Quantum Information Theory
Spring semester, 2017

Assignment 10
Assigned: Friday, June 2, 2017
Due: Friday, June 11, 2017 M. Skoglund

Problem 10.1: Describe the procedure for constructing a CSS quantum code.

Problem 10.2: Describe the procedure for constructing a quantum stabilizer code.

Problem 10.3: Show that the 9 qubit Shor code can correct any error affecting only one of the qubits.

Problem 10.4: A valid parity-check matrix for the (classical) $[7,4,3]$ *Hamming code* is obtained as

$$H = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Let $\mathcal{C}_1$ be this code, and $\mathcal{C}_2 = \mathcal{C}_1^\perp$ the corresponding dual code. Verify that the pair $(\mathcal{C}_1, \mathcal{C}_2)$ can be used to construct a valid CSS quantum code; the *Steane code*. Also specify, explicitly, a basis for this code.

Problem 10.5: [10.32 in NC] Verify that

$$\begin{aligned} &g_1 = \sigma_0^{\otimes 3} \otimes \sigma_1^{\otimes 4}, \quad g_2 = \sigma_0 \otimes \sigma_1^{\otimes 2} \otimes \sigma_0^{\otimes 2} \otimes \sigma_1^{\otimes 2} \\ &g_2 = \sigma_1 \otimes \sigma_0 \otimes \sigma_1 \otimes \sigma_0 \otimes \sigma_1 \otimes \sigma_0 \otimes \sigma_1, \quad g_4 = \sigma_0^{\otimes 3} \sigma_3^{\otimes 4} \\ &g_5 = \sigma_0 \otimes \sigma_3^{\otimes 2} \otimes \sigma_0^{\otimes 2} \otimes \sigma_3^{\otimes 2}, \quad g_2 = \sigma_3 \otimes \sigma_0 \otimes \sigma_3 \otimes \sigma_0 \otimes \sigma_3 \otimes \sigma_0 \otimes \sigma_3 \end{aligned}$$

generate the Steane code, as a stabilizer code.

Problem 10.6: [10.49 in NC] Verify that

$$\begin{aligned} &g_1 = \sigma_1 \otimes \sigma_3^{\otimes 2} \otimes \sigma_1 \otimes \sigma_0, \quad g_2 = \sigma_0 \otimes \sigma_1 \otimes \sigma_3^{\otimes 2} \otimes \sigma_1 \\ &g_3 = \sigma_1 \otimes \sigma_0 \otimes \sigma_1 \otimes \sigma_3^{\otimes 2}, \quad g_4 = \sigma_3 \otimes \sigma_1 \otimes \sigma_0 \otimes \sigma_1 \otimes \sigma_3 \end{aligned}$$

generate a stabilizer code that can correct an arbitrary single qubit error.

Problem 10.7: [10.2 in NC] Prove the Gilbert–Varshamov bound for CSS quantum codes. That is, prove that there exists a CSS code of length n and dimension k that can correct up to t errors as long as

$$\frac{k}{n} \geq 1 - 2h\left(\frac{2t}{n}\right)$$

where $h(x) = -x \log x - (1-x)\log(1-x)$

Problem 10.8: Consider the group S generated by $\{g_\ell\}_{\ell=1}^L$ and such that $-I = -\sigma_0$ is not in S. Verify that the generators $\{g_\ell\}_{\ell=1}^L$ are independent iff the rows of the corresponding check matrix are linearly independent.