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Fictitious FEM for the Wave Equation

Simon Sticko

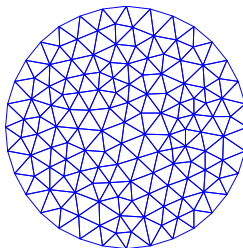
Uppsala University
Department of Information Technology
Division of Scientific Computing

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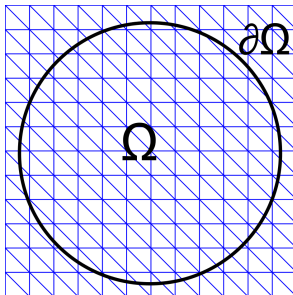
Problem



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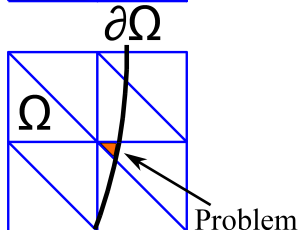
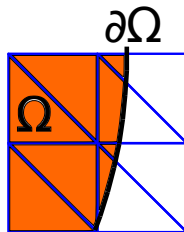
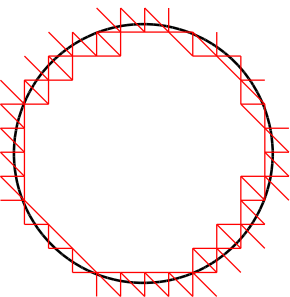
- ▶ Fictitious: Domain not aligned with mesh.
- ▶ Why?
- ▶ Wave equation:
 $u_{tt} = \nabla^2 u, u|_{\Omega} = 0$



How?



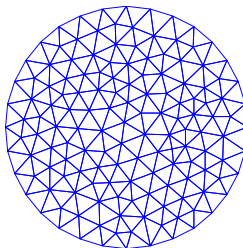
- ▶ $u_h = \sum_j \xi_j \phi_j \quad \Rightarrow \text{system: } M \ddot{\xi} = A \xi$
 $A_{ij} = \int_{\Omega} \nabla \phi_i \cdot \nabla \phi_j dA \quad M_{ij} = \int_{\Omega} \phi_i \phi_j dA$
- ▶ A, M : very ill-conditioned
- ▶ Stabilizing term J acting on edges in cut-zone, [Burman & Hansbo, 2012].
- ▶ $A + J, M + h^2 J$: well-conditioned!
- ▶ Solve: $(M + h^2 J) \ddot{\xi} = (A + J) \xi$



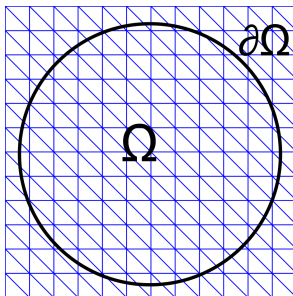
Boundary conditions



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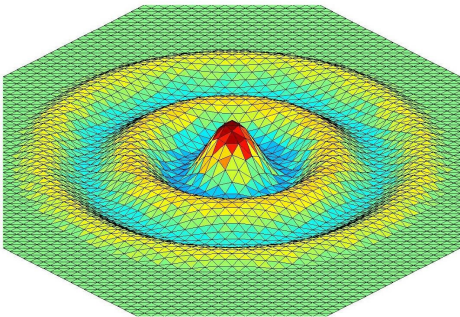


- ▶ Nitsche's Method
- ▶ B.C. through penalty term:
- ▶ line integral along $\partial\Omega$

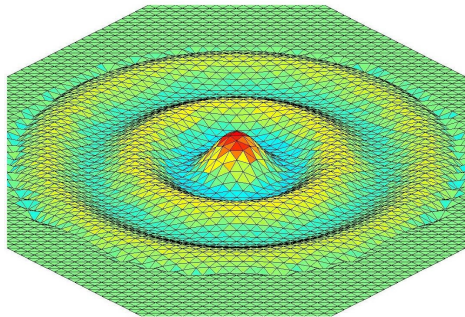


Membrane Test Case:

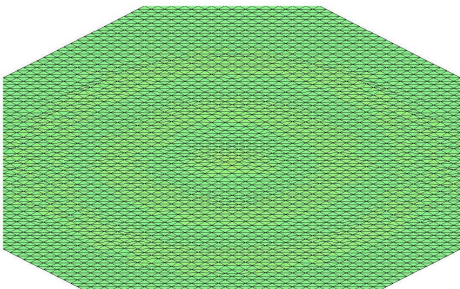
$t=0$



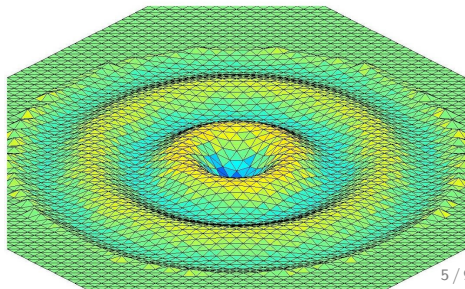
$t=0.05$



$t=0.1$



$t=0.15$

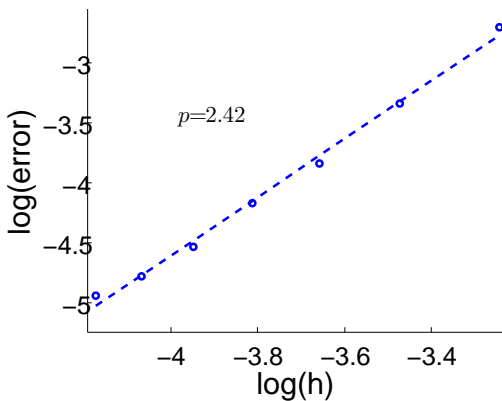


L_2 -Convergence



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- ▶ $\|u_h - u\|_{L_2} = Ch^p$
- ▶ $p = 2.4?$



Summary



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- ▶ Wave equation
- ▶ Unfitted Mesh
- ▶ Stabilizing term
- ▶ Weak boundary condition through Nitsche



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Thank you!