CPU and GPU performance of large scale numerical simulations in Geophysics

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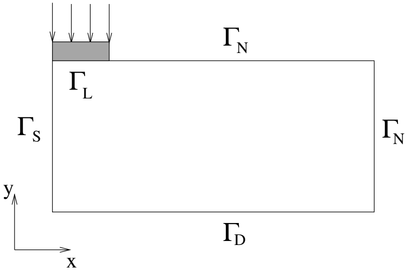
Simplified model - geometry 2D



$$\begin{cases} -\nabla \cdot (2\mu\varepsilon(\mathbf{u})) - \nabla(\mathbf{u} \cdot \nabla p_0) + (\nabla \cdot \mathbf{u})\nabla p_0 - \mu \nabla p = \mathbf{f} \\ \mu \nabla \cdot \mathbf{u} - \frac{\mu^2}{\lambda} p = 0 \end{cases}$$

$$\mathcal{A} \begin{bmatrix} \mathbf{u_h} \\ p_h \end{bmatrix} \equiv \begin{bmatrix} M & B^T \\ B & -C \end{bmatrix} \begin{bmatrix} \mathbf{u_h} \\ p_h \end{bmatrix} = \begin{bmatrix} \mathbf{l_h} \\ \mathbf{s_h} \end{bmatrix}, \qquad \mathcal{D} = \begin{bmatrix} [M] & 0 \\ B & [\hat{S}] \end{bmatrix}$$

What is taking the time

For each action of the preconditioner:

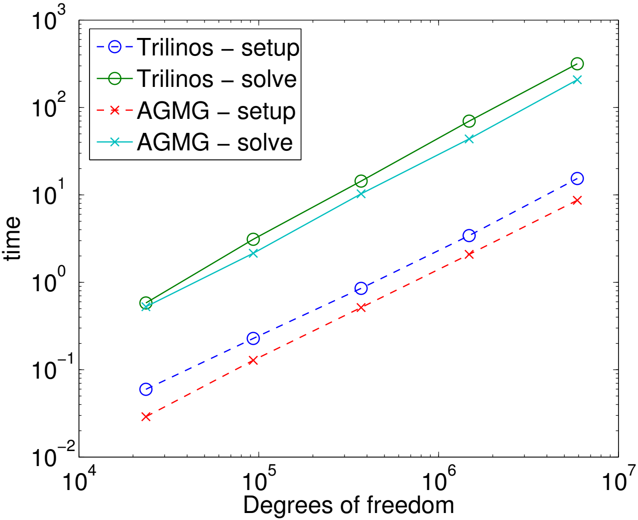
1. Solve $Mw_1 = v_1$
2. Compute $r = Bw_1 - v_2$
3. Solve $\hat{S}w_2 = r$

DOF	Solve M	Solve $\widetilde{S}$	Total solve time
23 603	0.625 (90%)	0.0286 (10%)	0.687
93 283	3.49 (93%)	0.0983 (7%)	3.72
370 883	15.3 (93%)	0.441 (7%)	16.3
1 479 043	77.2 (94%)	2.22 (6%)	81.8
5 907 203	350 (94%)	10.3 (6%)	370

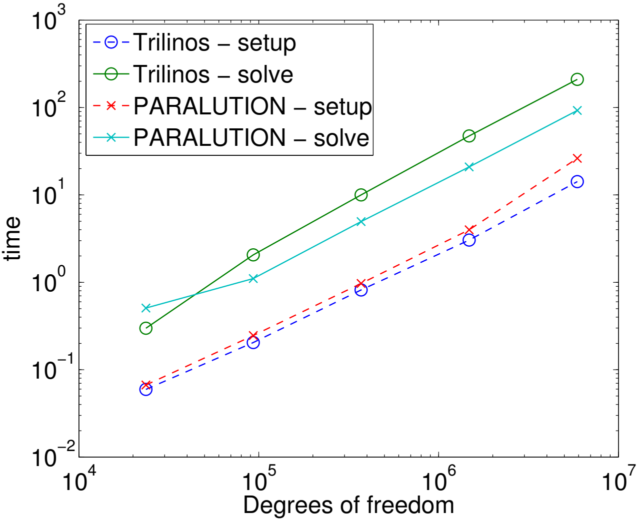
Implementation summary

- Quadrilateral finite elements with quadratic basis function for the displacement and bilinear basis functions for the pressure.
- FEM from `deal.II`
- Linear Algebra:
  - `Trilinos`
  - `PARALUTION`
  - `AGMG`
- Parallel tools: `OpenMp`
- Iterative scheme: GMRES
  - Outer solver tolerance: 10^{-7}
  - Inner solver tolerance: 10^{-1}

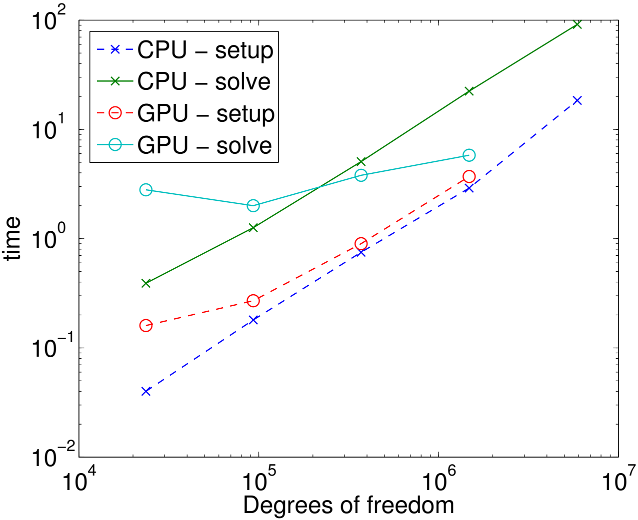
Trilinos - AGMG, comparison in time



Trilinos - PARALUTION, comparison in time



PARALUTION; CPU - GPU, comparison in time



Thank you

The model (Full Complexity)

$$\underbrace{\nabla \cdot \boldsymbol{\sigma}}_{(A)} - \underbrace{\nabla(\rho_0 \mathbf{u} \cdot \nabla \Phi_0)}_{(B)} - \underbrace{\rho_1 \nabla \Phi_0}_{(C)} - \underbrace{\rho_0 \nabla \Phi_1}_{(D)} = \mathbf{0} \quad \text{in } \Omega \subset \mathbb{R}^3,$$

$$\nabla \cdot (\nabla \Phi_1) - 4\pi G \rho_1 = 0,$$

$$\rho_1 + \rho_0 \nabla \cdot \mathbf{u} + \mathbf{u} \cdot \frac{\partial \rho_0}{\partial \mathbf{r}} = 0.$$

(A) force from spatial gradients in stress

(B) *advection of pre-stress*

(C) internal buoyancy

(D) self-gravitation effects