = \Big(t_f^d - C_{1f}^d, \ldots, t_f^d - C_{n_f^d f}^d, \ldots, t_f^d - C_{Jf}^d \Big) \in \mathbb{R}^{Q_2}$, with $Q_2 = J$,

$$\mathbf{h}_f^3(\mathbf{y}_f) = \Big(-\gamma_{1f}^u, \dots, -\gamma_{n_f^u f}^u, \dots, -\gamma_{If}^u \Big) \in \mathbb{R}^{Q_3}, \text{ with } Q_3 = I,$$
$$\mathbf{h}_f^4(\mathbf{y}_f) = \Big(-\gamma_{1f}^d, \dots, -\gamma_{n_f^d f}^d, \dots, -\gamma_{Jf}^d \Big) \in \mathbb{R}^{Q_4}, \text{ with } Q_4 = J;$$

6. $\mathbf{b}^1 = \mathbf{b}^2 = 0$ and $\mathbf{b}^3 = \Big(-\gamma_{\text{th}}^u, \dots, -\gamma_{\text{th}}^u\Big) \in \mathbb{R}^{Q_3}, \mathbf{b}^4 = \Big(-\gamma_{\text{th}}^d, \dots, -\gamma_{\text{th}}^d\Big) \in \mathbb{R}^{Q_4}$.

Assumptions 1 and 2 from [125] can be proved without problems, but not Assumption 3. Since we have 4 different inequality constraints, we need to prove that every one holds, because we can imagine a concatenation of the inequality constraints with dimension $Q_1 + Q_2 + Q_3 + Q_4$ that fulfils Assumption 3.

Let $\tilde{\mathbf{y}}$ be any vector in $\text{conv}(\mathcal{Y}_f)$ and let us first analyse the constraint related to $\mathbf{h}_f^1(\mathbf{y}_f)$. From the definition of $\tilde{\mathbf{h}}(\tilde{\mathbf{y}})$ [125, Eq. (28)], we can express it as $\tilde{\mathbf{h}}_f^1(\tilde{\mathbf{y}}) = \sum_{k=1}^{y+1} \alpha^k \mathbf{h}_f^1(\mathbf{y}^k)$, for some $\mathbf{y}^k \in \mathcal{Y}_f$ and α^k such that $\tilde{\mathbf{y}} = \sum_{k=1}^{y+1} \alpha^k \mathbf{y}$, $\sum_{k=1}^{y+1} \alpha^k = 1$, $\alpha^k \geq 0$. For problem (C.12), we have

$$\begin{aligned}
\tilde{\mathbf{h}}_f^1(\tilde{\mathbf{y}}) &= \sum\nolimits_k \alpha^k \Big(t_f^{u,k} - C_{1f}^{u,k}, \dots, t_f^{u,k} - C_{n_f^u f}^{u,k}, \dots, t_f^{u,k} - C_{If}^{u,k} \Big), && \text{(C.31a)}\\
&= \sum\nolimits_k \Big(\alpha^k\, t_f^{u,k} - \alpha^k\, C_{1f}^{u,k}, \dots, \alpha^k\, t_f^{u,k} - \alpha^k\, C_{n_f^u f}^{u,k}, \dots, \alpha^k\, t_f^{u,k} - \alpha^k\, C_{If}^{u,k} \Big), && \text{(C.31b)}\\
&= \sum\nolimits_k \Big(\alpha^k\, t_f^{u,k} - 0, \dots, \alpha^k\, t_f^{u,k} - \alpha^k\, C_{n_f^u f}^{u,k}, \dots, \alpha^k\, t_f^{u,k} - 0 \Big), && \text{(C.31c)}\\
&\geq \sum\nolimits_k \Big(0, \dots, -\alpha^k\, C_{n_f^u f}^{u,k}, \dots, 0 \Big), && \text{(C.31d)}\\
&\geq \Big(0, \dots, -C_{n_f^u f}^{u}, \dots, 0 \Big), && \text{(C.31e)}\\
&= \mathbf{h}_f^1(\mathbf{y}), && \text{(C.31f)}
\end{aligned}$$

where $\mathbf{z}_f^u = (0, \dots, 1, \dots, 0)_{i \in \mathcal{I}}$ and $t_f^u = 0$, with the other vectors of the solution $\mathbf{y}$ defined in accordance with any feasible solution. Note that (C.31c) follows from the assumption that a feasible solution for x_{if}^u is to assign the user $n_f^u = \arg\max_{i \in \mathcal{I}} C_{if}^{u,k}$, which leaves $x_{if}^u = 0$ for $i \neq n_f^u$ and if we take the minimum value of t_f, we will have (C.31d). Moreover, since $\sum_{k=1}^{y+1} \alpha^k = 1$, $\alpha^k \geq 0$, we have that $-\alpha^k\, C_{n_f^u f}^{u,k} > -C_{n_f^u f}^{u,k}$. Thus, we have proved that $\tilde{\mathbf{h}}_f^1(\tilde{\mathbf{y}}) \geq \mathbf{h}_f^1(\mathbf{y})$, and a similar proof can be done for $\mathbf{h}_f^2$.

Now, we have that for $\mathbf{h}_f^3$

$$\begin{aligned}
\tilde{\mathbf{h}}_f^3(\tilde{\mathbf{y}}) &= \sum\nolimits_k \alpha^k \Big(-\gamma_{1f}^{u,k}, \dots, -\gamma_{n_f^u f}^{u,k}, \dots, -\gamma_{If}^{u,k} \Big), && \text{(C.32a)}\\
&= \sum\nolimits_k \Big(-\alpha^k 0, \dots, \alpha^k - \gamma_{n_f^u f}^{u,k}, \dots, -\alpha^k 0 \Big), && \text{(C.32b)}\\
&\geq \Big(0, \dots, -\gamma_{n_f^u f}^{u}, \dots, -0 \Big), && \text{(C.32c)}\\
&= \mathbf{h}_f^3(\mathbf{y}), && \text{(C.32d)}
\end{aligned}$$

and in a similar manner as before, we have proved that $\tilde{\mathbf{h}}_f^3(\tilde{\mathbf{y}}) \geq \mathbf{h}_f^3(\mathbf{y})$. The same idea can be applied to $\mathbf{h}_f^4(\mathbf{y})$ and therefore, all inequalities fulfil Assumption 3 of [125].

Therefore, since we proved that Assumptions 1-3 hold for the modified MINLP (C.30) and together with Proposition 4, we have

$$p^\star - d^\star \leq (2(I+J)+1)\Big(\min\{t^u_{\max}, t^d_{\max}\} + \rho_f\Big), \tag{C.33}$$

where the inequality follows from $\sup_f f(y_f) = 0$ and $\inf_f f(y_f) = -\min\left\{\max_{i,f} C^u_{if}, \max_{j,f} C^d_{jf}\right\}$. Inequality (C.33) ensures that the numerator of the relative duality gap is bounded above by a fixed number, due to power constraints the maximum spectral efficiency is bounded above, which is dependent of the total number of frequency channels F, and because I and J do not increase with F. Moreover, we note that when $F \to \infty$, $p^\star$ increases due to more resources available. Therefore, we conclude that the relative duality gap $(p^\star - d^\star)/p^\star$ diminishes as $F \to \infty$.

Paper D

On the Spectral Efficiency and Fairness in Full-Duplex Cellular Networks

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On the Sum Rate-Fairness Trade-Off in Full-Duplex Cellular Networks

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Abstract

To increase the spectral efficiency of wireless networks without requiring full-duplex capability of user devices, a potential solution is the recently proposed three-node full-duplex mode. To realize this potential, networks employing three-node full-duplex transmissions must deal with self-interference and user-to-user interference, which can be managed by frequency channel and power allocation techniques. Whereas previous works investigated either spectral efficient or fair mechanisms, a scheme that balances these two metrics among users is investigated in this paper. This balancing scheme is based on a new solution method of the multi-objective optimization problem to maximize the weighted sum of the per-user spectral efficiency and the minimum spectral efficiency among users. The mixed integer non-linear nature of this problem is dealt by Lagrangian duality. Based on the proposed solution approach, a low-complexity centralized algorithm is developed, which relies on large scale fading measurements that can be advantageously implemented at the base station. Numerical results indicate that the proposed algorithm increases the spectral efficiency and fairness among users without the need of weighting the spectral efficiency. An important conclusion is that managing user-to-user interference by resource assignment and power control is crucial for ensuring spectral efficient and fair operation of full-duplex networks.

D.1 Introduction

Due to recent advancements in antenna and digital baseband technologies, as well as radio-frequency/analog interference cancellation techniques, in-band full-duplex (FD) transmissions appear as a viable alternative to traditional half-duplex (HD) transmission modes [12]. The in-band FD transmission mode can almost double the spectral efficiency of conventional HD wireless transmission modes, especially in the low transmit power domain [12, 107]. However, due to the increasing demand for supporting the transmission of large data quantities in scarce spectrum scenarios [12, 127], and thanks to the continued advances in self-interference (SI) cancellation technologies, FD is being considered as a technology component beyond small cell and short range communications [10, 18].

A viable introduction of FD technology in cellular networks consists in making the base station (BS) FD-capable, while letting the user equipments (UEs) operate in HD mode. This transmission mode is termed three-node full-duplex (TNFD) [12], in which only one of the three nodes (i.e., two UEs and the cellular BS) must have FD and SI suppression capability. In a TNFD cellular network, the FD-capable BS transmits to its receiving UE, while receiving from another UE on the same frequency channel.

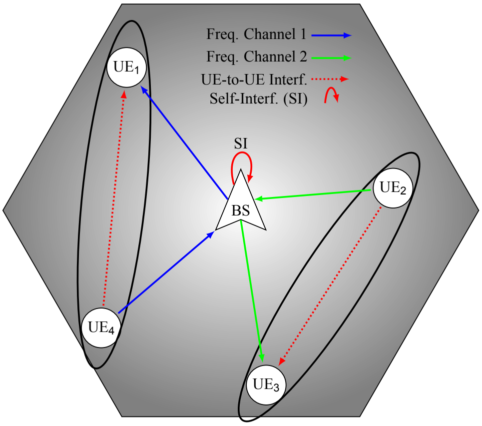

Figure D.1: *An example of cellular network employing FD with two UEs pairs. The BS selects pairs UE$_1$-UE$_4$ and UE$_2$-UE$_3$, represented by the ellipses, and jointly schedules them for FD transmission by allocating frequency channels in the UL and DL. To mitigate the UE-to-UE interference, it is advantageous to co-schedule DL/UL users for FD transmission that are far apart, such as UE$_1$-UE$_2$ and UE$_3$-UE$_4$.*

An example of a cellular network employing TNFD with two UEs pairs is illustrated in Figure D.1. Note that apart from the inherently present SI, FD operation in a cellular network must also deal with the UE-to-UE interference, indicated by the red dotted lines between UE$_1$-UE$_4$ and UE$_2$-UE$_3$. The level of UE-to-UE interference depends on the UEs locations and propagation environments and their transmission powers. To mitigate the negative effects of the interference on the spectral efficiency of the system, coordination mechanisms are needed [10]. Two key elements of such mechanisms are UE *pairing* and power allocation, that together determine which UEs are scheduled for simultaneous uplink (UL) and downlink (DL) transmissions, and at which power UL and DL UEs will transmit or receive. Consequently, it is crucial to design efficient and fair medium access control protocols and physical layer procedures capable of supporting adequate coordination mechanisms.

A typical and natural objective for many physical layer procedures for FD cellular networks proposed in the literature is to maximize the sum spectral efficiency [32, 79]. The authors in [79] consider a joint subcarrier and power allocation problem, but without taking into account the UE-to-UE interference. The work reported in [32] considers the application of TNFD transmission mode in a cognitive femto-cell scenario with bidirectional transmissions from UEs, and develops sum-rate optimal resource allocation and power control algorithms.

Another important objective is to improve the fairness and per-user quality of service (QoS) of FD cellular networks, as emphasized in [12,30,80,107]. In our previous work, we proposed a weighted sum spectral efficiency maximization, where the weights represent path-loss compensation and are thus related to the rate distribution and fairness in the system [107]. The results showed that FD cellular networks can outperform current HD mode if appropriate SI cancellation and pairing schemes are employed. A heterogeneous statistical QoS provisioning framework, focusing on the bidirectional FD link case without considering the implications of TNFD transmissions is developed in [80]. The work in [12] emphasizes the importance of fairness and that it may degrade by a factor of two compared with HD communications. However, the authors do not provide power control and channel allocation schemes that are developed with such objectives in mind. In contrast, our previous work [30] formulated the maximization of the minimum spectral efficiency problem and proposed a max-min fair power control and channel allocation solution.

However, the interplay between weighted sum spectral efficiency maximization and fairness for FD cellular networks has not been studied. Therefore, in this work we aim to fill this research gap by proposing a multi-objective optimization problem to maximize simultaneously both the weighted sum spectral efficiency and the minimum spectral efficiency of all users. Such an optimization problem poses technical challenges that are markedly different from those investigated in our previous works [30, 107]. In particular, we develop an original and new solution approach based on the use of the scalarization technique to convert the multi-objective into a single-objective mixed integer nonlinear programming (MINLP) problem that considers jointly user pairing and UL/DL power control. Due to the complexity of the MINLP problem proposed, our novel solution approach relies on Lagrangian duality and the associated centralized algorithm based on the dual problem. This centralized solution is tested in a realistic system simulator that indicates that the solution is near-optimal, and increases the sum spectral efficiency and fairness among the users. An important feature of the proposed solution is that there is no need to consider weights in the sum spectral efficiency as done in previous works from the literature [107]. This is advantageous, because defining the weights in a weighted sum objective function is typically cumbersome and difficult in practice.

The numerical results also indicate that measuring and taking into account UE-to-UE interference is crucial for both overall spectral efficiency and fairness. When UE-to-UE interference is neglected, as in [79], the results are approximately as good as using random assignment and equal power allocation among all users.

D.2 System Model and Problem Formulation

D.2.1 System Model

We consider a hexagonal single-cell cellular system in which the BS is FD capable, while the UEs served by the BS are HD capable, as illustrated by Figure D.1. In the figure, the BS is subject to SI, and the UEs in the UL (UE_2 and UE_4) cause UE-to-UE interference to co-scheduled UEs in the DL, that is to UE_3 and UE_1 respectively. The number of UEs in the UL and DL is denoted by I and J, respectively, which are constrained by the total number of frequency channels in the system F, i.e., $I \leq F$ and $J \leq F$. The sets of UL and DL users are denoted by $\mathcal{I} = \{1, \dots, I\}$ and $\mathcal{J} = \{1, \dots, J\}$, respectively.

In this paper, we assume that fading is slow and frequency flat, which is an adequate model from the perspective of power control in existing and forthcoming cellular networks [101, 104]. Let G_{ib} denote the path gain between transmitter UE i and the BS, G_{bj} denote the path gain between the BS and the receiver UE j, and G_{ij} denote the interfering path gain between the UL transmitter UE i and the DL receiver UE j.

The vector of transmit power levels in the UL by UE i is denoted by $\mathbf{p^u} = [P_1^u \dots P_I^u]$, whereas the DL transmit powers by the BS is denoted by $\mathbf{p^d} = [P_1^d \dots P_J^d]$. We define β as the SI cancellation coefficient to take into account the residual SI that leaks to the receiver. Then, the SI power at the receiver of the BS is βP_j^d when the transmit power is P_j^d.

As illustrated in Figure D.1, the UE-to-UE interference depends heavily on the geometry of the co-scheduled UL and DL users, which in turn is determined by the co-scheduling or *pairing* of UL and DL users on the available frequency channels. Therefore, UE pairing is a key function of the system. To capture the pairing of UE pairs, we define the pairing matrix, $\mathbf{X} \in \{0,1\}^{I \times J}$, such that

$$x_{ij} = \begin{cases} 1, & \text{if the UL UE}_i \text{ is paired with the DL UE}_j, \\ 0, & \text{otherwise.} \end{cases}$$

The signal-to-interference-plus-noise ratio (SINR) at the BS of transmitting user i and the SINR at the receiving user j of the BS are given by

$$\gamma_i^u = \frac{P_i^u G_{ib}}{\sigma^2 + \sum_{j=1}^{J} x_{ij} P_j^d \beta}, \; \gamma_j^d = \frac{P_j^d G_{bj}}{\sigma^2 + \sum_{i=1}^{I} x_{ij} P_i^u G_{ij}}, \tag{D.1}$$

respectively, where x_{ij} in the denominator of γ_i^u accounts for the SI at the BS, whereas x_{ij} in the denominator of γ_j^d accounts for the UE-to-UE interference caused by UE$_i$ to UE$_j$, and σ^2 is the noise power. The achievable spectral efficiency for each user is given by the Shannon equation for the UL and DL as $C_i^u = \log_2(1 + \gamma_i^u)$ and $C_j^d = \log_2(1 + \gamma_j^d)$, respectively.

In addition to the spectral efficiency, we weight the achievable spectral efficiencies by constant weights, which are denoted by α_i^u and α_j^d, respectively. The purpose of these weights is to allow the system designer to choose between the commonly used sum rate maximization and important fairness related criteria such as the well known *path loss compensation* typically employed in the power control of cellular networks [100]. The weights α_i^u and α_j^d can account for sum rate maximization by setting $\alpha_i^u = \alpha_j^d = 1$, or for path loss compensation by setting $\alpha_i^u = G_{ib}^{-1}$ and $\alpha_j^d = G_{bj}^{-1}$.

D.2.2 Problem Formulation

Our goal is to maximize both the weighted sum spectral efficiency and the minimum spectral efficiency of all users, jointly considering the assignment of UEs in the UL and DL (*pairing*). This multi-objective optimization problem can be transformed to a single-objective optimization problem through the scalarization technique [110, Sec. 4.7.4]. We choose $(1 - \mu)$ as the weight for the weighted sum spectral efficiency and μ for the minimum spectral efficiency, where $\mu \in [0, 1]$. Specifically, we formulate the problem as

$$\begin{aligned}
\underset{\mathbf{X},\mathbf{p}^u,\mathbf{p}^d}{\text{maximize}} \quad & \Big(1-\mu\Big)\Big(\sum_{i=1}^{I}\alpha_i^u C_i^u + \sum_{j=1}^{J}\alpha_j^d C_j^d\Big) + \mu \min_{\forall i,j}\{C_i^u, C_j^d\} && \text{(D.2a)}\\
\text{subject to} \quad & P_i^u \le P_{\max}^u,\ \forall\, i, && \text{(D.2b)}\\
& P_j^d \le P_{\max}^d,\ \forall\, j, && \text{(D.2c)}\\
& \sum_{i=1}^{I} x_{ij} \le 1,\ \forall\, j, && \text{(D.2d)}\\
& \sum_{j=1}^{J} x_{ij} \le 1,\ \forall\, i, && \text{(D.2e)}\\
& x_{ij} \in \{0,1\},\ \forall\, i,j. && \text{(D.2f)}
\end{aligned}$$

The optimization variables are $\mathbf{p}^u$, $\mathbf{p}^d$ and $\mathbf{X}$. Constraints (D.2b) and (D.2c) limit the transmit powers, whereas constraints (D.2d)-(D.2e) assure that only one UE in the DL can share the frequency resource with a UE in the UL and vice-versa. For the sake of clarity, we denote the solution to problem (D.2) as P-OPT.

Problem (D.2) belongs to the category of MINLP, which is known for its high complexity and computational intractability [128]. To find a near-to-optimal solution to problem (D.2) we establish an original approach using the dual problem, as described in Section D.3. However, the complexity of the dual problem solution might be prohibitive in practical cellular systems, which motivates the reformulation of the dual problem in Section D.4, whose proposed solution in Algorithm 8 is denoted C-HUN.

D.3 Solution Approach Based on Lagrangian Duality

D.3.1 Problem Transformation

As a first step of solving problem (D.2), we consider the standard equivalent hypograph [110, Sec. 3.1.7] form of problem (D.2), where the new variable t and two more constraints are introduced. Note that the hypograph simplifies the problem formulation, because it allows to use a linear function of the variable t instead of the minimum between two nonlinear functions with the other variables.

$$\begin{aligned}
\underset{\mathbf{X},\mathbf{p}^u,\mathbf{p}^d,t}{\text{maximize}} \quad & \Big(1-\mu\Big)\Big(\textstyle\sum_{i=1}^{I}\alpha_i^u C_i^u + \sum_{j=1}^{J}\alpha_j^d C_j^d\Big) + \mu t\\
\text{subject to} \quad & C_i^u \ge t, \forall i, && \text{(D.3a)}\\
& C_j^d \ge t, \forall j && \text{(D.3b)}\\
& \text{Constraints (D.2b)-(D.2f)},
\end{aligned}$$

where $t > 0$ is an additional variable with respect to (D.2). Notice that problem (D.3), similarly to problem (D.2), is a MINLP.

D.3.2 Solution for X and $\mathbf{p}^u, \mathbf{p}^d$

From problem (D.3), we form the *partial* Lagrangian function by taking into account constraints (D.3a)-(D.3b) and ignoring the integer (D.2d)-(D.2f) and power allocation constraints (D.2b)-(D.2c). To account for these constraints, it is assumed that $\mathbf{X} \in \mathcal{X}$ and $\mathbf{p}^u, \mathbf{p}^d \in \mathcal{P}$, where $\mathcal{X}$ and $\mathcal{P}$ are sets in which the assignment constraints and power allocation constraints are fulfilled, respectively. The Lagrange multipliers associated with problem (D.3) are $\boldsymbol{\lambda}^u,\ \boldsymbol{\lambda}^d$, where the superscripts u and d denote UL and DL, and the vectors have dimensions of $I \times 1$ and $J \times 1$, respectively.

The partial Lagrangian is a function of the Lagrange multipliers and the optimization variables $\mathbf{X}, \mathbf{p}^u, \mathbf{p}^d$ as follows:

$$L(\boldsymbol{\lambda}^u, \boldsymbol{\lambda}^d, \mathbf{X}, \mathbf{p}^u, \mathbf{p}^d) \triangleq -\Big(1-\mu\Big)\Big(\sum_{i=1}^{I} \alpha_i^u C_i^u + \sum_{j=1}^{J} \alpha_j^d C_j^d\Big) - \mu t + \sum\nolimits_{i=1}^{I} \lambda_i^u \Big(t - C_i^u\Big) + \sum\nolimits_{j=1}^{J} \lambda_j^d \Big(t - C_j^d\Big). \tag{D.4}$$

It is useful to rewrite the partial Lagrangian function as

$$L(\boldsymbol{\lambda}^u, \boldsymbol{\lambda}^d, \mathbf{X}, \mathbf{p}^u, \mathbf{p}^d) = t\Big(\sum\nolimits_{i=1}^{I} \lambda_i^u + \sum\nolimits_{j=1}^{J} \lambda_j^d - \mu\Big) - \sum_{i=1}^{I} \Big(\lambda_i^u + (1-\mu)\alpha_i^u\Big) C_i^u - \sum_{j=1}^{J} \Big(\lambda_j^d + (1-\mu)\alpha_j^d\Big) C_j^d. \tag{D.5}$$

Let $g(\boldsymbol{\lambda}^u, \boldsymbol{\lambda}^d)$ denote the dual function obtained by minimizing the partial Lagrangian function (D.5) with respect to the variables $\mathbf{X}, \mathbf{p}^u, \mathbf{p}^d$. Thus,

$$g(\boldsymbol{\lambda}^u, \boldsymbol{\lambda}^d) = \inf_{\mathbf{X}\in\mathcal{X}\ \mathbf{p}^u,\mathbf{p}^d\in\mathcal{P}} L(\boldsymbol{\lambda}^u, \boldsymbol{\lambda}^d, \mathbf{X}, \mathbf{p}^u, \mathbf{p}^d) \tag{D.6a}$$

$$g(\boldsymbol{\lambda}^u, \boldsymbol{\lambda}^d) = \begin{cases} \inf\limits_{\mathbf{X}\in\mathcal{X}\ \mathbf{p}^u,\mathbf{p}^d\in\mathcal{P}} \Big[\sum_i q_i^u(\mathbf{X}, \mathbf{p}^u, \mathbf{p}^d) + \\ \sum_j q_j^d(\mathbf{X}, \mathbf{p}^u, \mathbf{p}^d)\Big], & \text{if } \sum\limits_i \lambda_i^u + \sum\limits_j \lambda_j^d = \mu \\ -\infty, & \text{otherwise,} \end{cases} \tag{D.6b}$$

where it follows from equality (D.6b) that the linear function $t\big(\sum_{i=1}^{I} \lambda_i^u + \sum_{j=1}^{J} \lambda_j^d - \mu\big)$ is lower bounded when it is identically zero, and

$$q_i^u(\mathbf{X}, \mathbf{p}^u, \mathbf{p}^d) \triangleq -\Big(\lambda_i^u + (1-\mu)\alpha_i^u\Big) C_i^u, \tag{D.7a}$$

$$q_j^d(\mathbf{X}, \mathbf{p}^u, \mathbf{p}^d) \triangleq -\Big(\lambda_j^d + (1-\mu)\alpha_j^d\Big) C_j^d. \tag{D.7b}$$

The infimum of the dual function (D.6b) is obtained when the SINR of the UL-DL pairs is maximized. We can therefore write an initial solution for the assignment x_{ij} as follows:

$$x_{ij}^{\star} = \begin{cases} 1, & \text{if } (i,j) = \arg\max\limits_{i,j} \Big(q_i^u + q_j^d\Big) \\ 0, & \text{otherwise,} \end{cases} \tag{D.8}$$

where, for simplicity, we denote an ordinary pair of UL-DL users as (i, j). With the assignment solution given by (D.8) and recalling that $x_{ij} \in \mathcal{X}$, an UL user can be uniquely associated with a DL user. However, $x^{\star}_{ij}$ is still tied through the SINRs in the UL and DL, γ^u_i and γ^d_j, respectively. With this, the solution for the assignment is still complex and – through q^u_i and q^d_j – is intertwined with the optimal power allocation.

Recall that from Eq. (D.6b) we must find the infimum of the sum between terms in Eqs. (D.7). Thus, with the initial solution for the assignment problem of finding $\mathbf{X}$, we can now evaluate the power allocation assuming that the pairs (i, j) are already formed. The power allocation problem is formulated as follows:

$$\underset{\mathbf{p}^u, \mathbf{p}^d}{\text{maximize}} \sum\nolimits_{i=1}^{I} \left(\lambda^u_i + (1-\mu)\alpha^u_i\right) C^u_i + \sum\nolimits_{j=1}^{J} \left(\lambda^d_j + (1-\mu)\alpha^d_j\right) C^d_j \tag{D.9a}$$

$$\text{subject to} \quad \mathbf{p}^u, \mathbf{p}^d \in \mathcal{P}, \tag{D.9b}$$

where the minimization of negative sums is converted to the maximization of positive sums. From the results of [101], it follows that the optimal transmit power allocation will have either P^u_i or P^d_j equal to P^u_{max} or P^d_{max}, given that i and j share a frequency channel and form a pair. Therefore, the optimal power allocation is found within the corner points of (P^u_i, P^d_j): $(0, P^d_{\text{max}})$, $(P^u_{\text{max}}, 0)$ or $(P^u_{\text{max}}, P^d_{\text{max}})$. If a user (either in the UL or DL) is not sharing the resource, i.e., assigned to a frequency channel alone, then its transmit power is simply P^u_{max} or P^d_{max}. With the assignment and power allocation solutions, we now need to find the optimal Lagrange multipliers $\boldsymbol{\lambda}^u$ and $\boldsymbol{\lambda}^d$.

D.3.3 Dual Problem Solution

We need to find the Lagrangian multipliers λ^u_i and λ^d_j, which also appear in the objective function of problem (D.9). Given the optimal power allocation problem (D.9), the dual function can be written as

$$g(\boldsymbol{\lambda}^u, \boldsymbol{\lambda}^d) = -\sum_{i=1}^{I} \lambda^u_i C^u_i - \sum_{j=1}^{J} \lambda^d_j C^d_j - h(C^u_i, C^d_j), \tag{D.10}$$

where the term $h(C^u_i, C^d_j)$ is the weighted sum of UL and DL spectral efficiencies, which are independent of λ^u_i and λ^d_j, and do not impact the dual problem. Therefore, the dual problem of (D.9) can be formulated as

$$\underset{\boldsymbol{\lambda}^u,\, \boldsymbol{\lambda}^u}{\text{minimize}} \quad \sum\nolimits_{i=1}^{I} \lambda^u_i C^u_i + \sum\nolimits_{j=1}^{J} \lambda^d_j C^d_j \tag{D.11a}$$

$$\text{subject to} \quad \sum\nolimits_{i=1}^{I} \lambda^u_i + \sum\nolimits_{j=1}^{J} \lambda^d_j = \mu, \tag{D.11b}$$

$$\lambda^u_i,\ \lambda^d_j \geq 0, \forall i, j, \tag{D.11c}$$

where the maximization of negative sums is converted to the minimization of positive sums. The dual problem (D.11) is a Linear Programming (LP) problem in the variables λ^u_i and λ^d_j.

It is convenient to rewrite the dual problem (D.11) in the standard LP form as [129, Sec. 4.2]:

$$\underset{\boldsymbol{\lambda}}{\text{minimize}} \quad \mathbf{c}^T\boldsymbol{\lambda} \tag{D.12a}$$

$$\text{subject to} \quad \mathbf{a}\boldsymbol{\lambda} = b, \tag{D.12b}$$

$$\boldsymbol{\lambda} \geq 0, \tag{D.12c}$$

where $\mathbf{c} = [C_1^u \ \ldots \ C_I^u \ C_1^d \ \ldots \ C_J^d]^T$, the variable vector is $\boldsymbol{\lambda} = [\lambda_1^u \ \ldots \ \lambda_I^u \ \lambda_1^d \ \ldots \ \lambda_j^d]^T$, the constraint vector $\mathbf{a} = \mathbf{1}^T$, and $b = \mu$. Since $\mathbf{a}$ has rank 1, we can separate the components of $\boldsymbol{\lambda}$ into two subvectors [129, Sec. 4.3], one consisting of $(I+J-1)$ nonbasic variables λ_N (all of which are zero), and another consisting of 1 basic variable λ_B, which is equal to b. Therefore, we have a single nonzero λ_B, either in the UL or DL, whose index B corresponds to the user with minimum spectral efficiency.

Therefore, using the results on the solution to the assignment problem (D.8), the optimal power allocation problem (D.9) from Section D.3.2, the dual problem (D.11) can be solved by checking exhaustively which pair of UL and DL users jointly solve Eq. (D.8), where the power allocation for each pair is within the corner points of set $\mathcal{P}$. Nevertheless, for a large number of users, this exhaustive search solution might not be practical due to the large number of iterations. Because of this property, we will reformulate the dual problem and propose a centralized solution in Section D.4.

D.4 Centralized Solution based on the Lagrangian Dual Problem

D.4.1 Insights from the Dual Problem

In Section D.3.3, we showed that the dual problem (D.11) maximizes the user with minimum spectral efficiency in the system. To this end, we can initially set one λ equal to μ and exhaustively check which one maximizes the power allocation problem (D.9). However, such exhaustive solution demands large number of iterations that depend on the number of simultaneously served UL and DL users. Consequently, such solution is not viable in practical systems.

Notice that the minimum spectral efficiency that a user can achieve is 0, because of the binary power control solution in problem (D.9). Therefore, whenever one user in the pair is not transmitting (has zero power), the λ associated with that user will be nonzero, which leads to the non-uniqueness of λ. To reduce the complexity on the search of the nonzero λ, we assume that for each pair there is a λ which equals μ, whose index corresponds to the user with the minimum spectral efficiency of that pair.

D.4.2 Centralized Solution to Reformulated Dual Problem

Based on the reasoning on the non-uniqueness of λ above and using the results from Section D.3, we reformulate the dual problem (D.11) to solve the assignment in Eq. (D.8). We propose a solution that aims at jointly maximizing the sum of the minimum spectral efficiency of the UL-DL pairs and the sum spectral efficiency. To this end, we rewrite the

Algorithm 8 Centralized Algorithm at the BS

1: Input: $\alpha_i^u,\ \alpha_i^u,\ G_{ibf},\ G_{bjf},\ G_{ijf},\ \beta,\ P_{\max}^u,\ P_{\max}^d$
2: **for** $i = 1$ **to** I **do**
3:     **for** $j = 1$ **to** J **do**
4:         Evaluate the corner point (P_i^u, P_j^d) that maximizes $s_{ij} = (1-\mu)(\alpha_i^u C_i^u + \alpha_j^d C_j^d) + \mu \min\{C_i^u, C_j^d\}$
5:     **end for**
6: **end for**
7: With s_{ij}, evaluate the optimal assignment using Hungarian algorithm
8: **Output**: $\mathbf{X}, \mathbf{p}^u, \mathbf{p}^d$

solution in Eq. (D.8) as an assignment problem given by

$$\underset{\mathbf{X}}{\text{maximize}} \quad \sum_{i=1}^{I} \sum_{j=1}^{J} s_{ij} x_{ij} \tag{D.13a}$$

$$\text{subject to} \quad \sum_{i=1}^{I} x_{ij} = 1,\ \forall j, \tag{D.13b}$$

$$\sum_{j=1}^{J} x_{ij} = 1,\ \forall i, \tag{D.13c}$$

$$x_{ij} \in \{0, 1\},\ \forall i, j, \tag{D.13d}$$

where the matrix $\mathbf{S} = [s_{ijf}] \in \mathbb{R}^{I \times J}$ can be understood as the benefit of pairing UL user i with DL user j. It is given by $s_{ij} = (1-\mu)(\alpha_i^u C_i^u + \alpha_j^d C_j^d) + \mu \min\{C_i^u, C_j^d\}$ for a pair (i, j) assigned to the same frequency. Constraint (D.13b) ensures that the DL users are associated with exactly one UL user. Similarly, constraint (D.13c) ensures that each UL user must be associated with a DL user.

Computing the optimal assignment as given by problem (D.13) requires checking $(I + J)!$ assignments [103, Section 1]. Alternatively, the Hungarian algorithm can be used in a fully centralized manner [103, Section 3.2], which has worst-case complexity of $O\Big((I + J)^3\Big)$. Algorithm 8 summarizes the steps to solve problem (D.13) using the Hungarian algorithm. The inputs to Algorithm 8 are all the path gain between UL users, DL users and the BS. The BS runs Algorithm 8 and acquires or estimates the channel gains, which are measured and feedback by the served UEs using signalling mechanisms standardized by 3rd Generation Partnership Project (3GPP) [104]. The most challenging measure to obtain is the UE-to-UE interference path gain for the pair (i, j), but due to recent advances in 3GPP for device-to-device communications, this measurement can be obtained by sidelink transmissions and receptions [130].

Once all inputs are available, the optimal power allocation for all possible pairs needs to be evaluated (see line 4). Algorithm 8 evaluates which corner point the pair (i, j) belongs to, and stores s_{ij} for later use. With s_{ij} at hand, the assignment problem (D.13) can be solved by using the Hungarian algorithm [103, Section 3.2] (see line 7). The outputs of the algorithm (see line 8) are the assignment matrix $\mathbf{X}$, and the optimal power allocation vectors $\mathbf{p}^u$, $\mathbf{p}^d$. The complexity of Algorithm 8 hinges on creating the matrix $\mathbf{S}$, which, recall, has a complexity $O(3IJ)$, and on the Hungarian algorithm, which has worst-case complexity of $O\Big((I + J)^3\Big)$.

Table D.6: *Simulation parameters*

Parameter	Value
Cell radius	$100\,\mathrm{m}$
Number of UL UEs $[I = J]$	$[4\ 25]$
Monte Carlo iterations	400
Carrier frequency	$2.5\,\mathrm{GHz}$
System bandwidth	$5\,\mathrm{MHz}$
Number of freq. channels $[F]$	$[4\ 25]$
LOS path-loss model	$34.96 + 22.7\log_{10}(d)$
NLOS path-loss model	$33.36 + 38.35\log_{10}(d)$
Shadowing st. dev. LOS and NLOS	$3\,\mathrm{dB}$ and $4\,\mathrm{dB}$
Thermal noise power $[\sigma^2]$	$-116.4\,\mathrm{dBm}$/channel
SI cancelling level $[\beta]$	$-100\ \mathrm{dB}$
Max power $[P^u_{\text{max}}] = [P^d_{\text{max}}]$	$24\,\mathrm{dBm}$

D.5 Numerical Results and Discussion

In this section we consider a single cell system operating in the urban micro environment [106]. The maximum number of frequency channels is $F = 25$ that corresponds to the number of available frequency channel blocks in a 5 MHz long term evolution (LTE) system [106]. The total number of served UE are $I + J = 8$ and $I + J = 50$, where we assume that $I = J$. We set the weights α_i^u and α_j^d based on either sum rate maximization (SR), or path loss compensation rule (PL). For SR, we set $\alpha_i^u = \alpha_j^d = 1$, whereas for PL we set $\alpha_i^u = G_{ib}^{-1}$ and $\alpha_j^d = G_{bj}^{-1}$. The parameters of the simulations are set according to Table D.6.

To evaluate the performance of the proposed centralized solution in Algorithm 8, we use the RUdimentary Network Emulator (RUNE) as a basic platform for system simulations and extended it to FD cellular networks [117]. The RUNE FD simulation tool allows to generate the environment of Table D.6 and to perform Monte Carlo simulations using either an exhaustive search algorithm to solve problem (D.2) or the centralized Hungarian solution.

Initially, we compare the optimality gap between the exhaustive search solution of problem (D.2), named P-OPT, and our proposed solution using Algorithm 8 with the optimal power allocation and the centralized Hungarian algorithm for the assignment, named C-HUN. In the following, we compare our proposed centralized solution with a basic FD solution with random assignment and equal power allocation for UL and DL users, named herein as R-EPA. In addition, we also consider a modified version of C-HUN that does not take into account UE-to-UE interference, named C-NINT. The motivation for C-NINT is to analyse how important the consideration of UE-to-UE interference is to the fairness and the sum spectral efficiency of the system.

Figure D.2 shows the objective function in Eq. (D.2a) between P-OPT and C-HUN as a measure of the optimality gap. We assume a small system with reduced number of users, 4 UL and DL users, and frequency channels, where we assume $\mu = [0.1\ 0.5\ 0.9]$ and with $\alpha_i^u = \alpha_j^d = 1$ to represent SR. Moreover, we consider a SI cancelling level of $\beta = -100\,\mathrm{dB}$. Notice that the difference between the P-OPT and C-HUN decreases when μ increases. For instance, for $\mu = 0.1$ the relative difference between P-OPT and C-HUN is approximately $33\,\%$, whereas for $\mu = 0.9$ this difference decreases to $26\,\%$. In addition, the value of the

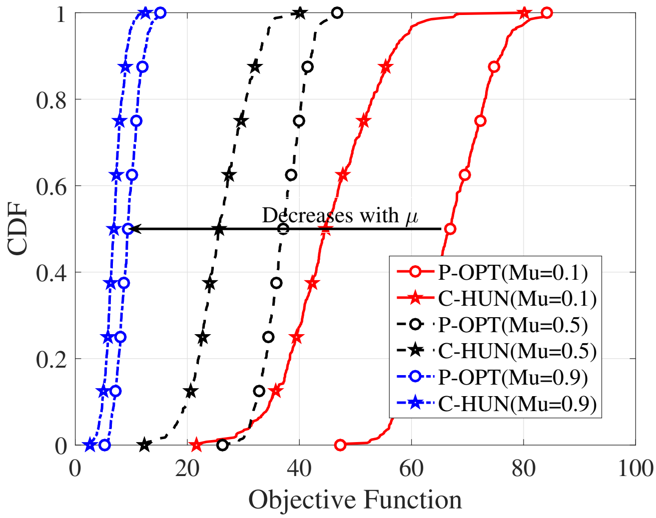

Figure D.2: *CDF of the objective function in Eq.* (D.2a) *for different values of μ. Notice that the optimality gap between P-OPT and C-HUN decreases with μ. Moreover, the objective function decreases with μ, which is expected because of the reduction of the term with sum spectral efficiency.*

objective function also decreases with μ because the term with the sum spectral efficiency also decreases.

Figure D.3 shows Jain's fairness index for the proposed solution C-HUN, the modified solution C-NINT that does not consider UE-to-UE interference, and the basic benchmark solution R-EPA. We assume a system fully loaded with 25 UL, DL users, and frequency channels, where we analyse the impact of the solutions for different weights of α_i^u and α_j^d, which are denoted SR for sum rate maximization, and PL for path-loss compensation. The value of μ is 0.9, which implies that we aim at a more fair scenario. The SI cancelling level is $-100\,\text{dB}$, i.e., $\beta = -100\,\text{dB}$. We notice that the difference between SR and PL for the proposed solution C-HUN is negligible, which implies that we can achieve similar levels of fairness without using weights on α_i^u and α_j^d. Conversely, there is a gain of approximately $16\,\%$ between C-HUN and C-NINT at the 50-th percentile irrespectively of the weights on α_i^u and α_j^d. In addition, there is practically no difference between C-NINT and R-EPA, which implies that using advanced solutions for pairing and power allocation without considering UE-to-UE interference bring losses to the system, and is as good as doing everything randomly and setting maximum power to all users. Thus, our proposed solution C-HUN is able to improve fairness in the system by approximately $16\,\%$ in comparison with the benchmark solution R-EPA.

Figure D.4 shows the sum spectral efficiency of the system for C-HUN, C-NINT, and R-EPA, where we assume the same parameters as the ones used for Figure D.3. As before, the difference between SR and PL for the proposed solution C-HUN is negligible, implying that also for the sum spectral efficiency there is practically no difference between using

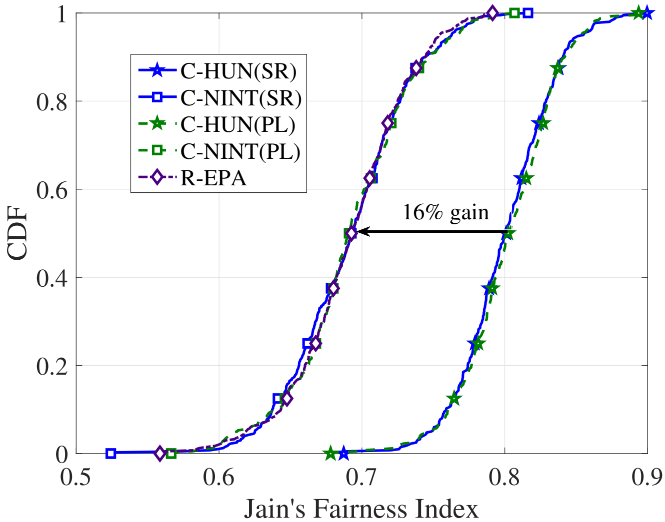

Figure D.3: *CDF of Jain's fairness index for $\mu = 0.9$ and different different weights of α_i^u and α_j^d. Notice that C-HUN achieves similar performance for SR and PL, which implies that for high values of μ SR is enough to achieve high fairness in the system. Also, C-NINT is as good as a R-EPA, but with higher complexity.*

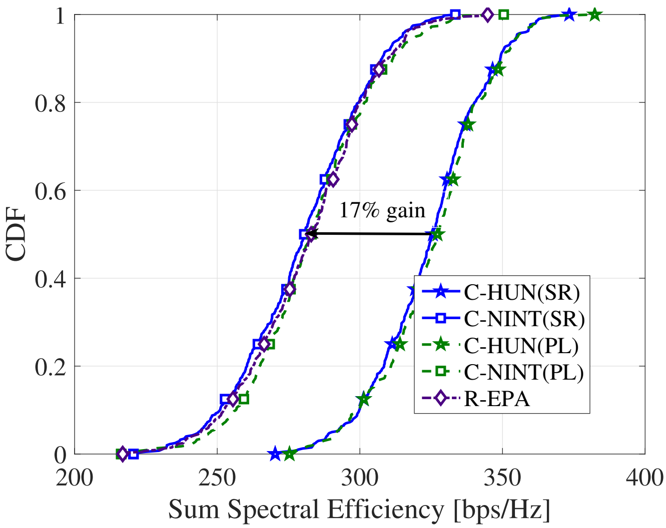

Figure D.4: *CDF of the sum spectral efficiency for all users. We notice that C-HUN is also able to improve the sum spectral efficiency with respect to C-NINT and R-EPA. Moreover, C-HUN with SR has practically the same performance as PL, implying that the weights on α_i^u and α_j^d are not necessary for high values of μ.*

weights on α_i^u and α_j^d. As noted earlier, there is a gain of approximately $17\,\%$ between C-HUN and C-NINT at the 50-th percentile for SR weights on α_i^u and α_j^d. Also, note that C-NINT for SR is slightly outperformed by R-EPA, and for PL, C-NINT is as good as R-EPA. This clearly shows that UE-to-UE interference needs to be taken into account if the system wants to maximize sum spectral efficiency. Therefore, C-HUN improves the sum spectral efficiency of the system by approximately $17\,\%$ in comparison with the benchmark solution R-EPA. Overall, we notice that when μ is high, there is no need to use weights on α_i^u and α_j^d to improve the sum spectral efficiency or/and fairness of the system. In addition, the UE-to-UE interference needs to be taken into account if the system also wants to improve the sum spectral efficiency or/and fairness. Regardless of how the assignment and power allocation are performed, if UE-to-UE interference is not taken into account, the results are approximately as good as using random assignment and equal power allocation among all users.

D.6 Conclusion

In this paper we investigated the multi-objective problem of balancing sum spectral efficiency and fairness among users in FD cellular networks. Specifically, we scalarized the problem to maximize the weighted sum spectral efficiency and the minimum spectral efficiency of the users, where now we can tune the weights to move towards sum spectral efficiency maximization or fairness. This problem was posed as a mixed integer nonlinear optimization, and given its high complexity, we resorted to Lagrangian duality. However, the solution of the dual problem was still prohibitive for networks with large number of users. Thus, we used the observations and results of the dual problem to propose a low-complexity centralized solution that can be implemented at the cellular base station. The numerical results showed that our centralized solution improved the sum spectral efficiency and fairness regardless of the weights on the sum spectral efficiencies of UL and DL users. Furthermore, the UE-to-UE interference needs to be taken into account, because otherwise irrespectively of how the assignment and power allocation are performed, the performance in terms of sum spectral efficiency and fairness will be close to a random assignment and equal power allocation among users.

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