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Distributed Macro Calibration in Sensor Networks

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Abstract— **In this paper a novel consensus-based distributed algorithm for blind macro-calibration in sensor networks is proposed. It is proved, on the basis of an originally developed methodology for treating higher order consensus schemes, that the algorithm achieves asymptotic agreement for sensor gains and offsets in the mean square sense and with probability one. In the case of a given reference, all sensors are asymptotically calibrated. Simulation results illustrate properties of the algorithm.**

I. Introduction

Recently, wireless sensor networks (WSN) have emerged as an important research area (see, *e.g.*, [1], [2], [3]). Diverse new applications have sparked the recognition of new classes of problems for the developers. Calibration represents one of the most important challenges in this respect, having in mind that numerous WSNs are today characterized by a large number of sensors. Relatively small sensor systems are built for *micro-calibration*, in which each device is individually tuned in a carefully controlled environment. Larger sensor networks, however, demand new methods of calibration, since many devices can often be in partially unobservable and dynamic environments, or may even be inaccessible. *Macro-calibration* is based on the idea to calibrate a network as a whole by observing only the overall system response, thus eliminating the need to directly calibrate each and every device. The usual prerequisite is to frame calibration as a parameter estimation problem, in which the parameters have to be chosen in such a way as to optimize the overall system response [4]. Automatic methods for jointly calibrating sensor networks in the field, without dependence on controlled stimuli or high-fidelity groundtruth data, is of significant interest. This problem is referred to as *blind calibration* [5]. One approach to blind calibration of sensor networks is to begin by assuming that the deployment is very dense, so that neighboring nodes have (in principle) nearly identical readings. There are also methods trying to cope with situations in which sensor network deployments may not meet the density requirements [6].

In this paper we propose a novel collaborative blind macro-calibration method for sensor networks based on distributed on-line estimation of the parameters of local linear calibration functions (adjusting both gains and offsets). It is assumed that the sensors form a network based only on communications between neighboring nodes, that the real measured signal is not directly accessible and that no reference sensor is identified. It will be demonstrated that the overall network behavior can be treated as a generalized *consensus* problem, in which all the equivalent sensor gains and offsets should converge asymptotically to equal values. Classical results related to different versions of the dynamic consensus algorithm are not applicable to this case (see, *e.g.*, [7]). Note also that, to the authors' knowledge, consensus has been applied to the calibration problems only in [8], [9], but within different contexts.

Using basic arguments derived from stability of diagonally dominant dynamic systems decomposed into non-overlapping subsystems [10], [11], it is proved that the proposed algorithm provides asymptotic consensus in the mean square sense and with probability one under mild conditions involving signal properties and real sensor characteristics. In the case when at least one node is selected as reference, the algorithm provides convergence to the desired parameters in the mean square sense and with probability one. Simulation results illustrate the efficiency of the proposed algorithm.

The outline of the paper is as follows. In Section II we formulate the calibration problem and introduce the basic algorithm. Section III is devoted to the algorithm's convergence analysis under different assumptions on the measured signals and network structure. In Section IV we present simulation results.

II. Problem Formulation and the Basic Algorithm

Consider n distributed sensors measuring the same discrete-time signal $x(t)$, $t = \ldots, -1, 0, 1, \ldots$, which is supposed to be a realization of a random process $\{x(t)\}$. Assume that the i-th sensor generates at its output the signal

$$y_i(t) = \alpha_i x(t) + \beta_i \tag{1}$$

where the gain α_i and the offset β_i are unknown constants.

By sensor calibration we consider application of the *calibration function* which produces the overall output

$$z_i(t) = a_i y_i(t) + b_i = a_i \alpha_i x(t) + a_i \beta_i + b_i = g_i x(t) + f_i. \tag{2}$$

The calibration parameters a_i and b_i have to be chosen in such a way as to set the equivalent gain g_i as close as possible to one and the equivalent offset f_i as close as possible to zero.

We assume that the observed sensors form a network with a predefined structure, represented by a directed graph $\mathcal{G} = (\mathcal{U}, \mathcal{V})$, where $\mathcal{U}$ is the set of nodes (one node corresponds

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to one sensor) and $\mathcal{V}$ the set of arcs. The adjacency matrix $C = [c_{ij}]$, $i, j = 1, \ldots, n$, is defined in such a way that $c_{ij} = 1$ when the j-th sensor can send its message to the i-th sensor; otherwise, $c_{ij} = 0$.

The aim of this paper is to propose an algorithm for distributed real-time estimation of the calibration parameters a_i and b_i which provides: a) asymptotically equal outputs $z_i(t)$ of all the sensors in the case when no reference signal or ideal sensor is given or identified; b) ideal asymptotic calibration of all the sensors ($g_i = 1$ and $f_i = 0$) in the case when at least one sensor is a priori known to have ideal (or desired) characteristics. In the first case it is expected that the majority of well calibrated sensors correct the behavior of those that are not, on the basis of global consensus.

a) Assuming first that no reference is given, the distributed calibration algorithm is derived starting from minimization of the set of instantaneous criteria

$$J_i = \sum_{j \in \mathcal{N}_i} \gamma_{ij} (z_j(t) - z_i(t))^2, \tag{3}$$

$i = 1, \ldots, n$, where $\mathcal{N}_i$ is the set of neighboring nodes of the i-th node (the sensors able to send information to the i-th sensor), and γ_{ij} are nonnegative scalar weights reflecting the relative importance of the neighboring nodes. If θ_i=$[a_i \;\; b_i]^T$, we obtain that

$$\mathrm{grad}_{\theta_i} J_i = \sum_{j \in \mathcal{N}_i} \gamma_{ij} (z_j(t) - z_i(t)) \begin{bmatrix} y_i(t) \\ 1 \end{bmatrix}. \tag{4}$$

The last equation gives rise to the standard possibility to replace θ_i by its estimate $\hat{\theta}_i(t)$ and $\mathrm{grad}_{\theta_i} J_i$ by its realizations, and to construct in such a way the following recursions of stochastic gradient type:

$$\hat{\theta}_i(t+1) = \hat{\theta}_i(t) + \delta_i(t) \sum_{j \in \mathcal{N}_i} \gamma_{ij} \epsilon_{ij}(t) \begin{bmatrix} y_i(t) \\ 1 \end{bmatrix}, \tag{5}$$

where $\hat{\theta}_i(t)$=$[\hat{a}_i(t) \;\; \hat{b}_i(t)]^T$, $\delta_i(t) > 0$ is a time varying gain influencing convergence properties of the algorithm, $\epsilon_{ij}(t) = \hat{z}_j(t) - \hat{z}_i(t)$ and $\hat{z}_i(t) = \hat{a}_i(t) y_i(t) + \hat{b}_i(t)$, with the initial conditions $\hat{\theta}_i(0) = [1 \;\; 0]^T$, $i = 1, \ldots, n$. Notice that each iteration of the algorithm subsumes reception of the current outputs of the neighboring nodes, as well as the local measurement. The main idea is to ensure that the estimates of all the local gains $\hat{g}_i(t) = \hat{a}_i(t)\alpha_i$ and offsets $\hat{f}_i(t) = \hat{a}_i(t)\beta_i + \hat{b}_i(t)$ tend asymptotically to the same values $\bar{g}$ and $\bar{f}$, implying $\hat{z}_j(t) = \hat{z}_i(t)$, $i, j = 1, \ldots, n$, *i.e.*, minimization of all the criteria J_i.

Introduce

$$\hat{\rho}_i(t) = \begin{bmatrix} \hat{g}_i(t) \\ \hat{f}_i(t) \end{bmatrix} = \begin{bmatrix} \alpha_i & 0 \\ \beta_i & 1 \end{bmatrix} \hat{\theta}_i(t), \tag{6}$$

and

$$\epsilon_{ij}(t) = \begin{bmatrix} x(t) & 1 \end{bmatrix} (\hat{\rho}_j(t) - \hat{\rho}_i(t)), \tag{7}$$

so that (5) becomes

$$\hat{\rho}_i(t+1) = \hat{\rho}_i(t) + \delta_i(t) \sum_{j \in \mathcal{N}_i} \gamma_{ij} \Phi_i(t) (\hat{\rho}_j(t) - \hat{\rho}_i(t)), \tag{8}$$

where

$$\Phi_i(t) = \begin{bmatrix} \alpha_i y_i(t) x(t) & \alpha_i y_i(t) \\ [1 + \beta_i y_i(t)] x(t) & 1 + \beta_i y_i(t) \end{bmatrix} \tag{9}$$
$$= \begin{bmatrix} \alpha_i \beta_i x(t) + \alpha_i^2 x(t)^2 & \alpha_i \beta_i + \alpha_i^2 x(t) \\ (1 + \beta_i^2) x(t) + \alpha_i \beta_i x(t)^2 & 1 + \beta_i^2 + \alpha_i \beta_i x(t) \end{bmatrix},$$

with the initial conditions $\hat{\rho}_i(0) = [\alpha_i \;\; \beta_i]^T$, $i = 1, \ldots, n$. Recursions (8) can be represented in the following compact form

$$\hat{\rho}(t+1) = [I + (\Delta(t) \otimes I_2) B(t)] \hat{\rho}(t), \tag{10}$$

where $\hat{\rho}(t) = [\hat{\rho}_1(t)^T \cdots \hat{\rho}_n(t)^T]^T$, $\Delta(t) = \mathrm{diag}\{\delta_1(t), \cdots, \delta_n(t)\}$,

$$B(t) = \Phi(t)(\Gamma \otimes I_2),$$

$\Phi(t) = \mathrm{diag}\{\Phi_1(t), \ldots, \Phi_n(t)\}$, $\otimes$ denotes the Kronecker's product, I_2 is the 2×2 unit matrix and

$$\Gamma = \begin{bmatrix} -\sum_{j, j \neq 1} \gamma_{1j} & \gamma_{12} & \cdots & \gamma_{1n} \\ \gamma_{21} & -\sum_{j, j \neq 2} \gamma_{2j} & \cdots & \gamma_{2n} \\ & & \cdots & \\ \gamma_{n1} & \gamma_{n2} & \cdots & -\sum_{j, j \neq n} \gamma_{nj} \end{bmatrix},$$

where $\gamma_{ij} = 0$ if $j \notin \mathcal{N}_i$. The initial condition is $\hat{\rho}(0) = [\hat{\rho}_1(0)^T \cdots \hat{\rho}_n(0)^T]^T$, in accordance with (8). The desirable asymptotic value of $\hat{\rho}(t)$ should be based on such a specific type of consensus which implies that the components of $\hat{\rho}(t)$ with odd indices (representing gains) and the components with even indices (representing offsets) have equal values.

b) In the case when it is a priori known that one of the sensors has ideal (or desirable) characteristics, the whole calibration network can be "pinned" to that sensor. Choosing the k-th sensor, $k \in \{1, \ldots, n\}$, as ideal, we simply eliminate the k-th recursion, *i.e.*, in (5) we set

$$\hat{\theta}_k(t+1) = \hat{\theta}_k(t) \tag{11}$$

with $\hat{\theta}_k(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\alpha_k = 1, \beta_k = 0$, and leave the remaining recursions unchanged (any predefined α_k and β_k can be chosen). The corresponding modification in the compact form (10) simply consists of setting to zero all the block matrices in the k-th block row of $B(t)$. It will be proved below that the resulting algorithm ensures convergence of $\hat{\rho}_i(t)$, $i = 1, \ldots, n$, $i \neq k$, to the same ideal vector $\hat{\rho}_k(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

III. Convergence Analysis

We are concerned with the structural properties of the algorithm and we assume no communication and/or measurement errors; also, we assume that:

A1) $\delta_i(t) = \delta = \mathrm{const}$, $i = 1, \ldots, n$;

A2) $\{x(t)\}$ is i.i.d., with $E\{x(t)\} = \bar{x} < \infty$ and $E\{x(t)^2\} = s^2 < \infty$.

Assumption A2) is not essential. It only allows a more direct insight into the basic structural properties of the algorithm, and will be relaxed at the end of the section.

Based on A1) and A2) we obtain

$$\bar{\rho}(t+1) = (I + \delta\bar{B})\bar{\rho}(t), \tag{12}$$

where $\bar{\rho}(t) = E\{\rho(t)\}$, $\bar{\rho}(0) = \rho(0)$, $\bar{B} = \bar{\Phi}(\Gamma \otimes I_2)$ and $\bar{\Phi} = E\{\Phi(t)\} = \mathrm{diag}\{\bar{\Phi}_1 \dots \bar{\Phi}_n\}$, with

$$\bar{\Phi}_i = \begin{bmatrix} \alpha_i\beta_i\bar{x} + \alpha_i^2 s^2 & \alpha_i\beta_i + \alpha_i^2\bar{x} \\ (1+\beta_i^2)\bar{x} + \alpha_i\beta_i s^2 & 1 + \beta_i^2 + \alpha_i\beta_i\bar{x} \end{bmatrix}.$$

We first pay attention to the asymptotic properties of (12). The well known results related to the classical consensus schemes, *e.g.*, [7], cannot be directly applied here, having in mind the specific structure of $\bar{B}$ composed of 2×2 block matrices. Our analysis will be based on several basic lemmas derived using the results related to the diagonal dominance of matrices decomposed into blocks [10], [11] .

Lemma 1: [10], [12] A matrix $A = [A_{ij}]$, where $A_{ij} \in \mathcal{C}^{m\times m}$, $i, j = 1, \dots n$, has quasi-dominating diagonal blocks if the test matrix $W \in R^{n\times n}$, with the elements

$$w_{ij} = 1 \;\; (i = j); \quad w_{ij} = -\|A_{ii}^{-1}A_{ij}\| \;\; (i \neq j)$$

is an M-matrix ($\|.\|$ denotes an operator norm). As a consequence, A is nonsingular. If $A - \lambda I$ has quasi-dominating diagonal blocks for all $\lambda \in \mathcal{C}_+$, then A is Hurwitz ($\mathcal{C}_+$ denotes the set of complex numbers with nonnegative real parts). ■

Lemma 2: If A has quasi-dominating diagonal blocks and A_{ii}, $i = 1, \dots, n$, are Hurwitz, A is also Hurwitz.

Proof: If A_{ii} is Hurwitz, then there exists a positive definite matrix D, such that $A_{ii}D + DA_{ii}^* = -Q_D$, where Q_D is positive definite. Define the following operator norm of a matrix $X \in C^{m\times m}$

$$\|X\| = \sup_{x\neq 0} \|Xx\|_D / \|x\|_D,$$

where $x \in C^m$, and $\|x\|_D = (x^*D^{-1}x)^{\frac{1}{2}}$, while $D > 0$ is such that $Q_D > 0$. Using this norm in the definition of the corresponding matrix W in Lemma 1, for its off-diagonal elements we have

$$\lambda_{max}(A_{ij}^*A_{ii}^{-1*}D^{-1}A_{ii}^{-1}A_{ij}) = \max_{x\neq 0} \frac{x^*A_{ij}A_{ij}^*x}{x^*A_{ii}DA_{ii}^*x}. \tag{13}$$

According to Lemma 1, A is Hurwitz if $A - \lambda I$ has quasi-dominating diagonal blocks for all $\lambda \in \mathcal{C}_+$, which is satisfied if the following holds

$$x^*(A_{ii} - \lambda I)D(A_{ii} - \lambda I)^*x \geq x^*A_{ii}DA_{ii}^*x \tag{14}$$

for all $\lambda \in \mathcal{C}_+$ since this guarantees that the corresponding matrix $W(\lambda)$ (with the above norm) is an M-matrix for all λ. Let $\lambda = \sigma + j\mu$ be a complex number with a nonnegative real part. Then, we have

$$\begin{aligned} H &= x^*(A_{ii} - \lambda I)D(A_{ii} - \lambda I)^*x \\ &\geq x^*A_{ii}DA_{ii}^*x + x^*(A_{ii}Dj - DA_{ii}^*j\mu)x\mu \\ &\quad - x^*\sigma(A_{ii}D + DA_{ii}^*)x + \mu^2\lambda_{min}(D)x^*x. \end{aligned} \tag{15}$$

As $x^*(A_{ii}D - DA_{ii}^*)x = 0$ and $x^*\sigma(A_{ii}D + DA_{ii}^*)x \leq 0$ for $\sigma \geq 0$ according to the assumption of the Lemma, we have that $H \geq x^*A_{ii}DA_{ii}^*x$. Hence, the result follows. ■

We now come back to the matrix $\bar{B}$ in (12), and analyze its properties under the following standard assumption:

A3) the graph $\mathcal{G}$ has a spanning tree.

This assumption implies, according to the results in, *e.g.*, [7], that the matrix Γ has one eigenvalue at the origin and the other eigenvalues have negative real parts.

Lemma 3: Let assumption A3) be satisfied and let the i-th node be a center node of $\mathcal{G}$. Then, the matrix $\Gamma' \in R^{(n-1)\times(n-1)}$, obtained from Γ by deleting its i-th row and its i-th column, is nonsingular.

Proof: Let $W^\Gamma = [w_{ij}^\Gamma]$, where $w_{ij}^\Gamma = 0$ for $i = j$, and $w_{ij}^\Gamma = (\sum_{j=1, j\neq i}^n \gamma_{ij})^{-1}\gamma_{ij}$ for $i \neq j$. This matrix is row stochastic and cogredient (amenable by permutation transformations) to

$$W_c^\Gamma = \begin{bmatrix} W_1^\Gamma & 0 \\ W_2^\Gamma & W_0^\Gamma \end{bmatrix}, \tag{16}$$

where $W_1^\Gamma \in R^{n_1\times n_1}$ is an irreducible matrix, $W_2^\Gamma \in R^{n_2\times n_1} \neq 0$ and $W_0^\Gamma \in R^{n_2\times n_2}$ is such that $\max_i \lambda_i \{W_0^\Gamma\} < 1$. Eliminating one of the center nodes from $\mathcal{G}$ means deleting the i-th row and the i-th column of W_c^Γ, where $1 \leq i \leq n_1$. As matrix W_1^Γ in (16) corresponds to a closed strong component of $\mathcal{G}$, it is easy to observe that deleting one node from it (together with the corresponding edges) results into a graph containing, in general, κ closed strong components ($\kappa \geq 1$). However, there is at least one row in each of the weighted adjacency matrices of these closed strong components in which the sum of all the elements becomes strictly less than one (as a consequence of the elimination of the edges leading to at least one node per the resulting strong component). Using the arguments from [10], [13], it is possible to conclude that the matrix $I - W_1^{\Gamma-}$, where $W_1^{\Gamma-}$ is obtained from W_1^Γ after deleting its i-th row and i-th column, is an M-matrix. Consequently, in general, one concludes that $I - W_c^{\Gamma-}$, where $W_c^{\Gamma-}$ is obtained from W_c^Γ after deleting its i-th row and i-th column, is also and M-matrix, and, therefore, Γ' is nonsingular according to Lemma 1. ■

Consequently, matrix $\bar{B}$ from (12) has at least two eigenvalues at the origin. In order to analyze its remaining eigenvalues, select one node of the graph $\mathcal{G}$ from the set of center nodes, *i.e.*, of the nodes from which all the nodes in the graph are reachable (suppose without loss of generality that its index is 1), and delete the corresponding two rows and two columns from $\bar{B}$. The remaining $(2n-2)\times(2n-2)$ matrix is $\bar{B}^- = [\bar{B}_{ij}^-]$, $i, j = 1, \dots, n-1$, where $\bar{B}_{ij}^- = -\sum_{k=2, k\neq i+1}^n \gamma_{i+1,k}\bar{\Phi}_{i+1}$ for $i = j$ and $\bar{B}_{ij}^- = \gamma_{i+1,j+1}\bar{\Phi}_{i+1}$ for $i \neq j$. According to Lemma 1, the corresponding test matrix is $W^{\Gamma-} = [w_{ij}^{\Gamma-}]$, $i, j = 1, \dots, n-1$, where $w_{ij}^{\Gamma-} = 1$ for $i = j$ and $w_{ij}^{\Gamma-} = -(\sum_{k=2, k\neq i+1}^n \gamma_{i+1,k})^{-1}\gamma_{i+1,j+1}$ for $i \neq j$.

Lemma 4: Let Assumption A3) be satisfied and let

A4) $-\bar{\Phi}_i$ is Hurwitz, $i = 1, \dots, n$.

Then, matrix $\bar{B}$ in (12) has two eigenvalues at the origin and the remaining eigenvalues have negative real parts.

Proof: Using the result of Lemma 3, we conclude, according to Lemma 1, that $\bar{B}^-$ has quasi-dominating diagonal blocks (W^- is in this case an M-matrix). According to Lemma 2, this fact together with Assumption A4) directly implies that $\bar{B}^-$ has all the eigenvalues with negative real parts. Thus, the result. ■

Lemma 5: Let $T = \left[i_1 \vdots i_2 \vdots T_{2n\times(2n-2)} \right]$, where $i_1 = [1\ 0\ 1\ 0\ \dots\ 1\ 0]^T$, $i_2 = [0\ 1\ 0\ 1\ \dots\ 0\ 1]^T$ and $T_{2n\times 2n-2}$ is an $2n \times (2n-2)$ matrix, such that span$\{T_{2n\times(2n-2)}\}$= span$\{\bar{B}\}$. Then, T is nonsingular and

$$T^{-1}\bar{B}T = \left[\begin{array}{c:c} 0_{2\times 2} & 0_{2\times(2n-2)} \\ \hdashline 0_{(2n-2)\times 2} & \bar{B}^* \end{array}\right], \tag{17}$$

where $\bar{B}^*$ is Hurwitz and $0_{i\times j}$ represents an $i\times j$ zero matrix.

Proof: The eigenvalue of $\bar{B}$ at the origin has both algebraic and geometric multiplicity equal to two: i_1 and i_2 represent two corresponding linearly independent eigenvectors. The rest of the proof follows from the Jordan decomposition of $\bar{B}$. Notice that

$$T^{-1} = \left[\begin{array}{c} \pi_1 \\ \hdashline \pi_2 \\ \hdashline S_{(2n-2)\times 2n} \end{array}\right], \tag{18}$$

where π_1 and π_2 are the left eigenvectors of $\bar{B}$ corresponding to the eigenvalue at the origin and $S_{(2n-2)\times 2n}$ is defined in accordance with (17). Thus, $\bar{B}^*$ is Hurwitz according to Lemma 4. ■

Theorem 1: Let Assumptions A1), A2), A3) and A4) be satisfied. Then there exists a positive number $\delta' > 0$ such that for all $\delta \le \delta'$ in (12) $\lim_{t\to\infty} \bar{\rho}(t) = \bar{\rho}_\infty = [\bar{\rho}_{\infty 1}^T \cdots \bar{\rho}_{\infty n}^T]^T$, with $\bar{\rho}_{\infty i}^T = \bar{\rho}_{\infty j}^T$, $i, j = 1, \dots, n$.

Proof: Using Lemma 5, we define $\tilde{\bar{\rho}}(t) = [\tilde{\bar{\rho}}_1(t)\ \ \tilde{\bar{\rho}}_2(t) \cdots \tilde{\bar{\rho}}_{2n}(t)]^T = T^{-1}\bar{\rho}(t)$. From (12) we obtain

$$\tilde{\bar{\rho}}(t+1)^{[1]} = \tilde{\bar{\rho}}(t)^{[1]}; \quad \tilde{\bar{\rho}}(t+1)^{[2]} = (I + \delta\bar{B}^*)\tilde{\bar{\rho}}(t)^{[2]}, \tag{19}$$

where $\tilde{\bar{\rho}}(t)^{[1]} = [\tilde{\bar{\rho}}_1(t)\ \ \tilde{\bar{\rho}}_2(t)]^T$, $\tilde{\bar{\rho}}(t)^{[2]} = [\tilde{\bar{\rho}}_3(t) \cdots \tilde{\bar{\rho}}_{2n}(t)]^T$. Having in mind the above results, we see immediately that for δ small enough all the eigenvalues of $I + \delta\bar{B}^*$ lie within the unit circle. Therefore, $\lim_{t\to\infty} \tilde{\bar{\rho}}(t)^{[2]} = 0$, so that

$$\lim_{t\to\infty} \tilde{\bar{\rho}}(t) = \tilde{\bar{\rho}}_\infty^T = [\tilde{\bar{\rho}}(0)^{[1]T} 0 \cdots 0]^T.$$

Consequently,

$$\bar{\rho}_\infty = T[\tilde{\bar{\rho}}(0)^{[1]T} 0 \cdots 0]^T = (i_1\pi_1 + i_2\pi_2)\bar{\rho}(0). \tag{20}$$

Having in mind the definition of i_1 and i_2, we conclude that $\bar{\rho}_{\infty 1} = \cdots = \bar{\rho}_{\infty(2n-1)}$ and $\bar{\rho}_{\infty 2} = \cdots = \bar{\rho}_{\infty(2n)}$. Obviously, this also shows that $\lim_{t\to\infty}(I + \delta\bar{B})^t = i_1\pi_1 + i_2\pi_2$. Thus, the result follows. ■

We analyze convergence of the basic recursion in (10) using the following lemma.

Lemma 6: Matrix $B(t)$ in (10) satisfies for all t

$$T^{-1}B(t)T = \left[\begin{array}{c:c} 0_{2\times 2} & 0_{2\times(2n-2)} \\ \hdashline 0_{(2n-2)\times 2} & B(t)^* \end{array}\right], \tag{21}$$

where T is defined in (18) and $B(t)^*$ is an $(2n-2)\times(2n-2)$ matrix.

Proof: It is possible to observe immediately that vectors i_1 and i_2 are eigenvectors for both $\bar{B}$ and $B(t)$, taking into account (12) and (10).

Let $w = [\, w_1 \cdots w_{2n}\,]$ be a left eigenvector of $\bar{B}$ corresponding to the zero eigenvalue. Then, $w\bar{B} = 0$ gives :

$$\begin{aligned} &-[w_{2i-1}(\alpha_i\beta_i + \beta_i^2\bar{x}) + w_{2i}(1 + \beta_i^2 + \alpha_i\beta_i\bar{x})]\cdot \\ &\cdot \textstyle\sum_{j=1,j\neq i}^n \gamma_{ji} + \sum_{l=1,l\neq i}^n [w_{2l-1}(\alpha_l\beta_l + \beta_l^2\bar{x}) + \\ &\qquad w_{2l}(1 + \beta_l^2 + \alpha_l\beta_l\bar{x})]\gamma_{li} = 0, \end{aligned} \tag{22}$$

$$\begin{aligned} &-[w_{2i-1}(\alpha_i\beta_i\bar{x} + \beta_i^2 s^2) + w_{2i}((1 + \beta_i^2)\bar{x} + \alpha_i\beta_i s^2)] \\ &\textstyle\sum_{j=1,j\neq i}^n \gamma_{ji} + \sum_{l=1,l\neq i}^n [w_{2l-1}(\alpha_l\beta_l\bar{x} + \beta_l^2 s^2) + \\ &\qquad w_{2l}((1 + \beta_l^2)\bar{x} + \alpha_l\beta_l s^2)]\gamma_{li} = 0, \end{aligned} \tag{23}$$

for $i = 1, \dots, n$. It is straightforward to conclude from (22) and (23) that $w\bar{B} = 0 \Longrightarrow wB(t) = 0$, having in mind that the components of $v(t) = wB(t)$ are

$$\begin{aligned} v_{2i-1}(t) = &-[w_{2i-1}(\alpha_i\beta_i + \beta_i^2 x(t)) + w_{2i}(1 + \beta_i^2 + \\ &\alpha_i\beta_i x(t))] \textstyle\sum_{j=1,j\neq i}^n \gamma_{ji} + \sum_{l=1,l\neq i}^n [w_{2l-1}(\alpha_l\beta_l + \\ &\beta_l^2 x(t)) + w_{2l}(1 + \beta_l^2 + \alpha_l\beta_l x(t))]\gamma_{li} = 0, \end{aligned} \tag{24}$$

$$\begin{aligned} v_{2i}(t) = &-[w_{2i-1}(\alpha_i\beta_i x(t) + \beta_i^2 x(t)^2) + \\ &w_{2i}((1 + \beta_i^2)x(t) + \alpha_i\beta_i x(t)^2)] \textstyle\sum_{j=1,j\neq i}^n \gamma_{ji} + \\ &\textstyle\sum_{l=1,l\neq i}^n [w_{2l-1}(\alpha_l\beta_l x(t) + \beta_l^2 x(t)^2) + \\ &w_{2l}((1 + \beta_l^2)x(t) + \alpha_l\beta_l x(t)^2)]\gamma_{li} = 0, \end{aligned} \tag{25}$$

$i = 1, \dots, n$. Therefore, we have $\pi_1 B(t) = 0$ and $\pi_2 B(t) = 0$, and the result follows taking into account (18). ■

Theorem 2: Let Assumptions A1), A2), A3) and A4) be satisfied. Then there exists a positive number $\delta'' > 0$ such that for all $\delta \le \delta''$

$$\lim_{t\to\infty} \hat{\rho}(t) = (i_1\pi_1 + i_2\pi_2)\hat{\rho}(0) \tag{26}$$

in the mean square sense and with probability one, where $i_1 = [1\ 0\ 1\ 0\ \dots\ 1\ 0]^T$, $i_2 = [0\ 1\ 0\ 1\ \dots\ 0\ 1]^T$, and π_1 and π_2 are the left eigenvectors of $\bar{B}$ corresponding to the eigenvalue at the origin.

Proof: Using Lemma 6, we define $\tilde{\rho}(i) = T^{-1}\hat{\rho}(t)$, where T is chosen according to Lemma 5, and obtain, similarly as in (19), that

$$\begin{aligned} \tilde{\rho}(t+1)^{[1]} &= \tilde{\rho}(t)^{[1]}; \\ \tilde{\rho}(t+1)^{[2]} &= (I + \delta B(t)^*)\tilde{\rho}(t)^{[2]}, \end{aligned} \tag{27}$$

where $\tilde{\rho}(t)^{[1]} = [\tilde{\rho}_1(t)\ \ \tilde{\rho}_2(t)]^T$, $\tilde{\rho}(t)^{[2]} = [\tilde{\rho}_3(t) \cdots \tilde{\rho}_{2n}(t)]^T$. Recalling that $\bar{B}^*$ in (18) is Hurwitz, we observe that there exists such a positive definite matrix R^* that

$$\bar{B}^{*T}R^* + R^*\bar{B}^* = -Q^*, \tag{28}$$

where Q^* is positive definite. Define $q(t) = E\{\tilde{\rho}(t)^{[2]T} R^* \tilde{\rho}(t)^{[2]}\}$, and let $\lambda_Q = \min_i \lambda_i\{Q^*\}$ and $k' = \max_i \lambda_i\{E\{B(t)^* \ B(t)^{*T}\}\}$ ($k' < \infty$ under the adopted assumptions). From (27) we obtain

$$q(t+1) = E\{\tilde{\rho}(t)^{[2]T} E\{(I+B(t)^*)^T R^* (I+B(t)^*)\}\tilde{\rho}(t)^{[2]}\} \tag{29}$$

and, further,

$$q(t+1) \leq (1 - \delta \frac{\lambda_Q}{\max_i \lambda_i\{R^*\}} + \delta^2 k' \frac{\max_i \lambda_i\{R^*\}}{\min_i \lambda_i\{R^*\}}) q(t), \tag{30}$$

having in mind that $E\{B(t)^*\} = \bar{B}^*$. Consequently, there exists such a δ'' that for $\delta < \delta''$, $i = 1, \ldots, n$, the term in the brackets at the right hand side of (30) is less than one. Therefore, $q(t)$ tends to zero exponentially, implying that $\tilde{\rho}(t)^{[2]}$ converges to zero in the mean square sense, and, with probability one (having in mind that the sequence $\{q(t)\}$ is summable). Coming back to the first equation in (27) we obtain the result in the same way as in Theorem 1. ■

The following theorem deals with the important case in which the network is "pinned" to a selected node taken as a reference.

Theorem 3: Let Assumptions A1), A2), A3) and A4) be satisfied. Assume also that the k-th node is one of the center nodes of $\mathcal{G}$ and that the corresponding sensor has ideal characteristics: $\rho_k = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Then it is possible to find such a positive number $\delta''' > 0$ that for all $\delta \leq \delta'''$, $i = 1, \ldots, n$, the algorithm (5) combined with (11) provides convergence of $\hat{\rho}_i(t)$, $i = 1, \ldots, n, i \neq k$, to ρ_k in the mean square sense and with probability one.

Proof: Assume without loss of generality that $k = 1$. From (8) we obtain, after introducing $r_i(t) = \hat{\rho}_i(t) - \rho_1$,

$$\begin{aligned} r_i(t+1) =& (1 - \delta \sum_{j \in \mathcal{N}_i} \gamma_{ij} \Phi_i(t)) r_i(t) \\ &+ \delta \sum_{j \in \mathcal{N}_i, j \neq 1} \gamma_{ij} \Phi_i(t) r_j(t), \end{aligned} \tag{31}$$

$i = 2, \ldots, n$, and, in a compact form,

$$r(t+1) = (I + \delta \Phi^-(t)(\Gamma^- \otimes I_2)) r(t) \tag{32}$$

where $r(t) = [r_2(t)^T \cdots r_n(t)^T]^T$, $\Phi^-(t) = \text{diag}\{\Phi_2(t), \ldots, \Phi_n(t)\}$ and $\Gamma^- = [\gamma_{ij}^-]$, $i, j = 1, \ldots, n-1$, where $\gamma_{ij}^- = -\sum_{k=2, k \neq i+1}^{n} \gamma_{i+1,k}$ for $i = j$ and $\gamma_{ij}^- = \gamma_{i+1,j+1}$ for $i \neq j$. According to Lemma 3, Γ^- is an M-matrix, having in mind that the first node is assumed to be a center node. As a consequence, $\bar{\Phi}^-(\Gamma^- \otimes I_2)$ is Hurwitz. Therefore, the methodology of the proofs of Theorems 1 and 2 can be directly applied, leading to the conclusion that $r(t)$ converges to zero in the mean square sense and with probability one for sufficiently small values of the gain $\delta > 0$. ■

Let us now analyze convergence of the proposed algorithm in the case of correlated signal $x(t)$:

A2') Process $\{x(t)\}$ is weakly stationary with $E\{x(t)\} = \bar{x}$, $E\{x(t)x(t-d)\} = m(d)$, $m(0) = s^2$, $|x(t)| \leq K < \infty$ (a.s.) and

$$a) \quad |E\{x(t)|\mathcal{F}_{t-\tau}\} - \bar{x}| = o(\tau), \ \text{(a.s.)} \tag{33}$$

$$b) \quad |E\{x(t)x(t-d)|\mathcal{F}_{t-\tau}\} - m(d)| = o(\tau), \ \text{(a.s.)} \tag{34}$$

for any fixed $d \in \{0, 1, 2, \ldots\}$, $\tau > d$, where $\mathcal{F}_{t-\tau}$ denotes the minimal σ-algebra generated by $\{x(t-\tau), x(t-\tau-1), \ldots, x(0)\}$ ($o(\tau)$ denotes a function that tends to zero as $\tau \to \infty$).

Theorem 4: Let Assumptions A1), A2'), A3) and A4) be satisfied. Then it is possible to find such a positive number $\delta'' > 0$ that for all $\delta \leq \delta''$, $i = 1, \ldots, n$, in (10) $\lim_{t \to \infty} \hat{\rho}(t) = (i_1\pi_1 + i_2\pi_2)\hat{\rho}(0)$ in the mean square sense and with probability one.

Proof: Following the proof of Theorem 2, we first compute $\tilde{\rho}(i) = T^{-1}\hat{\rho}(t)$, and obtain the same relations as in (27). Iterating back the second one, one obtains

$$\tilde{\rho}(t+1)^{[2]} = \prod_{s=t}^{t-\tau} (I + \delta B(s)^*) \tilde{\rho}(t-\tau)^{[2]}. \tag{35}$$

After calculating $E\{\tilde{\rho}(t+1)^{[2]T} R^* \tilde{\rho}(t+1)^{[2]}\}$ using (35), we extract the term linear in δ and replace $B(t)^* = \bar{B}^* + \tilde{B}(t)^*$, where $E\{\tilde{B}(t)^*\} = 0$. According to A4'),

$$\begin{aligned} &|E\{\tilde{\rho}(t-\tau)^{[2]T} E\{\tilde{B}(s)^* | \mathcal{F}_{t-\tau-1}\} \tilde{\rho}(t-\tau)^{[2]}\}| \\ &\leq \phi(s-t+\tau+1) q(t-\tau), \end{aligned} \tag{36}$$

where $\phi(t) > 0$, $\lim_{t \to \infty} \phi(t) = 0$. Therefore, it is possible to find such $\tau_0 > 0$ that for all $\tau \geq \tau_0$

$$(\tau+1)\lambda_{min}(Q^*) - \sum_{s=t}^{t-\tau} \phi(s) > \lambda_0 > 0, \tag{37}$$

since $\lambda_{min}(Q^*) > 0$ by definition. Therefore,

$$q(t+1) \leq (1 - \lambda_0 \delta + \sum_{s=2}^{2(\tau+1)} k_s \delta^s) q(t), \tag{38}$$

where $|k_s| < \infty$ due to signal boundedness. It follows from (38) that it is possible to find such a $\delta'' > 0$ that for all $\delta \leq \delta''$: $1 - \lambda_0 \delta + \sum_{s=2}^{2(\tau+1)} k_s \delta^s < 1$. The result follows now in the same way as in Theorem 2. ■

IV. Simulation Results

In order to illustrate properties of the proposed algorithm, a sensor network with ten nodes has been simulated. A fixed randomly selected communications structure has been adopted, as well as parameters α_i and β_i randomly selected around one and zero, with variance 0.3.

In Fig. 1 the equivalent gains $\hat{g}_i(t)$ and offsets $\hat{f}_i(t)$ generated by the proposed algorithm are presented for a preselected gain $\delta = 0.01$. It is clear that the consensus is achieved quickly, and that the asymptotic values are close to the optimal values. Fig. 2 depicts the situation when the first node is assumed to be a reference node with $\alpha_1 = 1$ and $\beta_1 = 0$. Convergence to the optimal values is obvious.

Fig. 3 is added as an illustration of the possibilities of the proposed algorithm in the important case when the measurements are corrupted by additive zero-mean noise, with variance randomly chosen within the interval $(0, 0.3)$. Time-varying decreasing gains $\delta_i(t) = 0.01/t^{0.6}$ have been

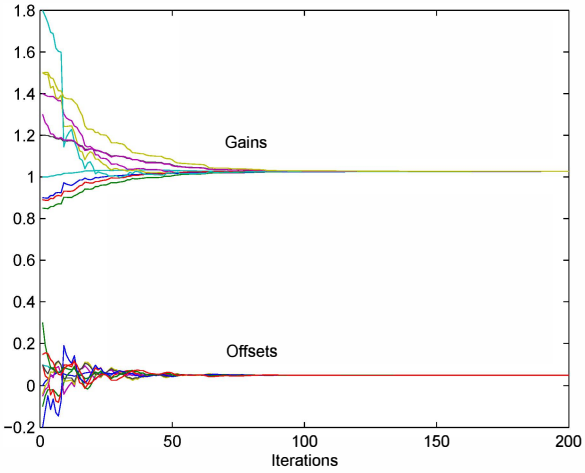

Fig. 1. Offset and gain estimates: no reference

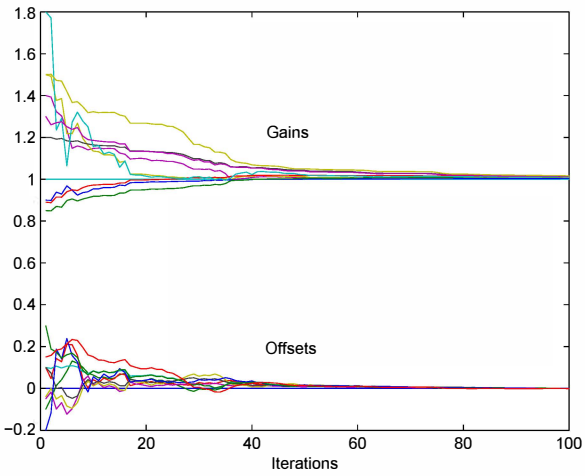

Fig. 2. Offset and gain estimates: reference included

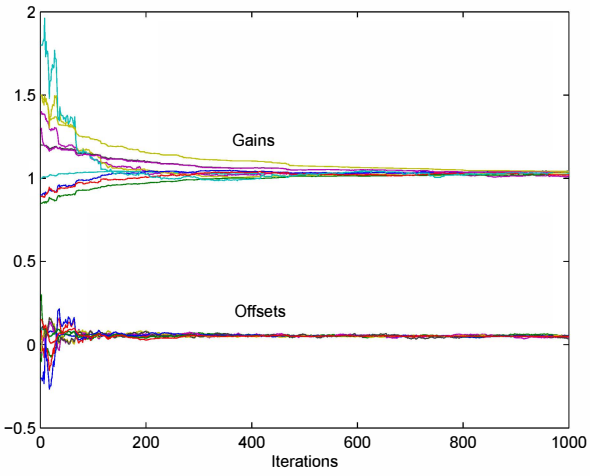

Fig. 3. Offset and gain estimates: noisy measurements

adopted, as well as a modification oriented towards eliminating nonzero correlation terms by introducing appropriate instrumental variables. This important scenario was treated in details in [14] and the obtained results appear to be very promising.

V. Conclusion

In this paper a distributed blind calibration algorithm based on consensus has been proposed for sensor networks. It is proved, on the basis of a novel methodology of treating higher order consensus schemes using the results related to diagonal dominance of matrices decomposed into blocks, that the algorithm achieves asymptotic agreement for sensor gains and offsets in the mean square sense and with probability one. When a reference is given, all offsets and gains converge to the given values in the mean square sense and with probability one.

The results open up a possibility of extending applicability of the proposed algorithm to the practically important case when communication errors and measurement noise are present [14], and to the case when the nodes are measuring spatially varying signals. Also, it is possible to assume that the obtained recursions at each node are asynchronous, which allows applicability of the proposed scheme to the important problem of time synchronization in sensor networks.

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