Short Course:

Topics on Cyber-Physical Control Systems

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KTH Royal Institute of Technology

- Sweden’s largest and oldest technical university
- 1/3 of of Sweden’s engineering education and research
- Located in the scientific and industrial hub of Stockholm:
  - Royal Academy of Sciences, Karolinska Inst, Stockholm U,…
  - Ericsson, ABB, Scania, Spotify, Skype, King, Mojang,…

ACCESS Linnaeus Center

- Cross-disciplinary research center on networks
- 36 faculty, 25 postdocs, >100 PhD students
- Focusing on the fundamentals and applications of networked systems

Course Outline

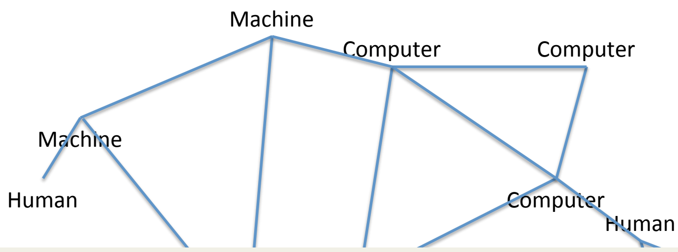
Jul 20: What is a cyber-physical system?

Jul 20: Event-based control of networked systems

Jul 22: Cyber-secure networked control systems

Aug 5?: IAS Lecture on "Cyber-physical control for sustainable freight transportation"

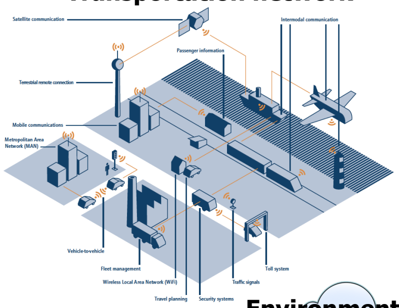
Cyber-physical Systems

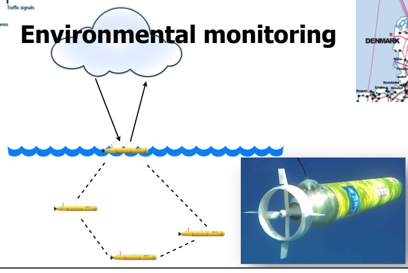


***Cyber-physical systems** are engineered systems whose operations are monitored and controlled by a computing and communication core embedded in objects and structures in the physical environment.*

Cyber-physical Systems Applications

Transportation network





Cyber-Physical Systems Challenges

Societal Scale

- Global and dense instrumentation of physical phenomena
- Interacting with a computational environment: closing the loop
- Security, privacy, usability

Distributed Services

- Self-configuring, self-optimization
- Reliable performance despite uncertain components, resilient aggregation

Programming the Ensemble

- Local rules with guaranteed global behavior
- Distributing control with limited information

Network Architectures

- Heterogeneous systems: local sensor/actuator networks and wide-area networks
- Self-organizing multi-hop, resilient, energy-efficient routing
- Limited storage, noisy channels

Real-Time Operating Systems

- Extensive resource-constrained concurrency
- Modularity and data-driven physics-based modeling

1000 Radios per Person

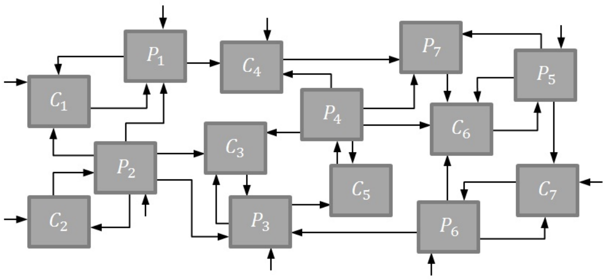
- Low-power processors, radio communication, encryption
- Coordinated resource management, spectrum efficiency

Sastry & J, 2010

Cyber-Physical **Control** Challenges

How to analyze, design, and implement control systems with

- Guaranteed **global objective** from local interactions
- **Physical dynamics** coupled with information interactions
- Tradeoff **computation-communication-control** complexities
- **Robustness to** external disturbances and other **uncertainties**



Event-based control of networked systems

Outline

- Introduction
- Stochastic event-based control
- Optimal event-based control
- Distributed event-based control
- Event-based anti-windup
- Conclusions

Acknowledgements

Presentation based on joint papers with students

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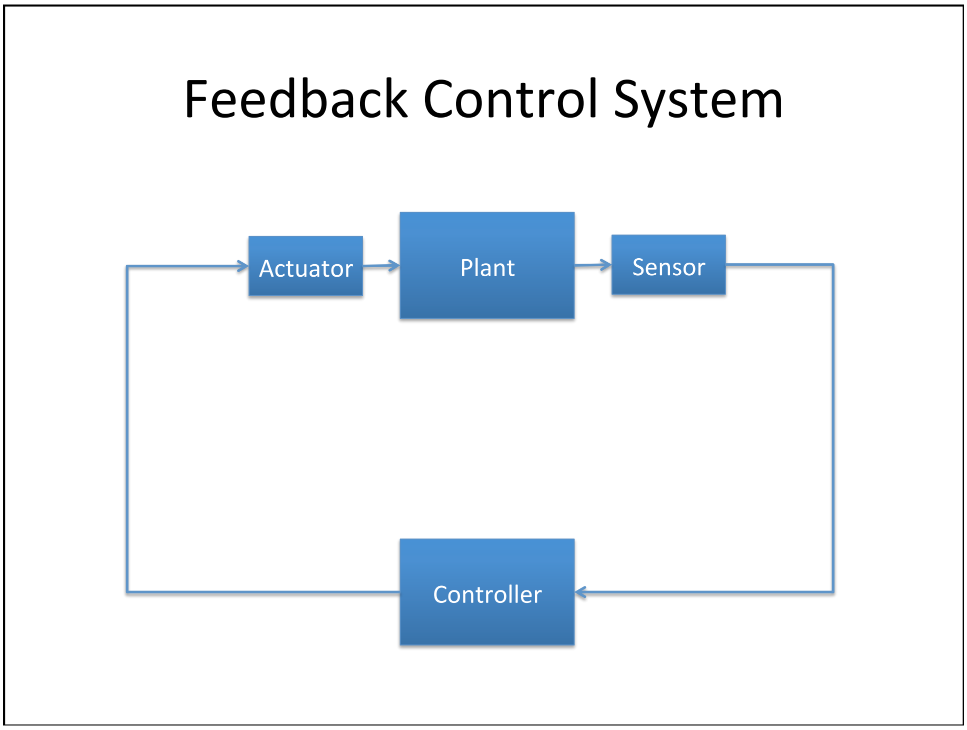
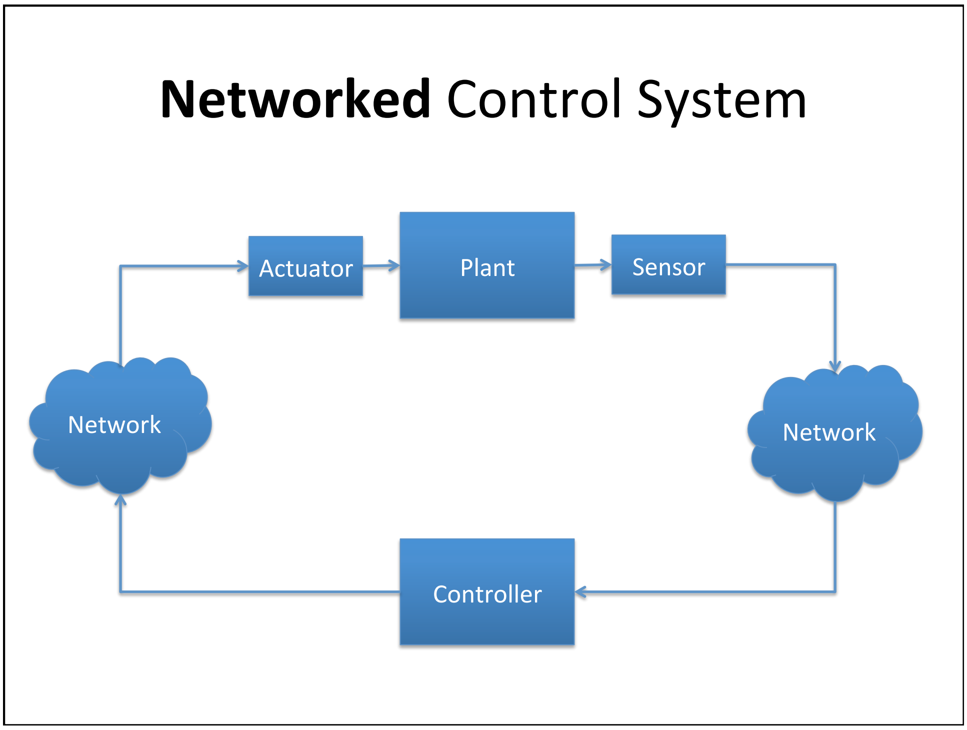
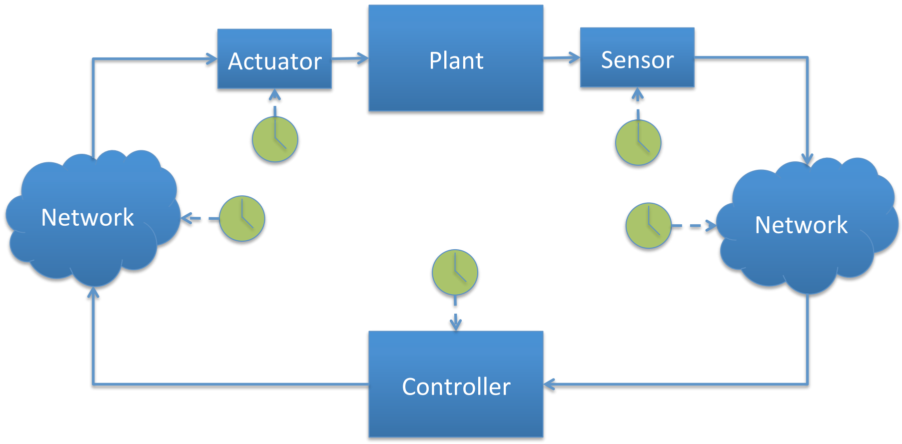
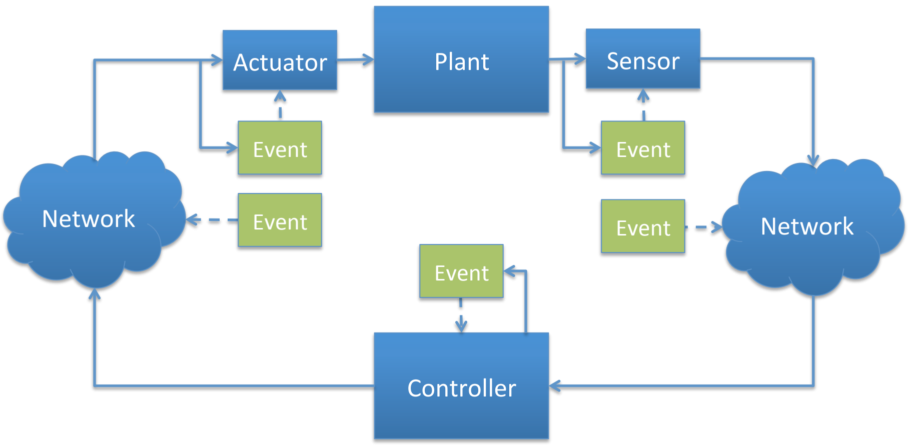

Feedback Control System
Actuator
Plant
Sensor
Controller


Networked Control System
Actuator
Plant
Sensor
Network
Network
Controller

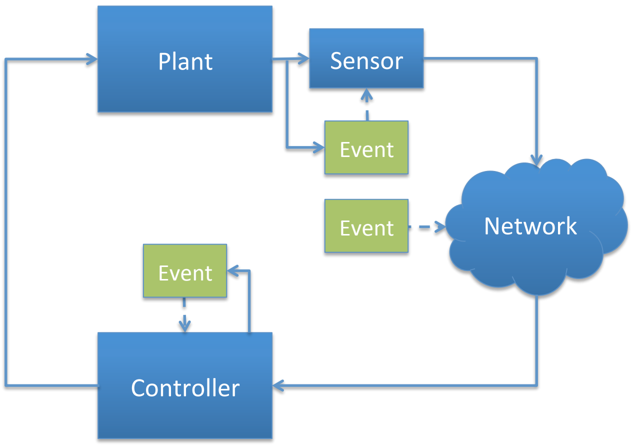
Time-Triggered Control System



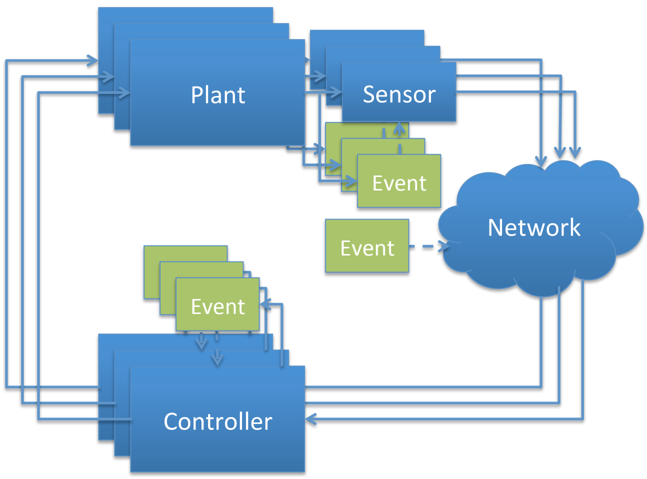
Event-Based Control System



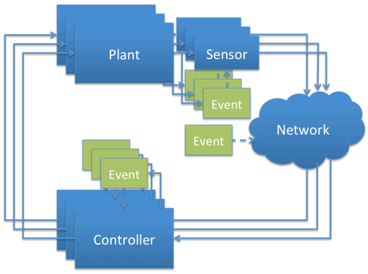
Event-Based Control System

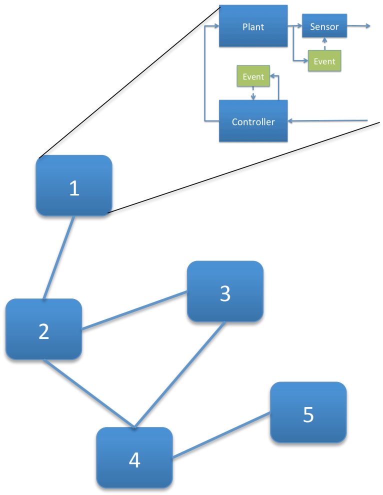


Event-Based Control System

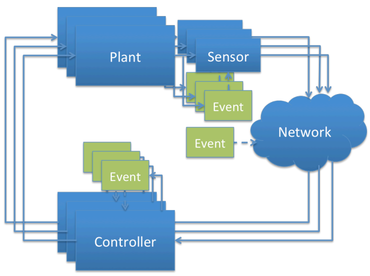


Event-Based Multi-Agent System





Goal: Guarantee Control Performance under Limited Resources



Resources

- Sensing
- Sensor communication
- Network
- Actuation
- (Computing)

Outline

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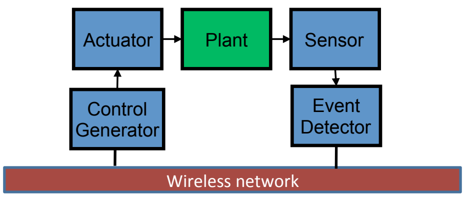
- Event-based anti-windup
- Conclusions

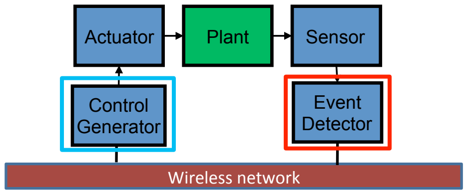
Event-based control loop



Åström, *2007,* Rabi and J., *WICON, 2008*

When to transmit?

- Event detector mechanism on sensor side
  - E.g., threshold crossing



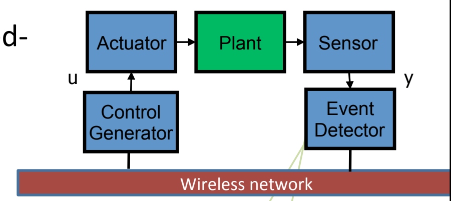
How to control?

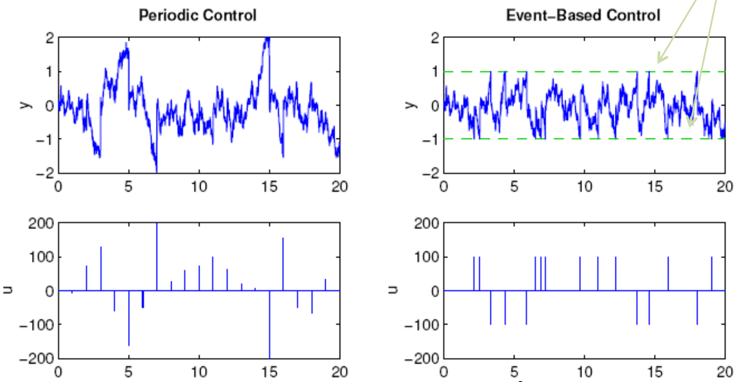
- Execute control law at actuator side
  - E.g., piecewise constant controls, impulse control

Rabi et al., *2008*

Example: Fixed threshold with impulse control

- Event-detector implemented as fixed-level threshold at sensor
- Event-based impulse control better than periodic impulse control



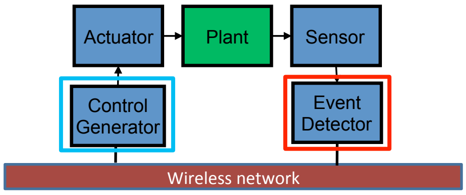


Åström & Bernhardsson, *IFAC*, 1999

Control generators and event detectors

1. Impulse
2. Zero order hold
3. Higher order hold

1. Fixed threshold
2. Time-varying
3. Adaptive



Plant model

Plant $$dx = udt + dv,$$

Stochastic differential equation, interpreted as

$$x(s+\tau) - x(\tau) = \int_{\tau}^{s+\tau} u(t)dt + \int_{\tau}^{s+\tau} dv(t)$$

with one ordinary (Lebesgue) integral and one stochastic (Ito) integral.

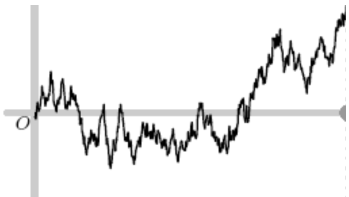
v is a Wiener process (or Brownian motion)

See Øksendal (2003) for an introduction to stochastic differential equations

Wiener process

A Wiener process v(t) fulfills

1. v(0)=0
2. v(t) is almost surely continuous
3. v(t) has independent increments with v(t)-v(s) ~ N(0,t-s) for t>s≥0



Remark The variance of a Wiener process is growing like

$$E(v(t+s)-v(t))^2 = |s|$$

Plant model

Plant $$dx = udt + dv,$$

Stochastic differential equation, interpreted as

$$x(s+\tau) - x(\tau) = \int_{\tau}^{s+\tau} u(t)dt + \int_{\tau}^{s+\tau} dv(t)$$

with one ordinary (Lebesgue) integral and one stochastic (Ito) integral.

When $s > 0$ is a small, the change of $x(\tau)$ is normally distributed with mean $su(\tau)$ and variance s.

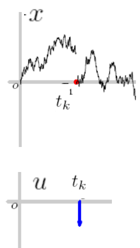
Plant model and control cost

Plant $dx = udt + dv,$

v is a Wiener process: $E(v(t+s) - v(t))^2 = |s|$

Cost function $V = \frac{1}{T} E \int_0^T x^2(t)dt.$

Periodic impulse control



Impulse applied at events t_k

$$u(t) = -x(t_k)\delta(t - t_k),$$

Periodic reset of state every event.

State grows linearly as

$$E(v(t+s) - v(t))^2 = |s|$$

between sample instances, because $dx = udt + dv,$

Average variance over sampling period h is $\frac{1}{2}h$ so the cost is $V_{PIH} = \frac{1}{2}h.$

Åström, 2007

Periodic ZoH control

Traditional sampled-data control theory gives that

$V = \frac{1}{h}\int_0^h Ex^2(t)\,dt$ is minimized for the sampled system

$$x(t+h) = x(t) + hu(t) + e(t),$$

with

$$u = -Lx = \frac{1}{h}\frac{3+\sqrt{3}}{2+\sqrt{3}}x$$

derived from

$$S = \Phi^T S\Phi + Q_1 - L^T RL, \quad L = R^{-1}(\Gamma^T S\Phi + Q_{12}^T), \quad R = Q_2 + \Gamma^T S\Gamma,$$

The minimum gives the cost

$$V_{PZOH} = \frac{3+\sqrt{3}}{6}h$$

Åström, 2007

Event-based impulse control with fixed threshold

Suppose an event is generated whenever

$$|x(t_k)| = a$$

generating impulse control

$$u(t) = -x(t_k)\delta(t - t_k),$$

One can show that the average time between two events is

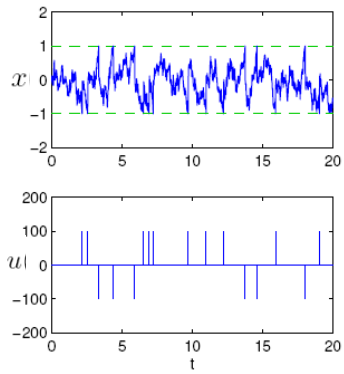
$$h_E := E(T_{\pm d}) = E(x^2_{T_{\pm d}}) = a^2$$

and that the pdf of x is triangular:

$$f(x) = (a - |x|)/a^2$$

The cost is

$$V_{EIH} = \frac{a^2}{6} = \frac{h_E}{6}$$

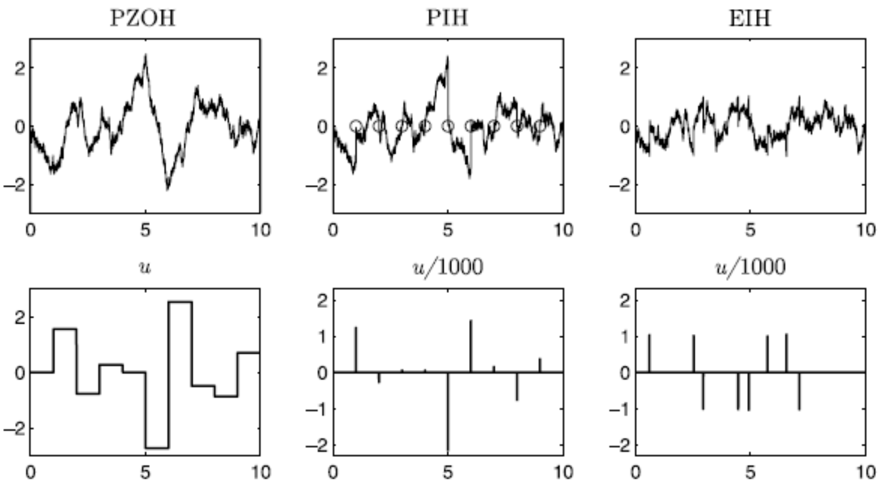


Åström, 2007

Pdf $f(x) = (a - |x|)/a^2$ is the solution to the forward Kolmogorov forward equation (or Fokker–Planck equation)

$$\frac{\partial f}{\partial t} = \frac{1}{2}\frac{\partial^2 f}{\partial x^2}(x) - \frac{1}{2}\frac{\partial f}{\partial x}(d)\delta_x + \frac{1}{2}\frac{\partial f}{\partial x}(-d)\delta_x, \qquad f(-a) = f(a) = 0,$$

Comparison



Åström, 2007

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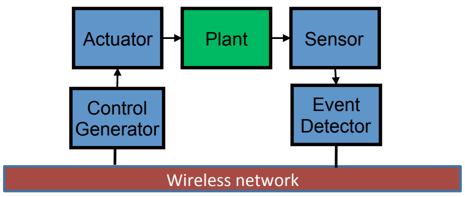
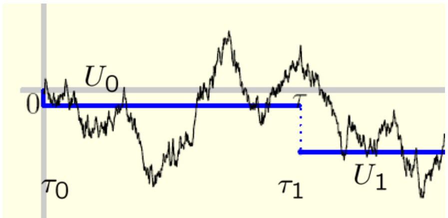
Event-based ZoH control with adaptive sampling



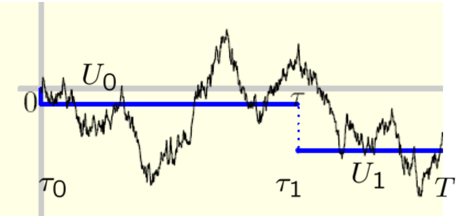


How choose $\{U_i\}$ and $\{\tau_i\}$ to minimize $V = \frac{1}{T} E \int_0^T x^2(t)dt$.

Rabi et al., *2008*

Optimal control with one sampling event

$$dx_t = u_t dt + dB_t$$



$$\min_{U_0,U_1,\tau} J = \min_{U_0,U_1,\tau} \mathbf{E} \int_0^T x_s^2 ds$$

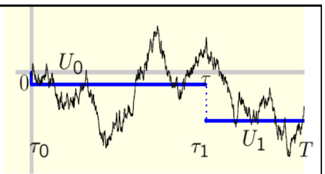
$$= \min_{U_0,U_1,\tau} \left[\mathbf{E} \int_0^\tau x_s^2 ds + \mathbf{E} \int_\tau^T x_s^2 ds \right]$$

A joint optimal control and optimal stopping problem

Rabi et al., *2008*

$$dx_t = u_t dt + dB_t$$

$$\min_{U_0,U_1,\tau} J = \min_{U_0,U_1,\tau} \mathbf{E} \int_0^T x_s^2 ds$$



If τ chosen deterministically (not depending on x_t) and $x_0 = 0$:

$$U_0^* = 0 \qquad U_1^* = -\frac{3x_{T/2}}{T} \qquad \tau^* = T/2$$

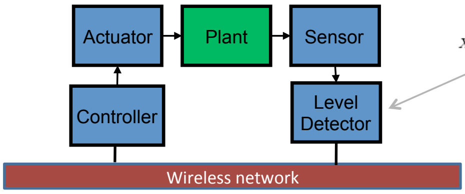
If τ is event-driven (depending on x_t) and $x_0 = 0$:

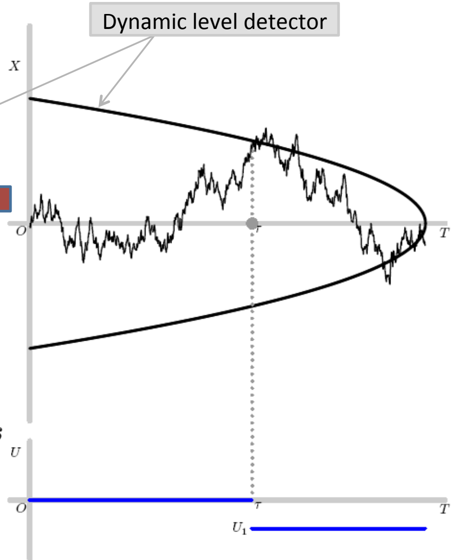
$$U_0^* = 0 \qquad U_1^* = -\frac{3x_{\tau^*}}{2(T-\tau^*)}$$

$$\tau^* = \inf\{t : x_t^2 \geq \sqrt{3}(T-t)\}$$

Envelope defines optimal level detector

Optimal level detector





$$dx_t = u_t dt + dB_t$$

$$\min_{U_0, U_1, \tau} J = \min_{U_0, U_1, \tau} \mathbf{E} \int_0^T x_s^2 ds$$

Proof

$$\min_{U_0, U_1, \tau} J = \min_{U_0, U_1, \tau} \mathbf{E} \int_0^T x_s^2 ds = \min_{U_0, U_1, \tau} \left[\mathbf{E} \int_0^\tau x_s^2 ds + \mathbf{E} \int_\tau^T x_s^2 ds \right]$$

$$\mathbf{E} \left\{ \int_\tau^T x_s^2 ds \middle| \tau, x_\tau, U_1 \right\} = \left[x_t = x_\tau + \int_\tau^t U_1 ds + \int_\tau^t dB_s \right]$$

$$= \int_\tau^T \mathbf{E} \left\{ \left[x_\tau^2 + U_1^2 (t-\tau)^2 + (B_t - B_\tau)^2 + 2 x_\tau U_1 (t - \tau) \right.\right.$$

$$\left.\left. + 2 x_\tau (B_t - B_\tau) + 2 U_1 (t-\tau)(B_t - B_\tau) \right] \right\} dt$$

$$= \left[\mathbf{E} B_t = 0,\ \mathbf{E} B_t^2 = t,\ \delta := T - \tau \right] = \delta x_\tau^2 + \frac{\delta^3}{3} U_1^2 + \frac{\delta^2}{2} + \delta^2 x_\tau U_1$$

$$= \frac{\delta}{4} x_\tau^2 + \delta \left(\frac{x_\tau \sqrt{3}}{2} + \frac{\delta U_1}{\sqrt{3}} \right)^2 + \frac{\delta^2}{2}$$

Hence, optimal control $U_1^* = U_1^*(x_\tau, T - \tau) = -\dfrac{3 x_\tau}{2(T-\tau)}$

$$J(U_0, U_1^*, \tau) = \mathbf{E}\int_0^\tau x_s^2 ds + \mathbf{E}\left\{\frac{T-\tau}{4}x_\tau^2 + \frac{(T-\tau)^2}{2}\right\}$$

If τ chosen deterministically (not depending on x_t) and $x_0 = 0$:

$$J(U_0, U_1^*, \theta) = \frac{\theta^3}{3}U_0^2 + \frac{\theta^2}{2} + \frac{T-\theta}{4}(U_0^2\theta^2 + \theta) + \frac{(T-\theta)^2}{2}$$

Hence,

$$U_0^* = 0 \qquad U_1^* = -\frac{3x_{T/2}}{T} \qquad \tau^* = T/2$$

which gives

$$J(U_0^*, U_1^*, \tau^*) = \frac{5T^2}{16}$$

If τ is event-driven (depending on x_t) and $x_0 = 0$:

$$J(U_0, U_1^*, \tau) = \mathbf{E}\int_0^\tau x_s^2 ds + \mathbf{E}\left\{\frac{T-\tau}{4}x_\tau^2 + \frac{(T-\tau)^2}{2}\right\} = \cdots$$

$$= \frac{T^2}{2} + \frac{U_0^2 T^3}{3} - \mathbf{E}\left\{\left(\frac{x_\tau\sqrt{3}}{2} + \frac{(T-\tau)U_0}{\sqrt{3}}\right)^2 (T-\tau)\right\}$$

$$= \frac{T^2}{2} - \frac{3}{4}\mathbf{E}\left\{x_\tau^2(T-\tau)\right\}$$

because from symmetry $U^* = 0$.

Find τ that maximizes $f(x_\tau, \tau) = \mathbf{E}\left\{x_\tau^2(T-\tau)\right\}$

Find τ that maximizes $f(x_\tau,\tau) = \mathbf{E}\left\{x_\tau^2(T-\tau)\right\}$

Suppose there exists smooth $g(x,t)$ such that

$$g(x,t) \geq x^2(T-t)$$
$$\frac{1}{2}g_{xx}(x,t) + g_t(x,t) = 0$$

Then, for $0 \leq t \leq \tau \leq T$,

$$f(x_\tau,\tau) = \mathbf{E}\left\{x_\tau^2(T-\tau)\right\} \leq \mathbf{E}\left\{g(x_\tau,\tau)\right\} = g(x_t,t) + \mathbf{E}\int_t^\tau dg(x_\tau,\tau)$$
$$= [\text{Ito formula}] = g(x_t,t) + \mathbf{E}\int_t^\tau \left(\frac{1}{2}g_{xx} + g_t\right)dt$$
$$= g(x_t,t)$$

Hence, g is an upper bound for the expected reward.

We next show that equality can be achieved.

$$g(x_t,t) = \frac{\sqrt{3}}{1+\sqrt{3}}\left(\frac{x_t^4}{6} + x_t(T-t)^2 + \frac{(T-t)^2}{2}\right)$$

is a solution to

$$\frac{1}{2}g_{xx}(x,t) + g_t(x,t) = 0$$

Moreover,

$$g(x_t,t) - x_t^2(T-t) = \frac{1}{2(1+\sqrt{3})}\left(\frac{x_t^4}{3} - \frac{2}{\sqrt{3}}x_t^2(T-t) + (T-t)^2\right)$$
$$= \frac{1}{2(1+\sqrt{3})}\left(\frac{x_t^4}{\sqrt{3}} - (T-t)^2\right) = 0$$

if $x_t^2 = \sqrt{3}(T-t)$.

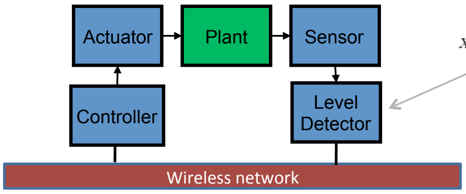
Hence, the optimal sampling time is

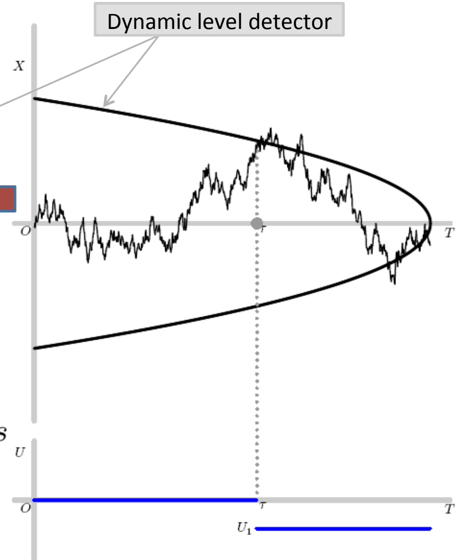
$$\tau^* = \inf\{t:\ x_t^2 \geq \sqrt{3}(T-t)\}$$

which gives

$$J(U_0^*, U_1^*, \tau^*) = \frac{T^2}{8}$$

Optimal level detector





$$dx_t = u_t dt + dB_t$$

$$\min_{U_0, U_1, \tau} J = \min_{U_0, U_1, \tau} \mathbf{E} \int_0^T x_s^2 ds$$

Multiple samples

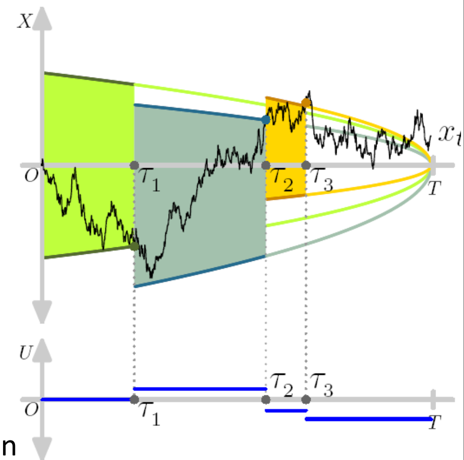
Extension to N>1 samples

$$J_N\left(x_0, \mathcal{U}, \{\tau\}_{i=1}^N\right) = \mathbb{E}\left[\int_0^T x_s^2 ds \middle| x_0\right]$$

through nested single sample problems

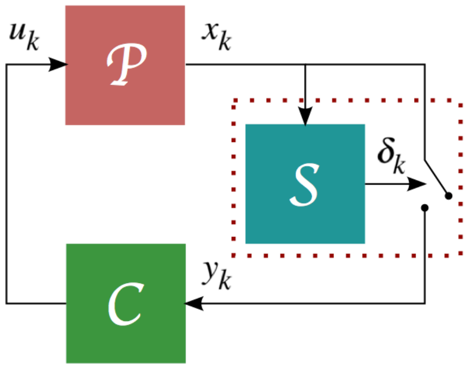
Extension to variable budget sampling, allowing number of samples to depend on x.

Extensions to lossy communication

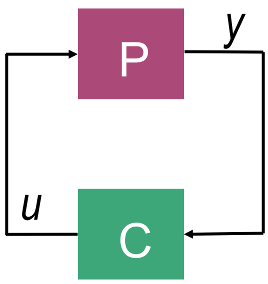


Rabi and J., *2009*

Joint Optimal Event-Generation and Control



Control without Event Scheduling: Classical LQG



The controller minimizing

$$J = \mathbb{E}\left[x_N^T Q_0 x_N + \sum_{s=0}^{N-1}(x_s^T Q_1 x_s + u_s^T Q_2 u_s)\right]$$

is given by

$$u_k = -L_k \hat{x}_{k|k} \ ,$$
$$L_k = (Q_2 + B^T S_{k+1} B)^{-1} B^T S_{k+1} A$$

where

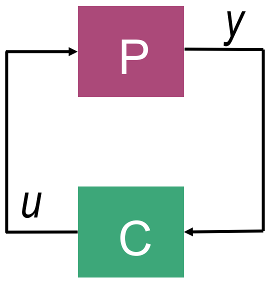
$$S_k = Q_1 + A^T S_{k+1} A - A^T S_{k+1} B (Q_2 + B^T S_{k+1} B)^{-1} B^T S_{k+1} A$$

$\hat{x}_{k|k} = \mathbb{E}[x_k | \{y\}_0^k \boldsymbol{u}_0^{k-1}]$ is the minimum mean-square error (MMSE) estimate

Kalman, 1960

Certainty Equivalence

Definition Certainty equivalence holds if the closed-loop optimal controller has the same form as the deterministic optimal controller with x_k replaced by the estimate $\hat{x}_{k|k} = \mathrm{E}[x_k | \mathbb{I}_k^{\mathbb{C}}]$.



Theorem[Bar-Shalom–Tse] Certainty equivalence holds if and only if $E[(x_k - E[x_k|I_k^c])^2|I_k^c]$ is not a function of past controls $\{u\}_0^{k-1}$ (no dual effect).

Here x_k is the plant state and I_k^c the information at the controller

Feldbaum, 1965; Åström, 1970; Bar-Shalom and Tse, 1974

Stochastic Control Formulation

Plant:

$$x_{k+1} = Ax_k + Bu_k + w_k$$

Scheduler:

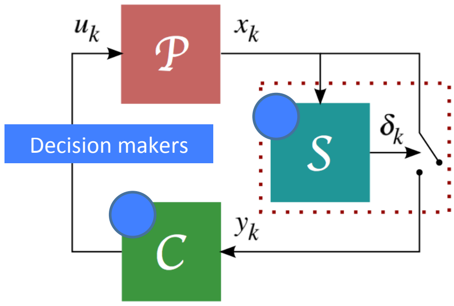
$$\delta_k = f_k(\mathbb{I}_k^{\mathbb{S}}) \in \{0,1\}$$

$$\mathbb{I}_k^{\mathbb{S}} = \left[\{x\}_0^k, \{y\}_0^{k-1}, \{\delta\}_0^{k-1}, \{u\}_0^{k-1}\right]$$

Controller:

$$u_k = g_k(\mathbb{I}_k^{\mathbb{C}})$$

$$\mathbb{I}_k^{\mathbb{C}} = \left[\{y\}_0^k, \{\delta\}_0^k, \{u\}_0^{k-1}\right]$$



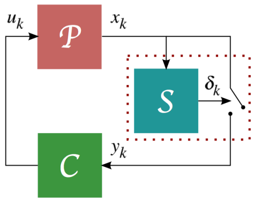
Cost criterion:

$$J(f,g) = \mathrm{E}[x_N^T Q_0 x_N + \sum_{s=0}^{N-1} (x_s^T Q_1 x_s + u_s^T Q_2 u_s)]$$

- Non-classical information pattern
- Hard to find optimal solutions in general
- Special cases lead to tractable problems

Cf., Witsenhausen, Hu & Chu, Varaiya & Walrand , Borkar, Mitter & Tatikonda, Rotkowitz etc

Example



Plant

$$x_{k+1} = x_k + u_k + w_k, \quad x_0 = 2, Ew_k^2 = 0.7^2$$

Certainty equivalent controller

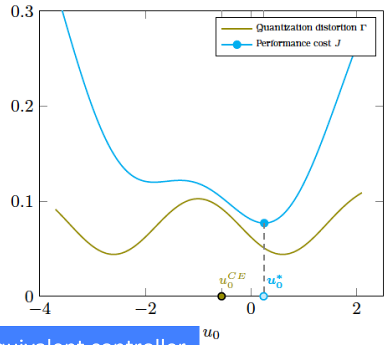
$$u_k^{\text{CE}} = -K_k^{\text{CE}}\left(E[x_k|\{y_k\}_0^k, \{u_k\}_0^{k-1}] + E[w_k|\{y_k\}_0^k, \{u_k\}_0^{k-1}]\right)$$

Event-generator encodes state as

$$\xi(x_k) = \begin{cases} 1, & \text{if } x_k \in (\infty, -\theta) \\ 2, & \text{if } x_k \in (-\theta, \theta) \\ 3, & \text{if } x_k \in (\theta, \infty) \end{cases}$$

Cost for time-horizon $N = 1$

$$J(u_0) = \sigma_w^2 + qu_0^2 + \left(p + \frac{qa^2}{q+1}\right)\mathbb{E}\left[x_1^2 | x_0, u_\right.$$

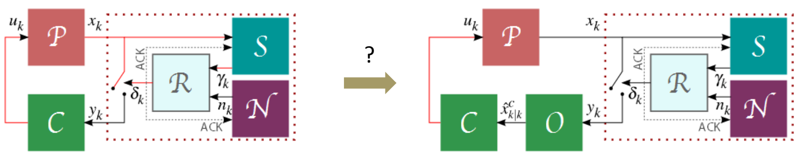


Optimal performance is not obtained by a certainty equivalent controller

Rabi et al, 2015

Condition for Certainty Equivalence

Corollary: The optimal controller for the system {$\mathcal{P}$,$\mathcal{S}$(f),$\mathcal{C}$(g)}, with respect to the cost J is certainty equivalent if the scheduling decisions are not a function of the applied controls.



Certainty equivalence achieved at the cost of optimality

Bar-Shalom & Tse, 1974; Ramesh et al., 2011

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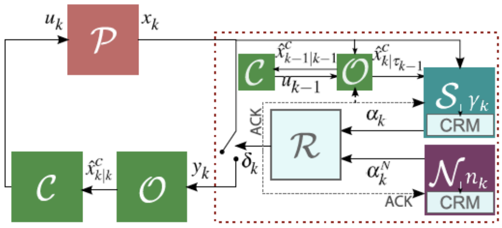
Architecture with Certainty Equivalent Controller



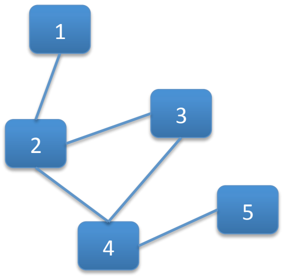
Ramesh et al., 2012, 2013

Outline

- Introduction
- Stochastic event-based control
- Optimal event-based control
- Distributed event-based control
- Event-based anti-windup
- Conclusions

Distributed Event-Based Control

- How to implement event-based control over a distributed system?
- Local control and communication, but global objective



Approach: Consider a prototype distributed control problem and study it under event-based communication and control

Average Consensus Problem

Multi-agent system model

- Group of N agents

$$\dot{x}_i(t) = u_i(t)$$

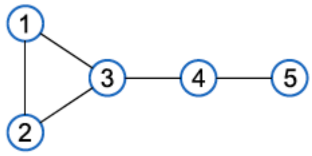
- Communication graph G
  A: undirected, connected

Adjacency matrix A with $a_{ij} = 1$ if agents i and j adjacent, otherwise $a_{ij} = 0$

Degree matrix D is the diagonal matrix with elements equal to the cardinality of the neighbor sets N_i

Objective: Average consensus

$$x_i(t) \overset{t \to \infty}{\longrightarrow} a = \frac{1}{N} \sum_{i=1}^{N} x_i(0)$$



Consensus protocol

$$u_i(t) = -\sum_{j \in N_i} (x_i(t) - x_j(t))$$

Closed-loop dynamics

$$\dot{x}(t) = -Lx(t)$$

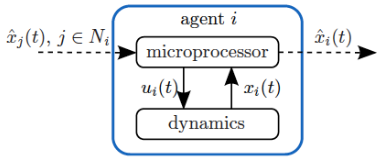
with Laplacian matrix $L = D - A$

Event-based implementation?

Olfati-Saber & Murray, 2004

Event-Based Average Consensus

Event-based scheduling of measurement broadcasts:



Event-based broadcasting

$$\hat{x}_i(t) = x_i(t_k^i),\ t \in [t_k^i, t_{k+1}^i[$$
$$0 \le t_0^i \le t_1^i \le t_2^i \le \cdots$$

- Consensus protocol

$$u_i(t) = -\sum_{j \in N_i} (\hat{x}_i(t) - \hat{x}_j(t))$$

- Measurement errors

$$e_i(t) = \hat{x}_i(t) - x_i(t)$$

- Closed-loop

$$\dot{x}(t) = -L\hat{x}(t) = -L(x(t) + e(t))$$

- Disagreement

$$\delta(t) = x(t) - a\mathbf{1}, \qquad \mathbf{1}^T\delta(t) \equiv 0$$

Seyboth et al, 2013

Trigger Function for Event-Based Control

Trigger mechanism: Define *trigger functions* $f_i(\cdot)$ and trigger when

$$f_i\left(t, x_i(t), \hat{x}_i(t), \bigcup_{j \in N_i} \hat{x}_j(t)\right) > 0$$

Defines sequence of events: $t_{k+1}^i = \inf\{t : t > t_k^i, f_i(t) > 0\}$

Extends [Tabuada, 2007] single-agent trigger function to multi-agent systems

Find f_i such that
- $|x_i(t) - x_j(t)| \to 0,\ t \to \infty$
- no Zeno (no accumulation point in time)
- few inter-agent communications

Cf., Dimarogonas et al., De Persis et al., Donkers et al., Mazo & Tabuada, Wang & Lemmon, Garcia & Antsaklis, Guinaldo et al.

Seyboth et al, 2013

Event-Based Control with Constant Thresholds

$$\dot{x}(t) = u(t), \qquad u(t) = -L\hat{x}(t) \qquad (1)$$

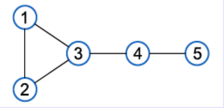
Theorem (constant thresholds)

Consider system (1) with undirected connected graph G. Suppose that

$$f_i(e_i(t)) = |e_i(t)| - c_0,$$

with $c_0 > 0$. Then, for all $x_0 \in \mathbb{R}^N$, the system does not exhibit Zeno behavior and for $t \to \infty$,

$$\|\delta(t)\| \leq \frac{\lambda_N(L)}{\lambda_2(L)} \sqrt{N} c_0.$$



Proof ideas:

- Analytical solution of disagreement dynamics yields

$$\|\delta(t)\| \leq e^{-\lambda_2(L)t}\|\delta(0)\| + \lambda_N(L)\int_0^t e^{-\lambda_2(L)(t-s)}\|e(s)\|ds$$

- Compute lower bound τ on the inter-event intervals

Seyboth et al, 2013

Event-Based Control with Exponentially Decreasing Thresholds

$$\dot{x}(t) = u(t), \qquad u(t) = -L\hat{x}(t) \qquad (1)$$

Theorem (exponentially decreasing thresholds)

Consider system (1) with undirected connected graph G. Suppose that

$$f_i(t, e_i(t)) = |e_i(t)| - c_1 e^{-\alpha t},$$

with $c_1 > 0$ and $0 < \alpha < \lambda_2(L)$. Then, for all $x_0 \in \mathbb{R}^N$, the system does not exhibit Zeno behavior and as $t \to \infty$,

$$\|\delta(t)\| \to 0.$$

Remarks

- Asymptotic convergence: $|x_i(t) - x_j(t)| \to 0$, $t \to \infty$
- $\lambda_2(L)$ is the rate of convergence for continuous-time consensus, so threshold need to decrease slower

Seyboth et al, 2013

Event-Based Control with Exponentially Decreasing Thresholds and Offset

$$\dot{x}(t) = u(t), \qquad u(t) = -L\hat{x}(t) \qquad (1)$$

Theorem (exponentially decreasing thresholds with offset)

Consider system (1) with undirected connected graph G. Suppose that

$$f_i(t, e_i(t)) = |e_i(t)| - (c_0 + c_1 e^{-\alpha t}),$$

with $c_0, c_1 \geq 0$, at least one positive, and $0 < \alpha < \lambda_2(L)$. Then, for all $x_0 \in \mathbb{R}^N$, the system does not exhibit Zeno behavior and for $t \to \infty$,

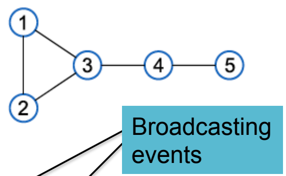
$$\|\delta(t)\| \leq \frac{\lambda_N(L)}{\lambda_2(L)} \sqrt{N} c_0.$$

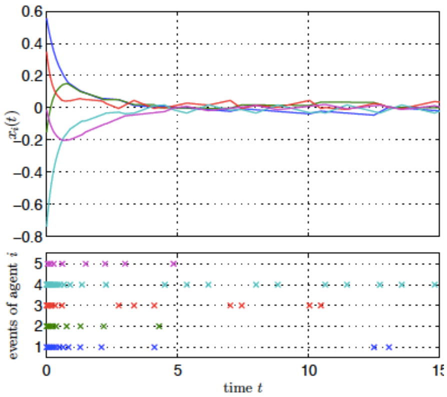
Seyboth et al, 2013

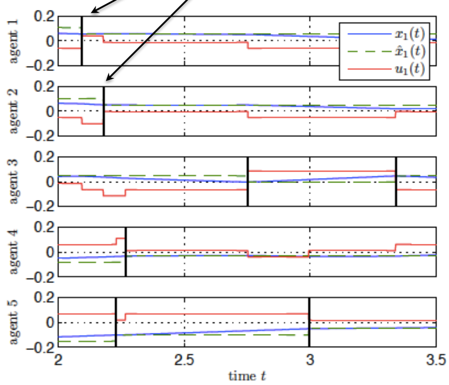
Example

$$\dot{x}(t) = -L\hat{x}(t)$$
$$|e_i(t)| \leq c_0 + c_1 e^{-\alpha t}$$

$$|e_i(t)| \leq 0.050$$



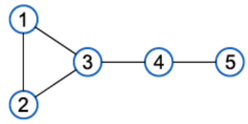




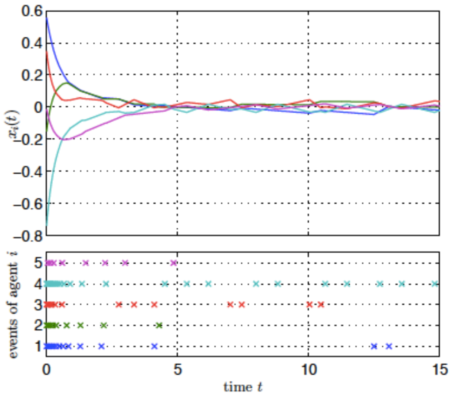
Seyboth et al, 2013

Example: Threshold Tuning

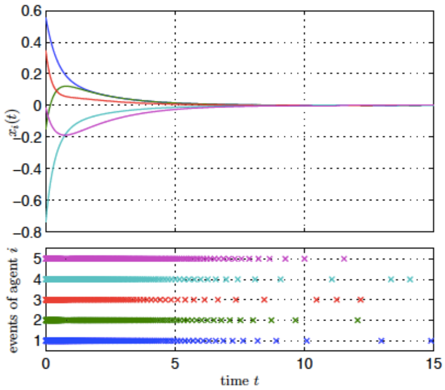
$$\dot{x}(t) = -L\hat{x}(t)$$
$$|e_i(t)| \leq c_0 + c_1 e^{-\alpha t}$$



$|e_i(t)| \leq 0.050$



$|e_i(t)| \leq 0.001$



Seyboth et al, 2013

Example: Threshold Tuning

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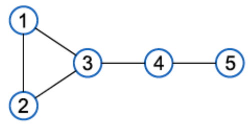
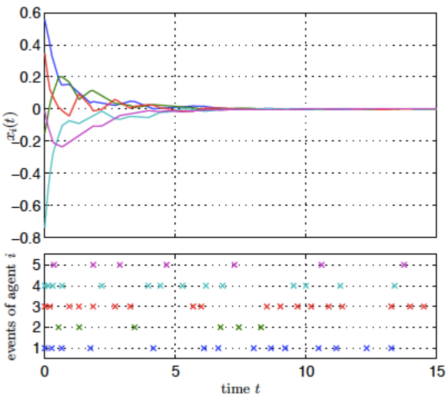
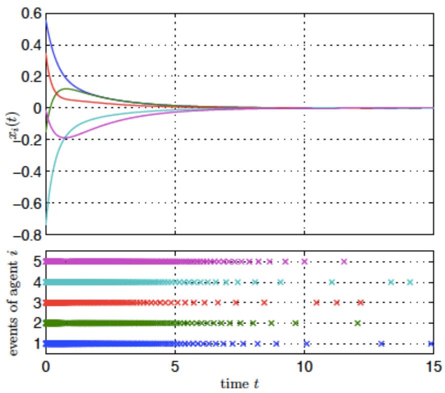


$|e_i(t)| \leq 0.001 + 0.249\, e^{-0.9\lambda_2(L)t}$



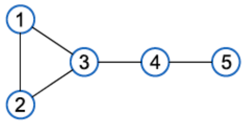
$|e_i(t)| \leq 0.001$



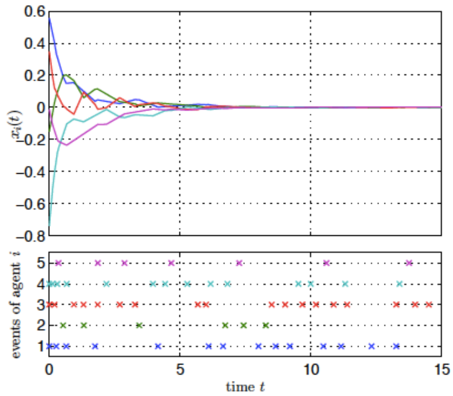
Seyboth et al, 2013

Example: Threshold Tuning

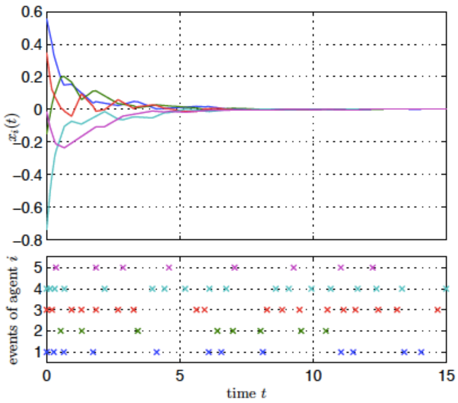
$$\dot{x}(t) = -L\hat{x}(t)$$
$$|e_i(t)| \leq c_0 + c_1 e^{-\alpha t}$$



$|e_i(t)| \leq 0.001 + 0.249\, e^{-0.9\lambda_2(L)t}$

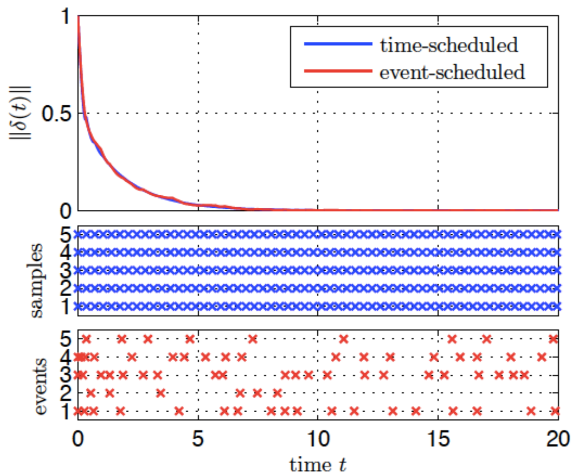


$|e_i(t)| \leq 0.250\, e^{-0.9\lambda_2(L)t}$

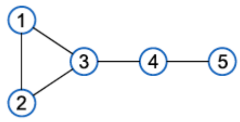


Seyboth et al, 2013

Example: Event- vs Time-Triggered Sampling



Graph:



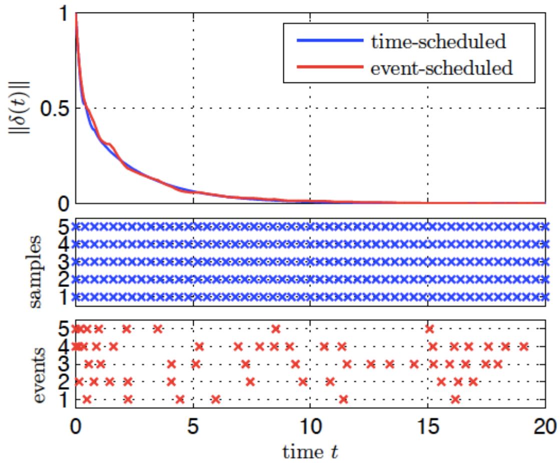
Sampling periods:

- Time-scheduling:
  $\tau_s = 0.350$
  $\tau_{max} = 0.480$
- Event-scheduling:
  $\tau_{mean} = 1.389$

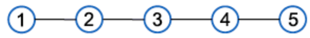
τ_{max} : largest stabilizing sampling period, see *G. Xie et al., ACC2009*

Seyboth et al, 2013

Example: Event- vs time-triggered sampling



Graph:

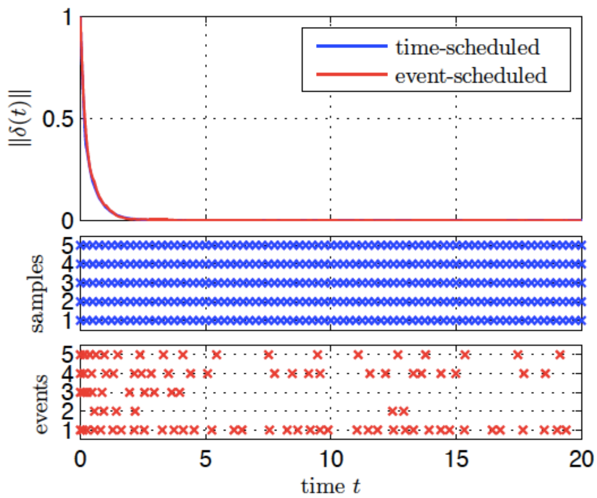


Sampling periods:

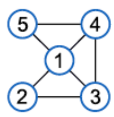
- Time-scheduling:
  $\tau_s = 0.400$
  $\tau_{max} = 0.553$
- Event-scheduling:
  $\tau_{mean} = 1.724$

Seyboth et al, 2013

Example: Event- vs time-triggered sampling



Graph:

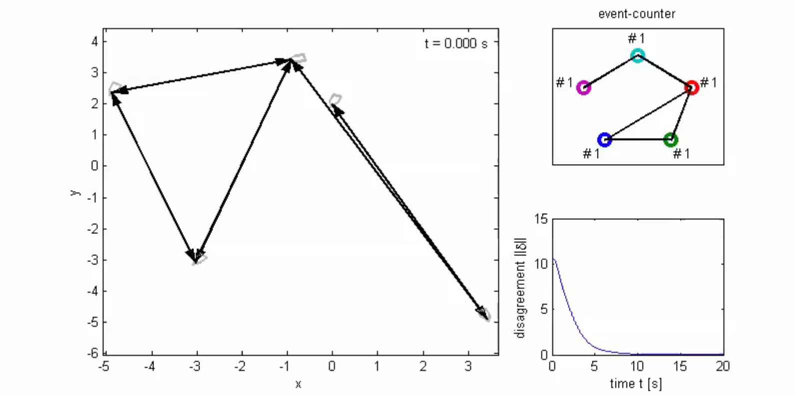


Sampling periods:

- Time-scheduling:
  $\tau_s = 0.250$
  $\tau_{max} = 0.400$
- Event-scheduling:
  $\tau_{mean} = 1.053$

Seyboth et al, 2013

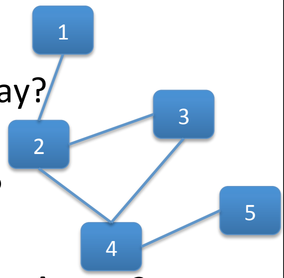
Event-Based Formation Control



- **Non-holonomic mobile robots under feedback linearization**
- **Event-based communication based on threshold for double-integrator network**

Seyboth et al, 2013

Extensions



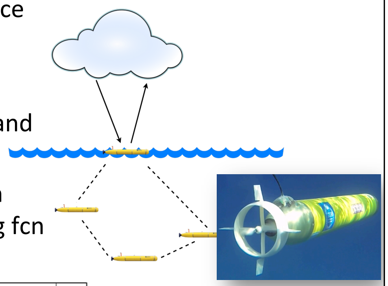
- How to **estimate λ_2(L)** in a distributed way?
  - Aragues et al., 2014
- How to handle **general** agent **dynamics**?
  - Guinaldo et al. 2013
- How to handle network **delays** and packet **losses**?
  - Guinaldo et al., 2014
- **Pinning** (leader-follower) control and switching networks
  - Adaldo et al., 2015
- Event-triggered **pulse width modulation**
  - Meng et al., 2015
- Event-triggered **cloud access**
  - Adaldo et al., 2015

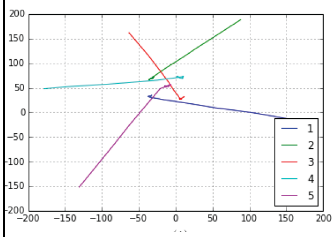
Event-triggered Cloud Access

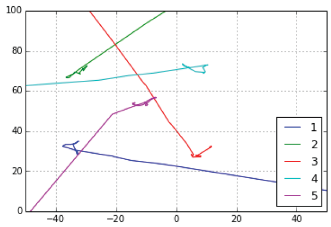
- Agent dynamics with unknown drift disturbance

$$\dot{x}_i(t) = u_i(t) + \omega_i(t), \quad i = 1, \ldots, N,$$

- Agents exchange state, control, disturbance, and timing data through a shared data base
- Schedule next data base access time based on dynamic estimates and event-based triggering fcn







Data base access times

k	$t_{1,k}$	$t_{2,k}$	$t_{3,k}$	$t_{4,k}$	$t_{5,k}$
0	0.00	0.00	0.00	0.00	0.00
1	5.01	6.21	7.41	8.51	10.11
2	12.72	14.72	16.72	18.81	21.31
3	23.32	25.82	28.02	30.41	32.61
4	34.92	37.23	39.63	41.92	44.22
5	46.53	48.84			

Adaldo et al., 2015

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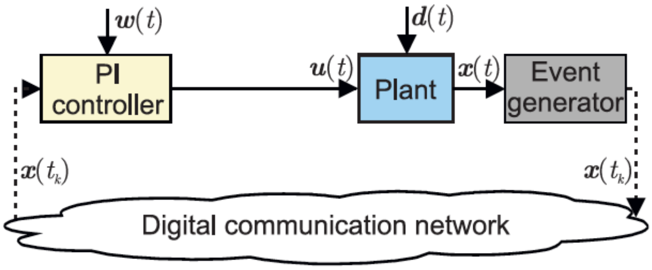
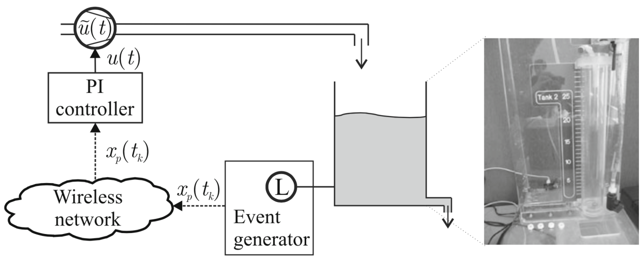
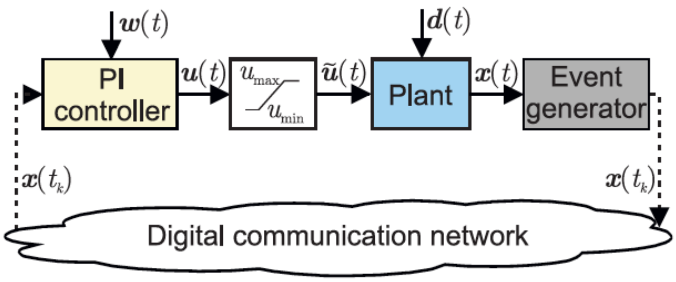
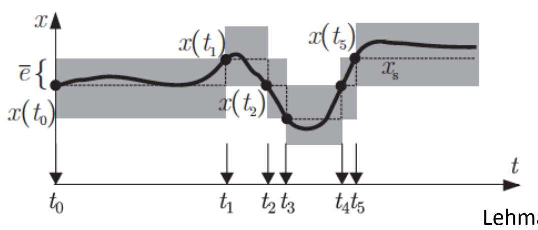
Event-Based Wireless PI Control





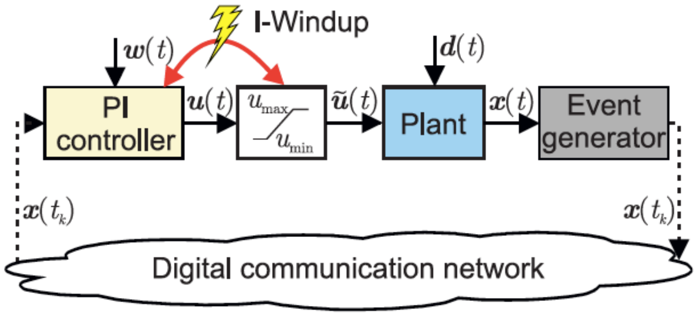
Event-Based PI Control with Saturation





Lehmann et al., 2012

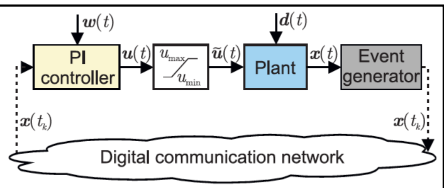
Event-Based PI Control with Saturation



- Industrial applications are generally affected by actuator limitations.

1. Does actuator saturation affect event-triggered PI control?
2. Under what conditions can we guarantee stability?
3. How to overcome potential effects of actuator saturation?

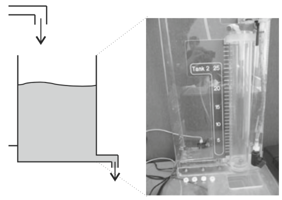
Example



- Plant:

$$\begin{aligned} \dot{x}(t) &= 0.1x(t) + \tilde{u}(t) + 0.1d(t), \quad x(0) = 0 \\ y(t) &= x(t) \end{aligned}$$

- Exogenous signals:



$$\begin{aligned} w(t) &= \bar{w} = 1.5 \\ d(t) &= \bar{d} = 0.1 \end{aligned}$$

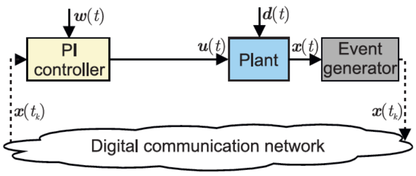
- Actuator saturation:

$$\tilde{u}(t) = \begin{cases} 0.4, & \text{for } u(t) > 0.4; \\ u(t), & \text{for } -0.4 \le u(t) \le 0.4; \\ -0.4, & \text{for } u(t) < -0.4; \end{cases}$$

- PI controller

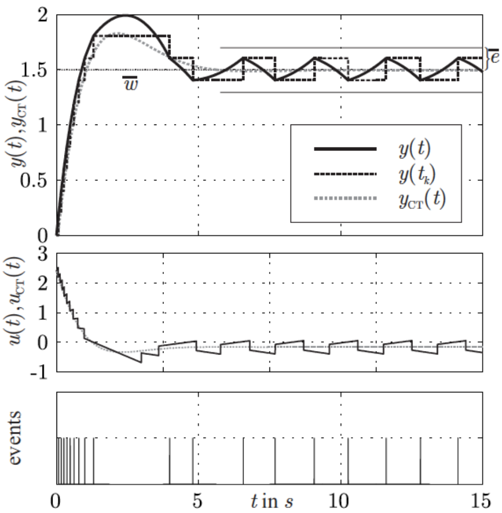
$$\begin{aligned} \dot{x}_I(t) &= y(t) - w(t), \quad x_I(0) = 0 \\ u(t) &= -x_I(t) - 1.6(y(t) - w(t)) \end{aligned}$$

Example: Without Saturation

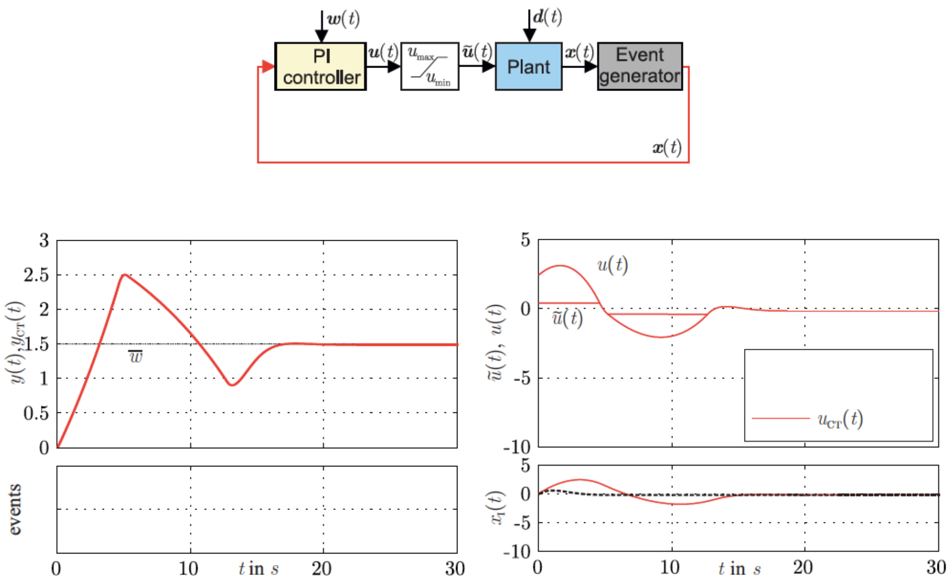


Event generator invokes a sensor transmission whenever output error reach a predefined fixed threshold:

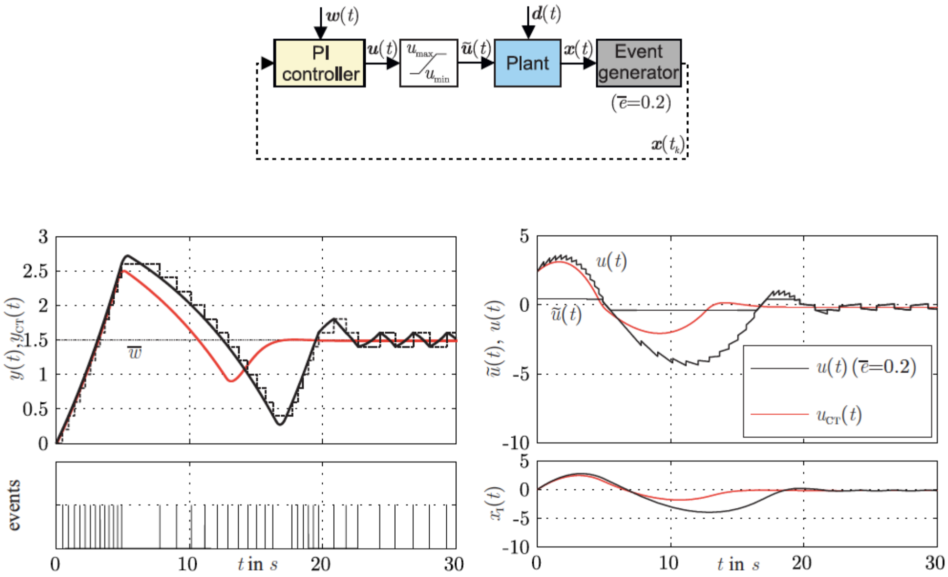
$$|e(t)| = |y(t) - y(t_k)| = \bar{e}.$$



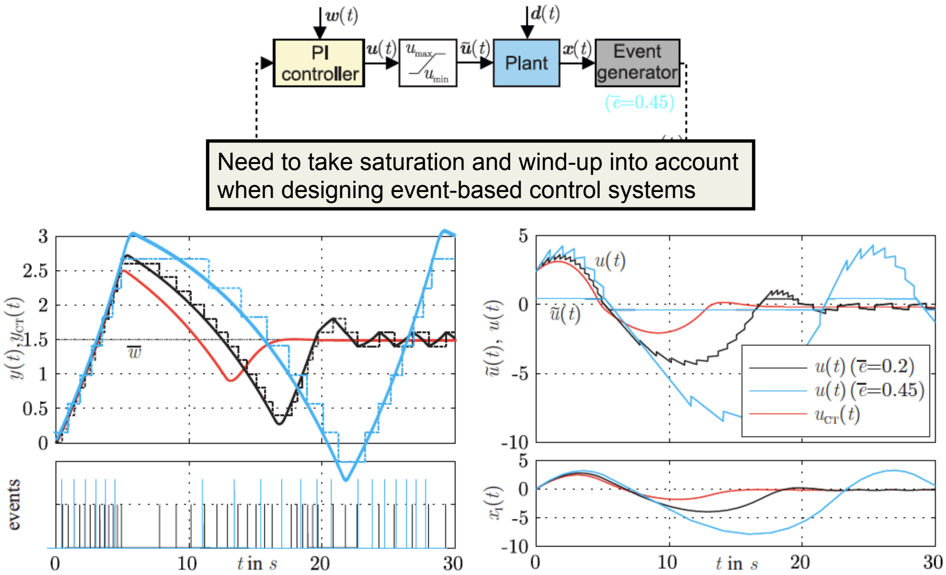
Motivating Example



Motivating Example



Motivating Example



Mathematical Model

- Plant:

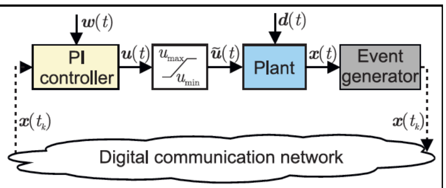
$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\tilde{\boldsymbol{u}}(t) + \boldsymbol{E}\boldsymbol{d}(t), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \\ \tilde{\boldsymbol{u}}(t) &= \mathbf{sat}(\boldsymbol{u}(t)) \\ \mathrm{sat}(u_i(t)) &= \begin{cases} u_0, & \text{for } u_i(t) > u_0 \\ u_i(t), & \text{for } -u_0 \le u(t) \le u_0 \quad \forall i \in \{1,2,..,m\} \\ -u_0, & \text{for } u_i(t) < -u_0 \end{cases}\end{aligned}$$

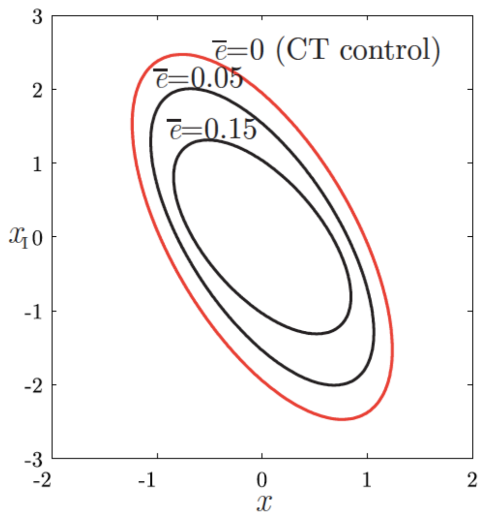
- Event generator: $\|\boldsymbol{x}(t) - \boldsymbol{x}(t_k)\| = \bar{e}$
- PI controller:

$$\begin{aligned}\dot{\boldsymbol{x}}_{\mathrm{I}}(t) &= \boldsymbol{x}(t) - \boldsymbol{e}(t) - \boldsymbol{w}(t), \quad \boldsymbol{x}_{\mathrm{I}}(0) = \boldsymbol{x}_0 \\ \boldsymbol{u}(t) &= \boldsymbol{K}_{\mathrm{I}}\boldsymbol{x}_{\mathrm{I}}(t) + \boldsymbol{K}_{\mathrm{P}}(\boldsymbol{x}(t) - \boldsymbol{e}(t) - \boldsymbol{w}(t))\end{aligned}$$

- **State error:** $e(t) = \boldsymbol{x}(t) - \boldsymbol{x}(t_k)$
- For the sake of simplicity: $\boldsymbol{w}(t) = \boldsymbol{d}(t) = \mathbf{0}$

Stability Regions

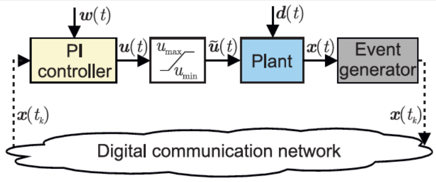




LMI condition to estimate region of stability

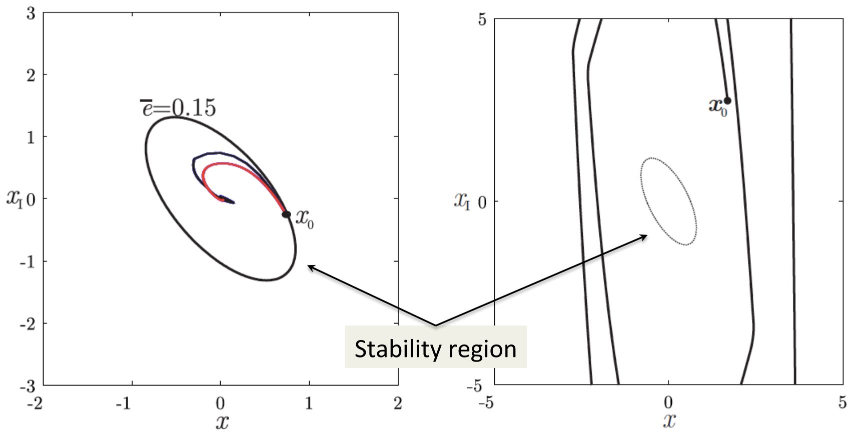
Lehmann et al., 2012

Simulations

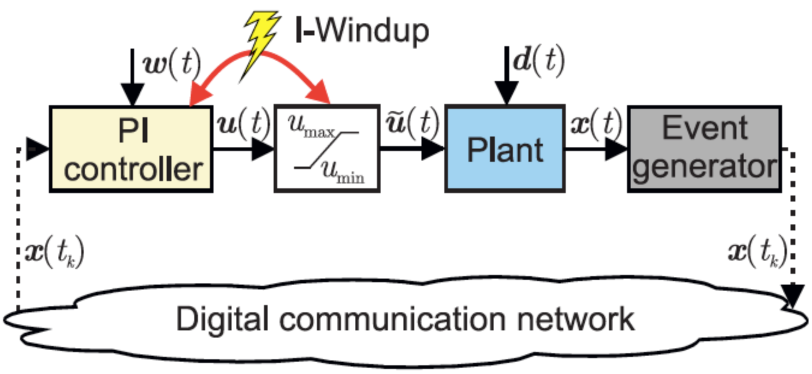


Stable trajectory

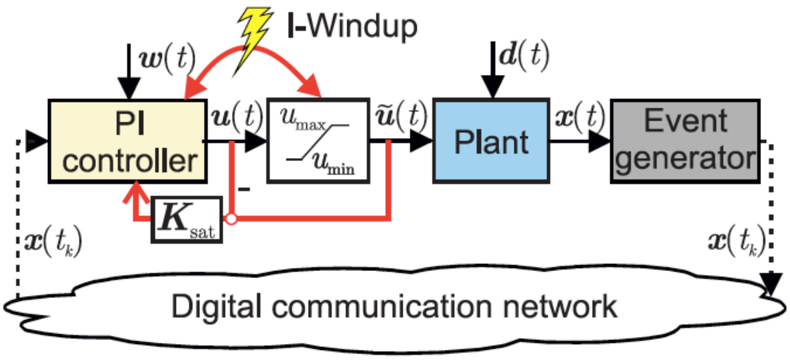
Unstable trajectory



Anti-Windup for Event-Based Control

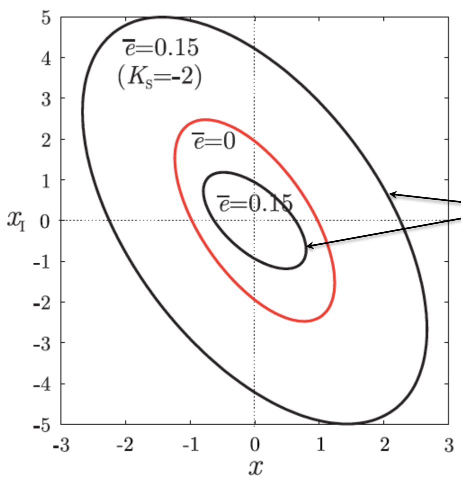


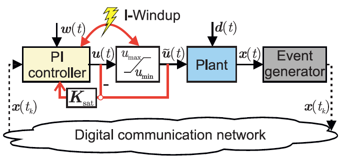
Anti-Windup for Event-Based Control



Cf., anti-windup for conventional control systems [Åström & Hägglund, 1995]

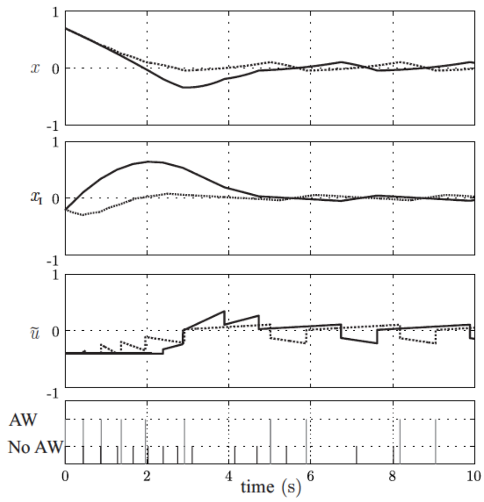
Stability Regions with Anti-Windup



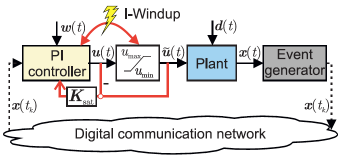


Anti-windup increases the region of stability

System Evolution with Anti-Windup

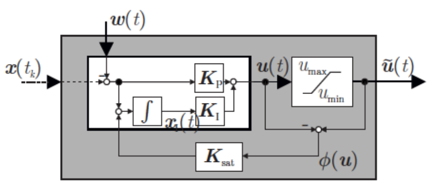


(solid line: no anti windup, dotted line: anti windup)

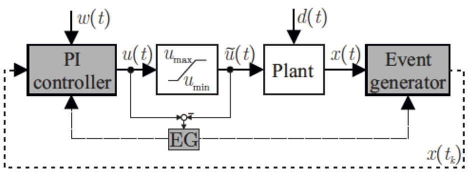


Anti-windup improves the system response

Event-Based Communication for Anti-Windup



Anti-windup event generated when actuator saturates (ETAW)



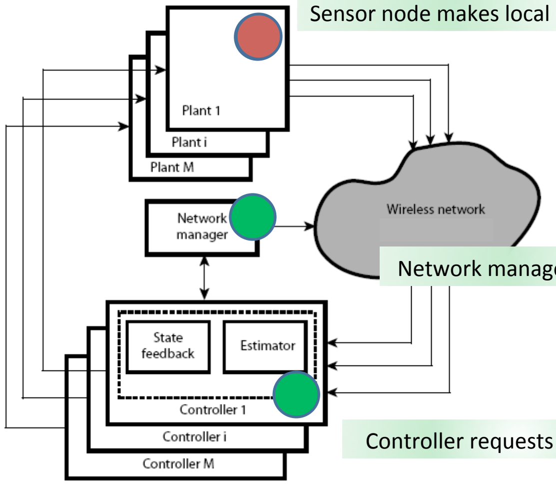
$|u(t) - u(t_k)| = \bar{e}_c$ if $\tilde{u}(t) - u(t) \neq 0$

Outline

- Introduction
- Stochastic event-based control
- Optimal event-based control
- Distributed event-based control
- Event-based anti-windup
- Conclusions

Open Problem on Event-Based Control: Where and When to Take Actions?

Sensor node makes local decisions on when to communicate

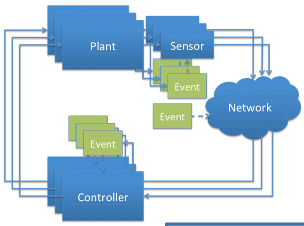


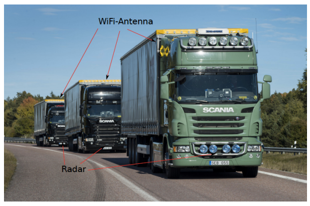
Network manager allocates communication slots

Controller requests sensor data

Conclusions

- Event-based control to handle limited **CPS** resources
- Hard to **jointly optimize** event condition and control law
- Certain **architectures** lead to strong results
- **Event-based control** of multi-agent systems
- Event-based **revisions** of classical control architectures: event-based anti-windup, feedforward, cascade control





http://people.kth.se/~kallej

Additional material

- Distributed event-based control
- Event-based anti-windup

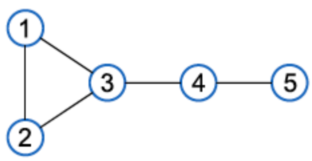
Extension to double-integrator agents

Multi-agent system model

- $\dot{\xi}_i(t) = \zeta_i(t)$
  $\dot{\zeta}_i(t) = u_i(t)$
- communication graph G

Objective: Average consensus

$$\zeta_i(t) \xrightarrow{t\to\infty} \frac{1}{N}\sum_{i=1}^{N} \zeta_i(0) = b$$
$$\xi_i(t) \xrightarrow{t\to\infty} \frac{1}{N}\sum_{i=1}^{N} \xi_i(0) + bt$$



Consensus protocol

$$u(t) = -L\xi(t) - \mu L\zeta(t)$$

Closed-loop dynamics

$$\begin{bmatrix}\dot{\xi}\\ \dot{\zeta}\end{bmatrix} = \underbrace{\begin{bmatrix}0 & I\\ -L & -\mu L\end{bmatrix}}_{\Gamma}\begin{bmatrix}\xi\\ \zeta\end{bmatrix}$$

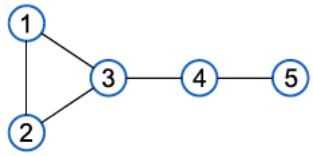
Event-based implementation

Multi-agent system model

- $\dot{\xi}_i(t) = \zeta_i(t)$
  $\dot{\zeta}_i(t) = u_i(t)$
- communication graph G

Consensus protocol

$u(t) = -L\xi(t) - \mu L\zeta(t)$



$$u(t) = -L\left(\hat{\xi}(t) + \text{diag}(t - t_k^1, ..., t - t_k^N)\hat{\zeta}(t)\right) - \mu L\hat{\zeta}(t)$$

$$\hat{\xi}_i(t) = \xi_i(t_k^i),\ \hat{\zeta}_i(t) = \zeta_i(t_k^i) \text{ for } t \in [t_k^i, t_{k+1}^i[$$

Measurement errors

- $e_{\xi,i}(t) = (\hat{\xi}_i(t) + (t - t_k^i)\hat{\zeta}_i(t)) - \xi_i(t)$
- $e_{\zeta,i}(t) = \hat{\zeta}_i(t) - \zeta_i(t)$

Event-based control for double-integrator agents

$$\begin{array}{l}\dot{\xi}(t) = \zeta(t) \\ \dot{\zeta}(t) = u(t)\end{array}, \quad u(t) = -L\left(\hat{\xi}(t) + \text{diag}(t - t_k^1, ..., t - t_k^N)\hat{\zeta}(t)\right) - \mu L\hat{\zeta}(t) \quad (2)$$

Theorem (double-integrator agents)

Consider system (2) with undirected connected graph G. Suppose that

$$f_i(t, e_{\xi,i}(t), e_{\zeta,i}(t)) = \left\| \begin{bmatrix} e_{\xi,i}(t) \\ \mu e_{\zeta,i}(t) \end{bmatrix} \right\| - \left(c_0 + c_1 e^{-\alpha t}\right),$$

with $c_0, c_1 \geq 0$, at least one positive, and $0 < \alpha < |\Re(\lambda_3(\Gamma))|$. Then, for all $\xi_0, \zeta_0 \in \mathbb{R}^N$, the system does not exhibit Zeno behavior and for $t \to \infty$,

$$\|\delta(t)\| \leq c_0 c_V \frac{\lambda_N(L)}{|\Re(\lambda_3(\Gamma))|}\sqrt{2N}.$$

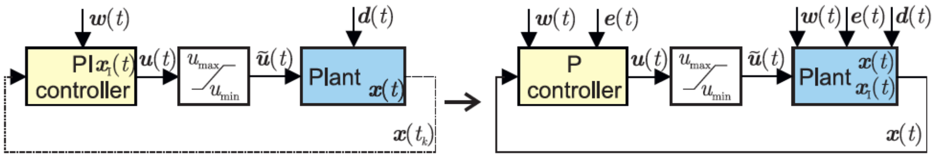
Mathematical model

Augmented state vector:

$$x_a(t) = \begin{pmatrix} x(t) \\ x_I(t) \end{pmatrix}$$

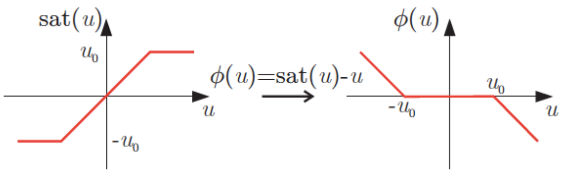
State-space model of the event-triggered PI-control loop:

$$\begin{aligned} \dot{x}_a(t) &= A_I x_a(t) + B_I \mathrm{sat}(K_I x_I(t) + K_P(x(t) - e(t))) - F_I e(t) \\ x_a(0) &= x_{a0} \end{aligned}$$



Transformation of saturation nonlinearity

$$\phi(u) = \mathrm{sat}(u) - u$$



Transformed state-space model of the event-triggered PI-control loop:

$$\begin{aligned} \dot{x}_a(t) &= \bar{A}_I x_a(t) + B_I \phi(K x_a(t) - K_P e(t)) - F_I e(t) \\ x_a(0) &= x_{a0} \end{aligned}$$

$$\bar{A}_I = \begin{pmatrix} A + BK_P & BK_I \\ I & O \end{pmatrix}; B_I = \begin{pmatrix} B \\ O \end{pmatrix}; F_I = \begin{pmatrix} BK_P \\ I \end{pmatrix}; K = \begin{pmatrix} K_P & K_I \end{pmatrix}$$

Nonlinearity transformation enables tighter stability conditions [Tarbouriech et al, 2006]

Theorem: Region of stability

If there exist a symmetric positive definite matrix $\boldsymbol{W}$, a positive definite diagonal matrix $\boldsymbol{S}$, a matrix $\boldsymbol{Z}$, a positive scalar η and two a priori fixed positive scalars τ_1 and τ_2 satisfying

$$\begin{bmatrix} \boldsymbol{W}\bar{\boldsymbol{A}}_{\mathrm{I}}^T + \bar{\boldsymbol{A}}_{\mathrm{I}}\boldsymbol{W} + \tau_1\boldsymbol{W} & \boldsymbol{B}_{\mathrm{I}}\boldsymbol{S} - \boldsymbol{W}\boldsymbol{K}^T - \boldsymbol{Z}^T & -\boldsymbol{F}_{\mathrm{I}} \\ \star & -2\boldsymbol{S} & -\boldsymbol{K}_{\mathrm{P}} \\ \star & \star & -\tau_2\boldsymbol{R} \end{bmatrix} < 0$$

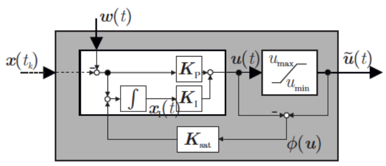
$$-\tau_1\delta + \tau_2\eta < 0$$

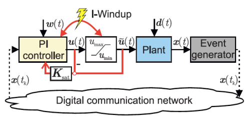
$$\begin{bmatrix} \boldsymbol{W} & \boldsymbol{Z}_i^T \\ \star & \eta u_0^2 \end{bmatrix} \geq 0, i \in 1, ..., m$$

then for $\boldsymbol{e} \in \mathcal{W} = \{\boldsymbol{e} : \boldsymbol{e}^T\boldsymbol{R}\boldsymbol{e} = \delta^{-1}\}$ $(\boldsymbol{R} = \boldsymbol{I}, \delta^{-1} = \bar{e}^2)$ the ellipsoid $\mathcal{E} = \{\boldsymbol{x}_{\mathrm{a}} : \boldsymbol{x}_{\mathrm{a}}^T\boldsymbol{P}\boldsymbol{x}_{\mathrm{a}} = \eta^{-1}\}$, with $\boldsymbol{P} = \boldsymbol{W}^{-1}$, is a region of stability.

- Computational tool to estimate region of stability for saturated event-based control
- Extends results for continuous-time systems [Tarbouriech; Zaccarian & Teel, 2011]

Anti-windup for event-based PI control





- Adapted dynamics of the controller state:

$$\dot{\boldsymbol{x}}_{\mathrm{I}}(t) \quad = \quad \boldsymbol{x}(t) - \boldsymbol{e}(t) - \boldsymbol{w}(t) + \boldsymbol{K}_{\mathrm{sat}}\phi(\boldsymbol{u}), \quad \boldsymbol{x}_{\mathrm{I}}(0) = \boldsymbol{x}_{\mathrm{I}0}$$

- Transformed state-space model of the event-triggered PI-control loop:

$$\dot{\boldsymbol{x}}_{\mathrm{a}}(t) \quad = \quad \bar{\boldsymbol{A}}_{\mathrm{I}}\boldsymbol{x}_{\mathrm{a}}(t) + \boldsymbol{B}_{\mathrm{I}}\phi(\boldsymbol{K}\boldsymbol{x}_{\mathrm{a}}(t) - \boldsymbol{K}_{\mathrm{P}}\boldsymbol{e}(t)) - \boldsymbol{F}_{\mathrm{I}}\boldsymbol{e}(t), \quad \boldsymbol{x}_{\mathrm{a}}(0) = \boldsymbol{x}_{\mathrm{a}0}$$

$$\bar{\boldsymbol{A}}_{\mathrm{I}} = \begin{pmatrix} \boldsymbol{A} + \boldsymbol{B}\boldsymbol{K}_{\mathrm{P}} & \boldsymbol{B}\boldsymbol{K}_{\mathrm{I}} \\ \boldsymbol{I} & \boldsymbol{O} \end{pmatrix}; \boldsymbol{B}_{\mathrm{I}} = \begin{pmatrix} \boldsymbol{B} \\ \boldsymbol{K}_{\mathrm{sat}} \end{pmatrix}; \boldsymbol{F}_{\mathrm{I}} = \begin{pmatrix} \boldsymbol{B}\boldsymbol{K}_{\mathrm{P}} \\ \boldsymbol{I} \end{pmatrix}; \boldsymbol{K} = \begin{pmatrix} \boldsymbol{K}_{\mathrm{P}} & \boldsymbol{K}_{\mathrm{I}} \end{pmatrix}$$