Wireless Event-Triggered Control

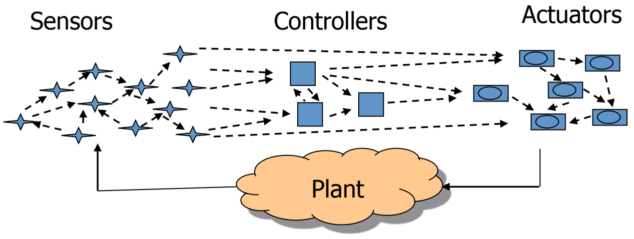
Karl Henrik Johansson
ACCESS Linnaeus Center
Royal Institute of Technology, Sweden

Based on joint work with Chithrupa Ramesh, José Araujo, Maben Rabi, Georg Seyboth, Henrik Sandberg, Carlo Fischione, Dimos Dimarogonas

Tutorial Session on Event-triggered and Self-triggered Control, IEEE CDC, Maui, 2012

Wireless control system

How to share common network resources while maintaining guaranteed closed-loop performance?



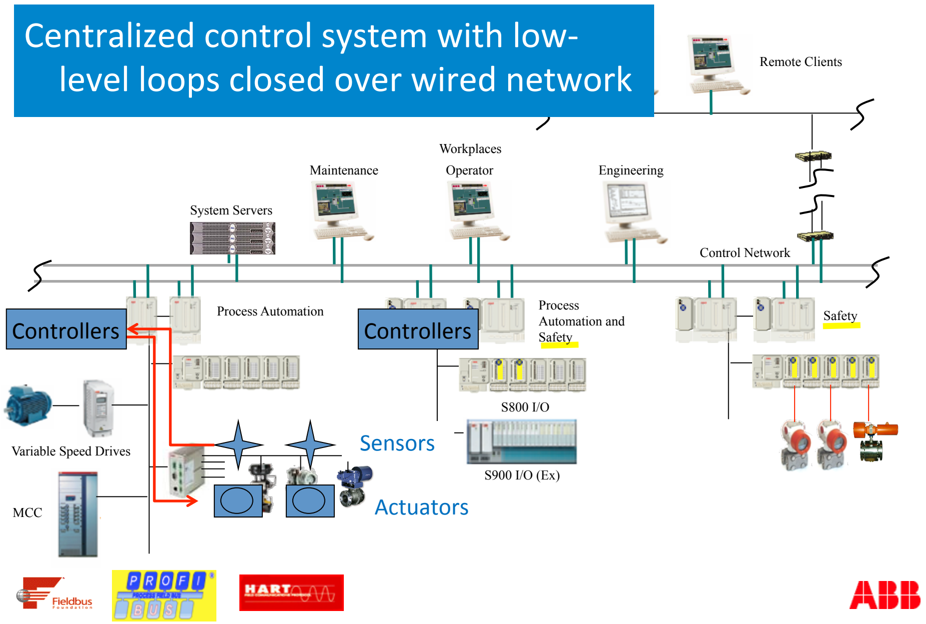
Idea: Utilize event- and self-triggered control to limit the use of network resources

Outline

- Motivating industrial applications
- Event-based scheduling for stochastic control
- Exploiting wireless network protocols
- Event-based control over lossy networks
- Extensions
- Conclusions

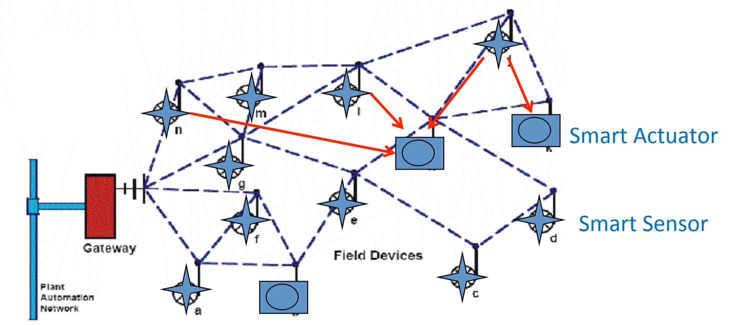
Today's industrial communication architecture

Centralized control system with low-level loops closed over wired network



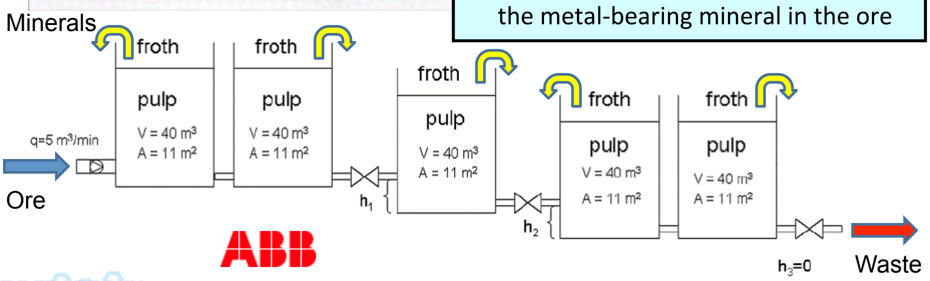
Towards wireless sensor and actuator network architecture

- Local control loops closed over **wireless multi-hop network**
- Potential for a dramatic change:
  - From fixed hierarchical centralized system to flexible distributed
  - Move intelligence from dedicated computers to sensors/actuators



Event-based control of froth flotation process

- Froth flotation process concentrates the metal-bearing mineral in the ore



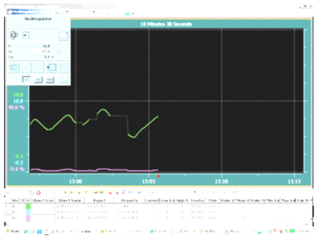
SOCRADES.EU ABB BOLIDEN

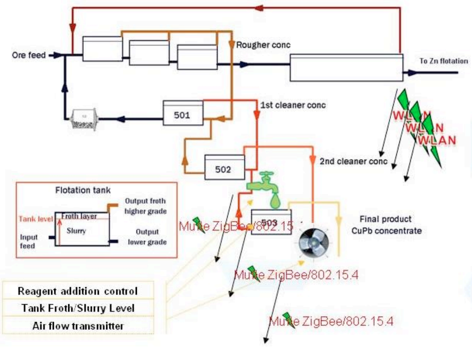
Wireless control of floatation process

The Boliden plant

Existing wired communication system

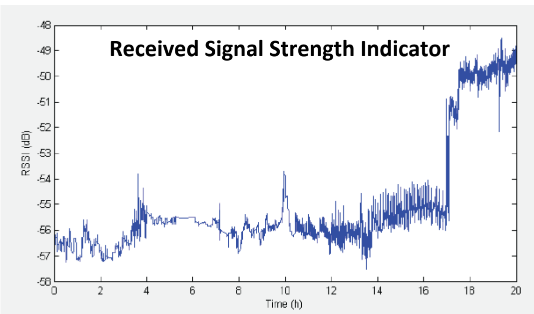
Wireless communication for tank level control





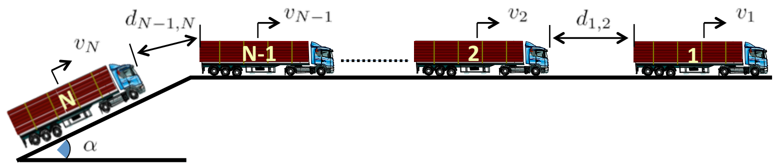
Radio Channel Measurements in Industrial Environment

- Rolling mill at Sandvik in Sweden
- Study of 2.45 GHz radio channel properties
- Slow but substantial RSSI variations due to mobile machines

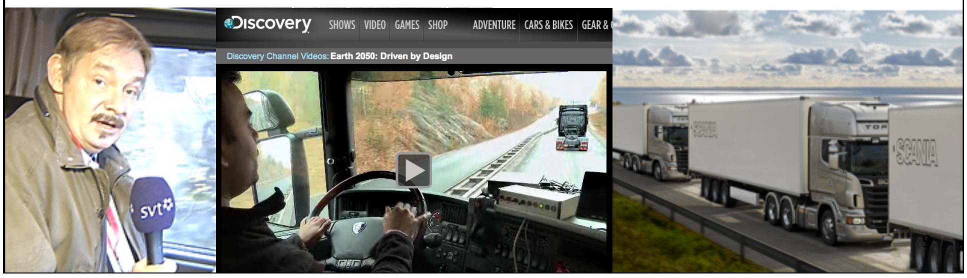


Ahlen et al, 2012

Event-based estimation in vehicle platooning



- Vehicles need **accurate estimates** of neighboring vehicles' states and actions
- Control performance is tightly coupled to how well data (position, velocity, breaking estimates) are communicated across the platoon
- Today's communication protocols are event-based (e.g., IEEE 801.11p)



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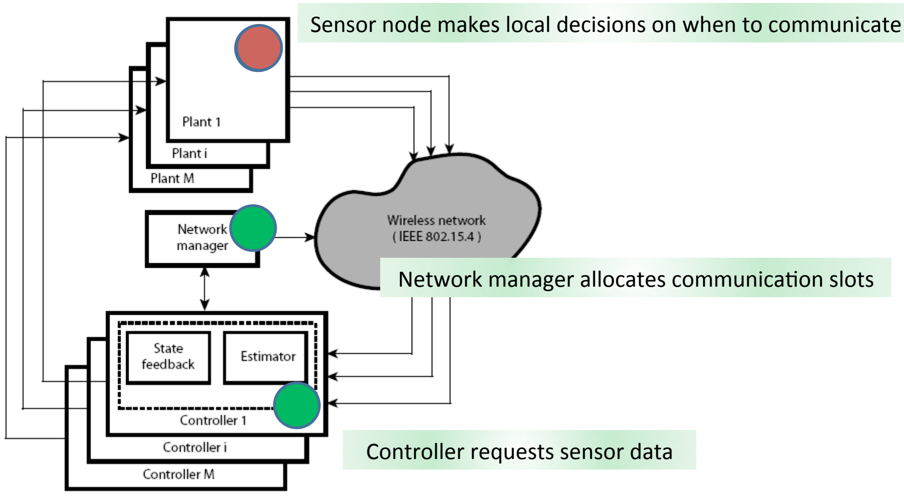
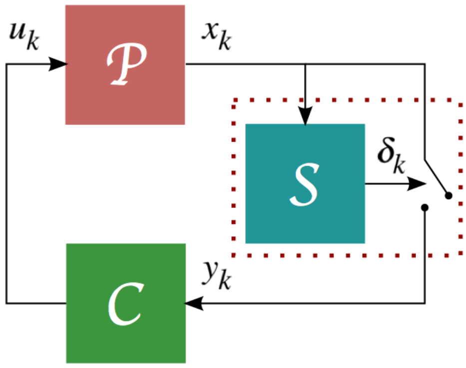
Where to take medium access decisions?



Is there a separation between event-based scheduling-estimation-control?



Stochastic control formulation

Plant:

$$x_{k+1} = Ax_k + Bu_k + w_k$$

Scheduler:

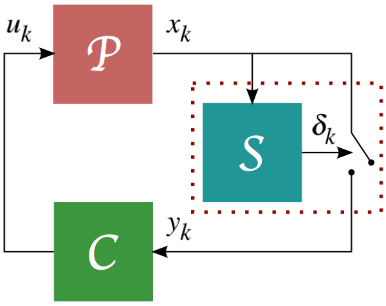
$$\delta_k = f_k(\mathbb{I}_k^{\mathbb{S}}) \in \{0,1\}$$

$$\mathbb{I}_k^{\mathbb{S}} = \left[\{x\}_0^k, \{y\}_0^{k-1}, \{\delta\}_0^{k-1}, \{u\}_0^{k-1}\right]$$

Controller:

$$u_k = g_k(\mathbb{I}_k^{\mathbb{C}})$$

$$\mathbb{I}_k^{\mathbb{C}} = \left[\{y\}_0^k, \{\delta\}_0^k, \{u\}_0^{k-1}\right]$$

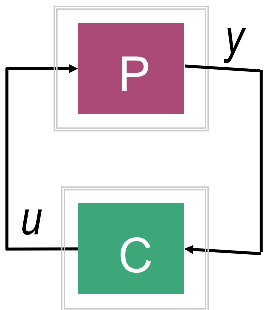


Cost criterion:

$$J(f,g) = \mathrm{E}[x_N^T Q_0 x_N + \sum_{s=0}^{N-1} (x_s^T Q_1 x_s + u_s^T Q_2 u_s)]$$

Certainty equivalence revisited

Definition Certainty equivalence holds if the closed-loop optimal controller has the same form as the deterministic optimal controller with x_k replaced by the estimate $\hat{x}_{k|k} = \mathrm{E}[x_k|\mathbb{I}_k^{\mathbb{C}}]$.



Theorem[Bar-Shalom–Tse] Certainty equivalence holds if and only if $E[(x_k - E[x_k|I_k^c])^2|I_k^c]$ is not a function of past controls $\{u\}_0^{k-1}$ (no dual effect).

Feldbaum, 1965; Åström, 1970; Bar-Shalom and Tse, 1974

Event-based scheduler

Plant:

$$x_{k+1} = Ax_k + Bu_k + w_k$$

Scheduler:

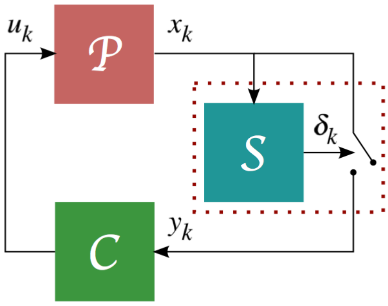
$$\delta_k = f_k(\mathbb{I}_k^{\mathbb{S}}) \in \{0,1\}$$

$$\mathbb{I}_k^{\mathbb{S}} = \left[\{x\}_0^k, \{y\}_0^{k-1}, \{\delta\}_0^{k-1}, \{u\}_0^{k-1}\right]$$

Controller:

$$u_k = g_k(\mathbb{I}_k^{\mathbb{C}})$$

$$\mathbb{I}_k^{\mathbb{C}} = \left[\{y\}_0^k, \{\delta\}_0^k, \{u\}_0^{k-1}\right]$$



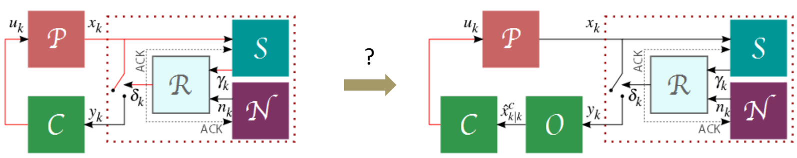
Corollary The control u_k for the optimal closed-loop system has a dual effect.

The separation principle does not hold for the optimal closed-loop system, so the design of the (event-based) scheduler, estimator, and controller is coupled

Ramesh et al., 2011

Conditions for Certainty Equivalence

Corollary: The optimal controller for the system $\{\mathcal{P},\mathcal{S}(f),\mathcal{C}(g)\}$, with respect to the cost J is certainty equivalent if and only if the scheduling decisions are not a function of the applied controls.



Certainty equivalence achieved at the cost of optimality

Ramesh et al., 2011

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Event-based control architecture

- Plant $\mathcal{P}$:

  $x_{k+1} = ax_k + bu_k + w_k$

- State-based Scheduler $\mathcal{S}$:

  $$\gamma_k = \begin{cases} 1, & |x_k - \hat{x}^s_{k|\tau_{k-1}}|^2 > \epsilon_d, \\ 0, & \text{otherwise.} \end{cases}$$

  $\hat{x}^s_{k|\tau_{k-1}} = a\hat{x}^c_{k-1|k-1} + bu_{k-1}$

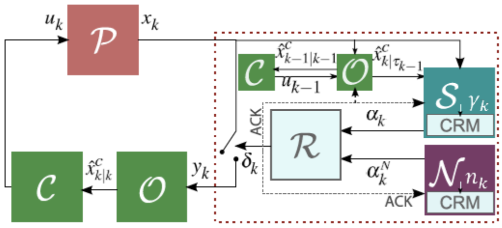
- CRM: $\mathbb{P}(\alpha_k{=}1|\gamma_k{=}1) = \mathbb{P}(\alpha^N_k{=}1|n_k{=}1) = p_\alpha$

  $\delta_k = \alpha_k(1 - \alpha^N_k)$

- Observer $\mathcal{O}$: $y^{(j)}_k = \delta^{(j)}_k x^{(j)}_k$

  $\hat{x}^c_{k|k} = \bar{\delta}_k(a\hat{x}^c_{k-1|k-1} + bu_{k-1}) + \delta_k x_k$

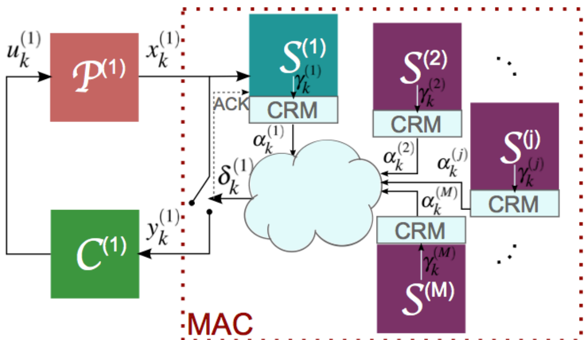
- Controller $\mathcal{C}$: $u_k = -L\hat{x}^c_{k|k}$



Ramesh et al., CDC, 2012, ThC01.3

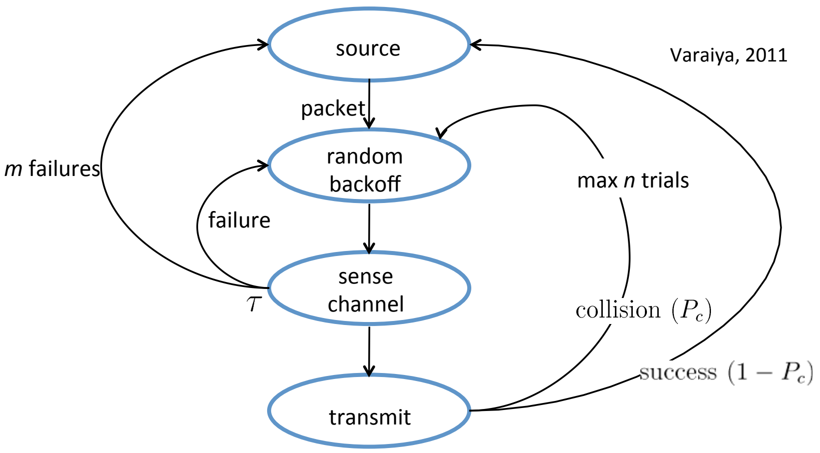
Integrating advanced contention resolution mechanisms

- Hard problem because of correlation between transmissions (and the plant states)
- Closed-loop analysis can still be done for classes of event-based schedulers and MAC's



Ramesh et al., CDC 2011

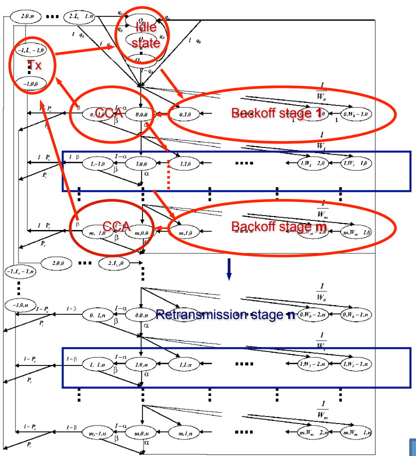
Contention resolution through CSMA/CA



- Every transmitting device executes this protocol
- For analysis, assume carrier sense events are independent [Bianchi, 2000]

CSMA/CA = Carrier Sense Multiple Access with Collision Avoidance

Detailed model of CSMA/CA in IEEE 802.15.4



- Markov state (*s*,*c*,*r*)
  - s: backoff stage
  - c: state of backoff counter
  - r: state of retransmission counter
- Model parameters
  - q_0: traffic condition (q_0=0 saturated)
  - m_0, m, m_b, n: MAC parameters
- Computed characteristics
  - α: busy channel probability during CCA1
  - β: busy channel probability during CCA2
  - P_c: collision probability

- Validated in simulation and experiment
- Reduced-order models for control design
- Detailed model for numerial evaluations

Park, Di Marco, Soldati, Fischione, J, 2009

Cf., Bianchi, 2000; Pollin et al., 2006

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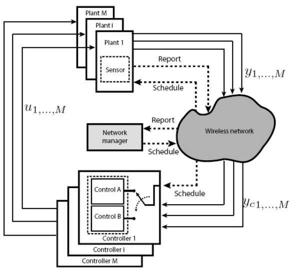
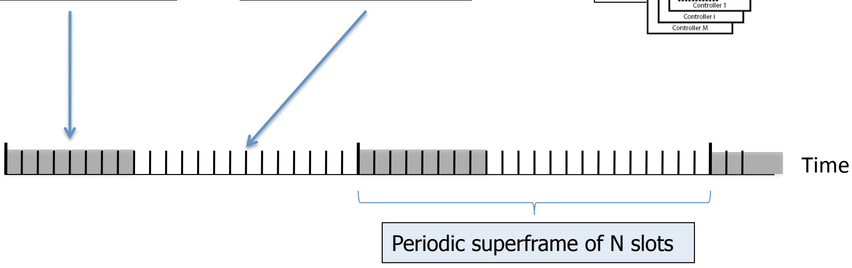
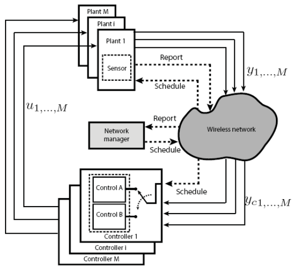
Slotted medium access



Many medium access protocols have slotted <u>contention-free</u> and <u>contention access</u> periods



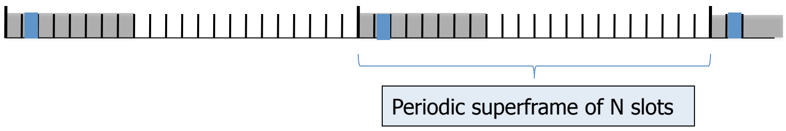
Hybrid MAC protocols



Exploit the mix of CFP's and CAP's for event- and self-triggered control

E.g., Araujo et al., 2010, Tiberi et al., 2010

Contention-free period for TDMA scheduled communication



Contention access period for random CSMA communication

TDMA = Time division multiple access, CSMA/CA = Carrier Sense Multiple Access with Collision Avoidance

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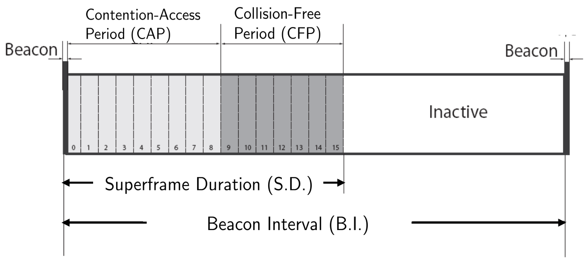
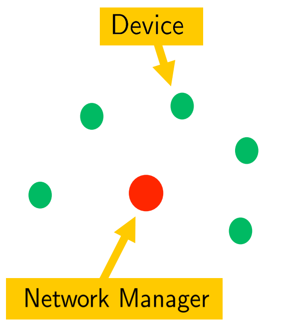
IEEE 802.15.4 MAC





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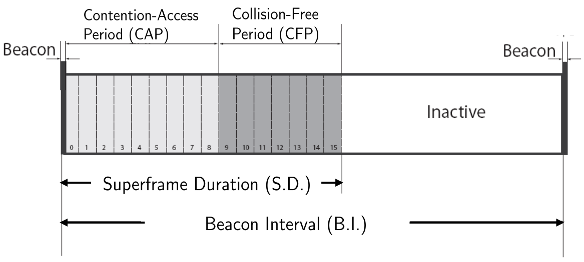
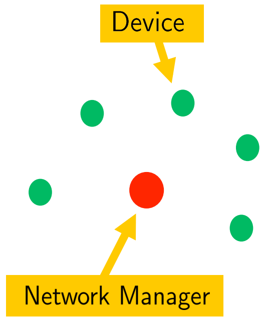
IEEE 802.15.4 MAC





- 16 slots for CAP and CFP
- Maximum 7 slots for CFP

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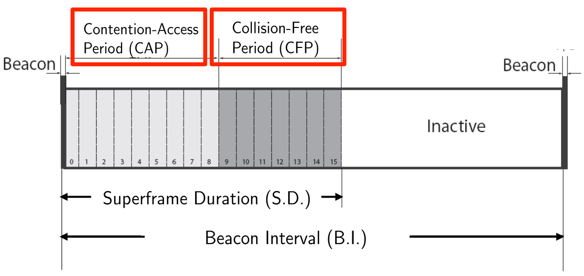
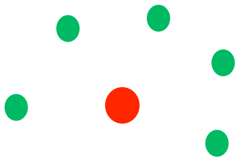
IEEE 802.15.4 MAC





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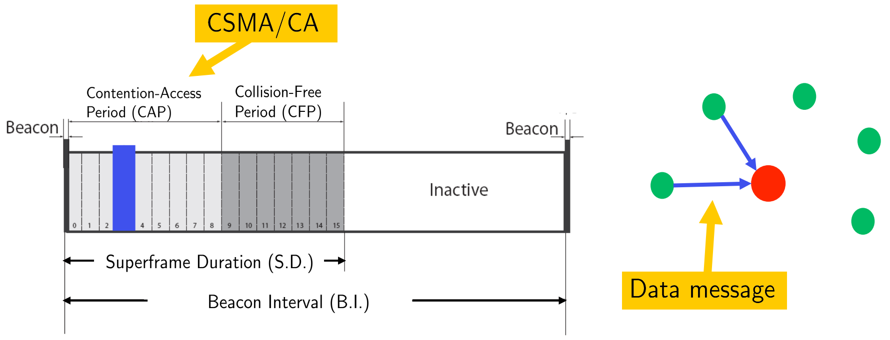
IEEE 802.15.4 MAC



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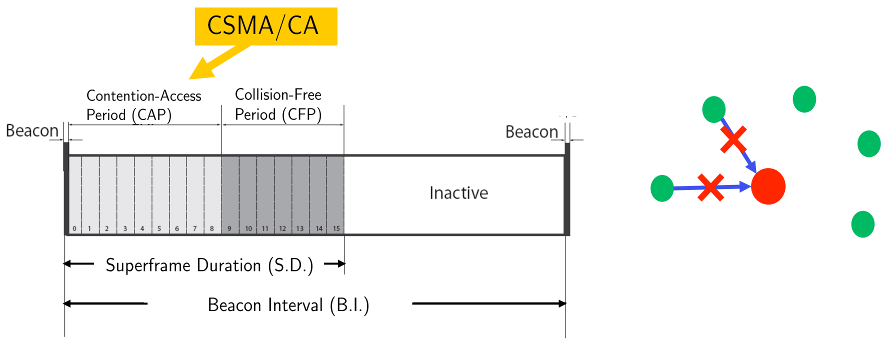

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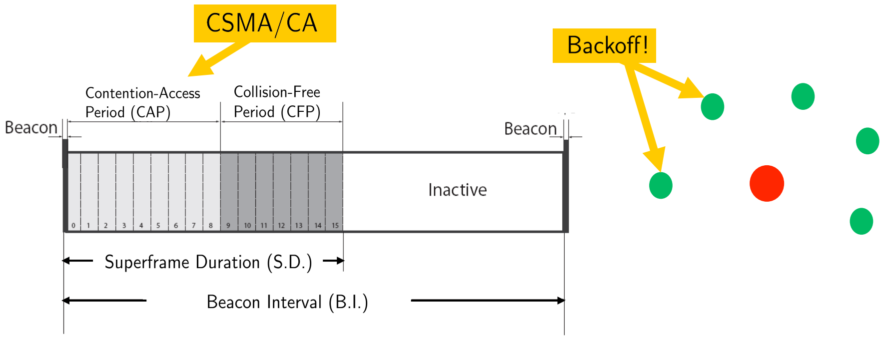
IEEE 802.15.4 MAC



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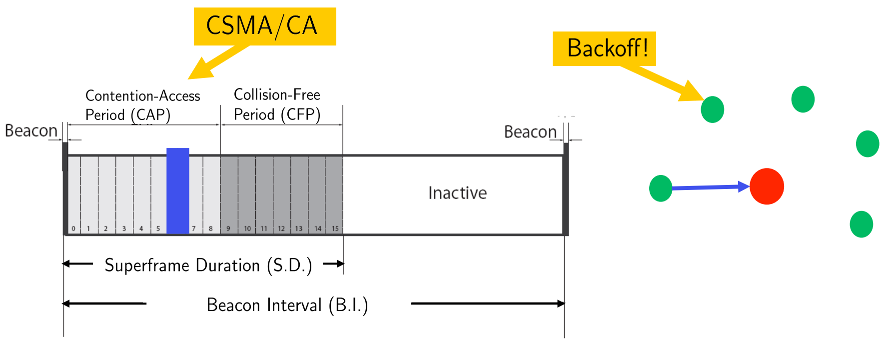
IEEE 802.15.4 MAC



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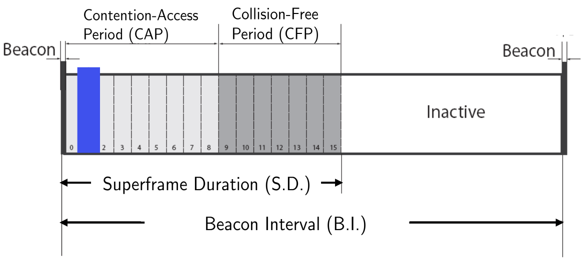
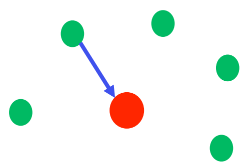
IEEE 802.15.4 MAC





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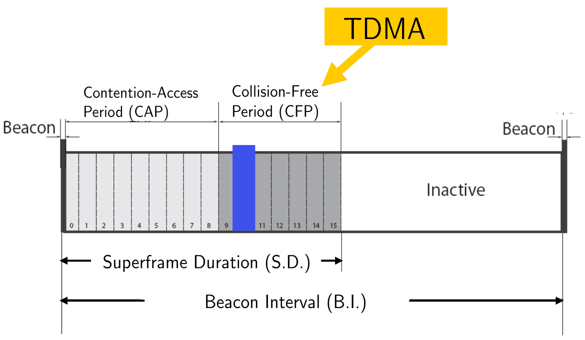
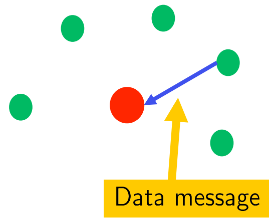
IEEE 802.15.4 MAC





- CFP slot allocation as First-Come First-Served

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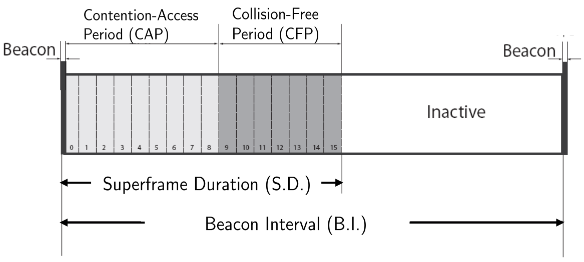
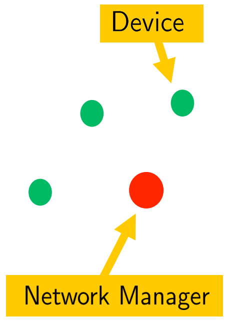
IEEE 802.15.4 MAC





- 16 slots for CAP and CFP
- Maximum 7 slots for CFP
- CFP slot allocation as First-Come First-Served

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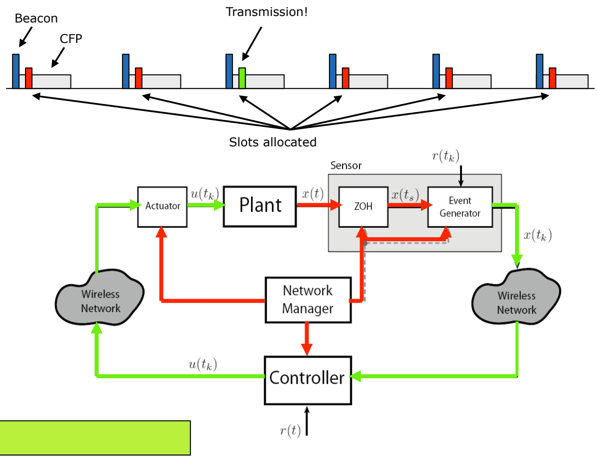
Event-based sensor communication

1. Fixed scheduling of sensing/actuation slots
2. Check triggering condition at every allocated slot
   - One-step ahead triggering condition
3. If triggering condition is true, transmit measurement and perform actuation



- **Robust to disturbances**

- **Unnecessary bandwidth utilization**
- **Energy spent on checking triggering condition**

Araujo, 2011

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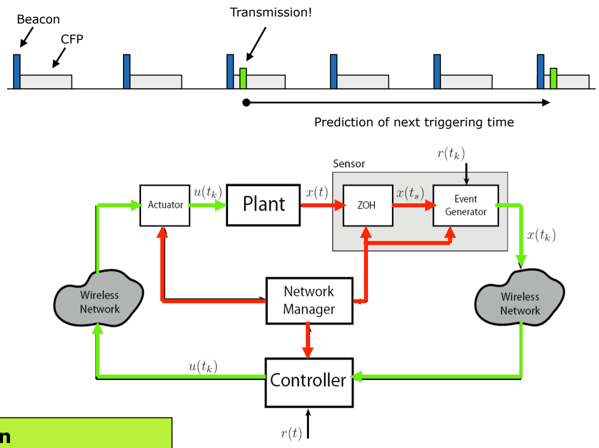
Predictive sensor communication

1. Scheduling of sensing/ actuation slots when required, at beacon times
2. If triggering condition is predicted to be true, transmit measurement and perform control action
3. At every transmission, predict and schedule the next triggering time
   - Set node to sleep until next transmission



- **Efficient bandwidth utilization**
- **Low energy consumption**

- **Less robust to disturbances**

Araujo, 2011 35

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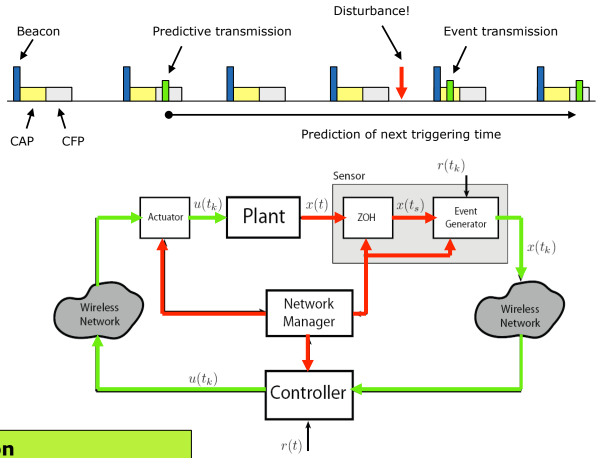
Hybrid sensor communication

1. Scheduling of slots as predictive scheme
2. Sensor node also checks triggering condition continuously (or during CAP)
3. If triggering condition is true, transmit measurement and perform control action



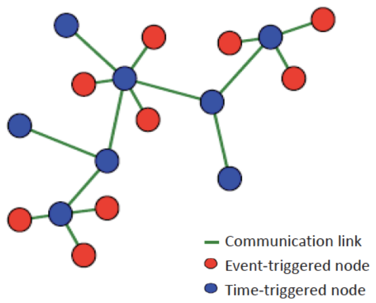
- **Efficient bandwidth utilization**
- **Robust to disturbances**

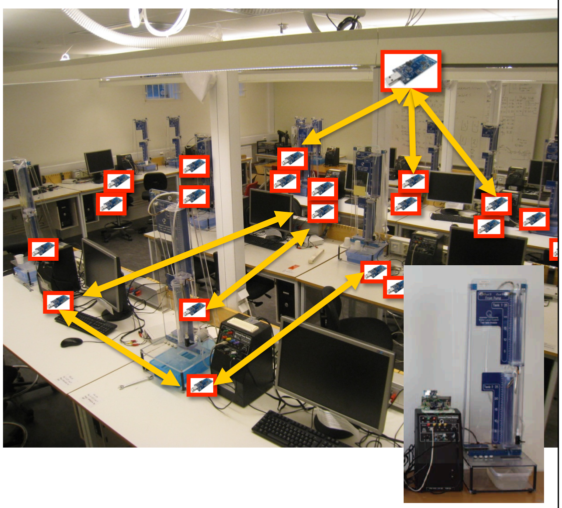
- **Energy spent on checking triggering condition**

Araujo, 2011 36

Multi-hop networks

- Routing decisions
- Time delays
- Hidden terminal problem





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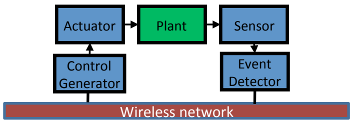
Event-based impulse control

Plant $dx_t = dW_t + u_t dt,\ x(0) = x_0,$

Sampling events $\mathcal{T} = \{\tau_0, \tau_1, \tau_2, \ldots\},$



Impulse control $u_t = \sum_{n=0}^{\infty} x_{\tau_n} \delta(\tau_n)$

Average sampling rate $R_\tau = \limsup_{M\to\infty} \frac{1}{M}\mathbb{E}\left[\int_0^M \sum_{n=0}^{\infty} \mathbf{1}_{\{\tau_n \le M\}} \delta(s-\tau_n)\, ds\right]$

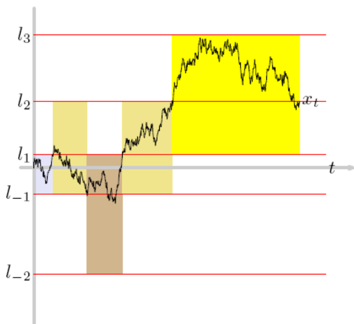
Average cost $J = \limsup_{M\to\infty} \frac{1}{M}\mathbb{E}\left[\int_0^M x_s^2 ds\right]$

Level-triggered control

Ordered set of levels $\mathcal{L} = \{\ldots, l_{-2}, l_{-1}, l_0, l_1, l_2, \ldots\}$ $l_0 = 0$

Multiple levels needed because we allow packet loss

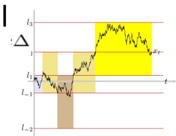
Sampling instances $\tau = \inf\left\{\tau \middle| \tau > \tau_i, x_\tau \in \mathcal{L}, x_\tau \notin x_{\tau_i}\right\}$



Level-triggered control

For Brownian motion, equidistant sampling is optimal

$$\mathcal{L}^* = \{k\Delta | k \in \mathbb{Z}\}$$



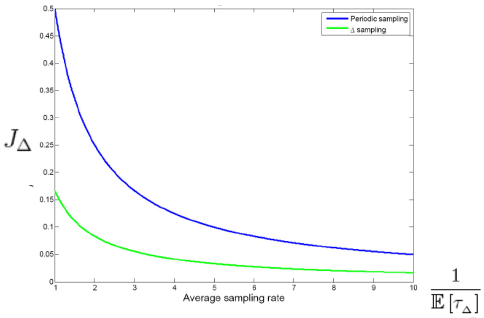
First exit time

$$\tau_\Delta = \inf\{\tau | \tau \geq 0, x_\tau \notin (\xi - \Delta, \xi + \Delta), x_0 = \xi\}$$

Average sampling rate $R_\Delta = \frac{1}{\mathbb{E}[\tau_\Delta]} = \frac{1}{\Delta^2},$

Average cost $J_\Delta = \frac{\mathbb{E}\left[\int_0^{\tau_\Delta} x_s^2 ds\right]}{\mathbb{E}[\tau_\Delta]} = \frac{\Delta^2}{6}.$

Comparison between time- and event-based control



$T = \Delta^2$ gives equal average sampling rate for periodic control and event-based control

Event-based impulse control is three times better than periodic

Åström & Bernhardsson, IFAC, 1999

What about the influence of communication losses?
Is event-based sampling still better?

Influence of i.i.d. packet loss

Times when packets are successfully received $\rho_i \in \{\tau_0 = 0, \tau_1, \tau_2, \ldots\}$,

$\{\rho_0 = 0, \rho_1, \rho_2, \ldots\}. \qquad \rho_i \geq \tau_i,$

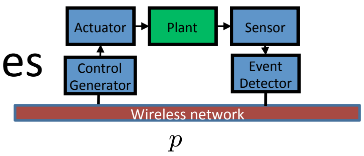
Average rate of packet reception

$$R_\rho = \limsup_{M\to\infty} \frac{1}{M}\mathbb{E}\left[\int_0^M \sum_{n=0}^{\infty} \mathbf{1}_{\{\rho_n \leq M\}} \delta(s-\rho_n)\, ds\right] = p \cdot R_\tau$$

Define the times between successful packet receptions $\rho_{(p,\Delta)}$

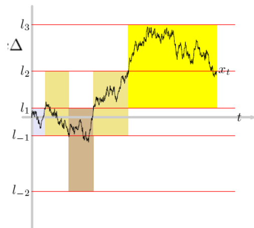
Average cost $J_p = \limsup_{T\to\infty} \frac{1}{T}\mathbb{E}\left[\int_0^T x_s^2 ds\right] = \frac{\mathbb{E}\left[\int_0^{\rho_{(p,\Delta)}} x_s^2 ds\right]}{\mathbb{E}\left[\rho_{(p,\Delta)}\right]}$

Event-based control with losses



Theorem

If packet losses are i.i.d. with probability p, then level-triggered sampling gives



$$J_p = \frac{\Delta^2(5p+1)}{6(1-p)}$$

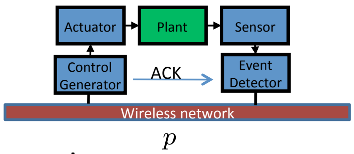
Event-based control better than periodic control if loss probability

$$p < 0.25$$

Rabi & J, 2009

Extensions to other medium models in Henningsson & Cervin, 2010; Blind & Allgöwer, 2011

Communication acknowledgements



If controller perfectly acknowledges packets to sensor, event detector can adjust its sampling strategy

Let $\Delta(l) = \sqrt{l+1}\Delta_0$

where $l \geq 0$ number of samples lost since last successfully transmitted packet

Gives that $\mathbb{E}\left[\tau_{i+1}^{\uparrow} - \tau_i^{\uparrow}\right]$ becomes independent of i.

Better performance than fixed $\Delta(l)$ for same sampling rate:

$$J_p^{\uparrow} = \frac{\Delta^2(1+p)}{6(1-p)} \leq \frac{\Delta^2(1+5p)}{6(1-p)} = J_p.$$

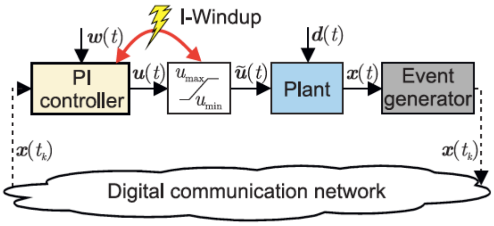
Rabi and J, *2009*

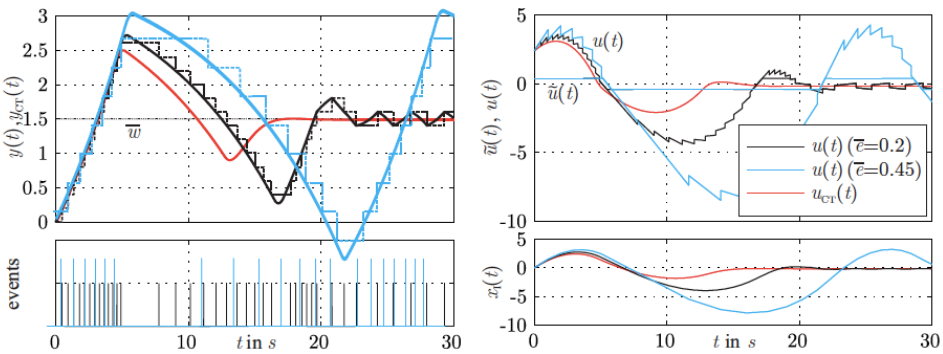
Outline

- Motivating industrial applications
- Event-based scheduling for stochastic control
- Exploiting wireless network protocols
- Event-based control over lossy networks
- Extensions
  - Event-based anti-windup
  - Event-based multi-agent systems
- Conclusions

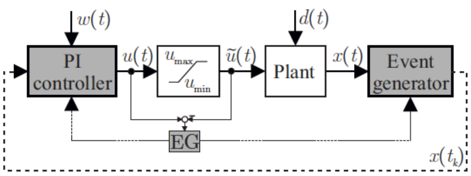
Windup in event-based control

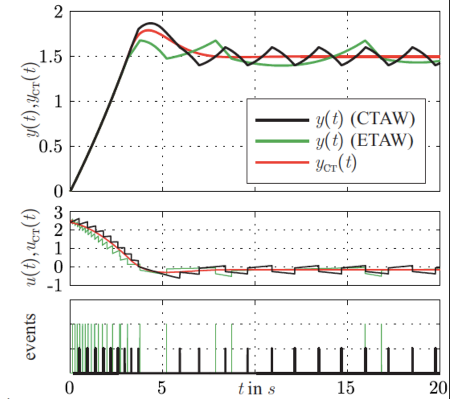
- Saturations cause windup
- Important for event-based control
- How to design anti-windup scheme?





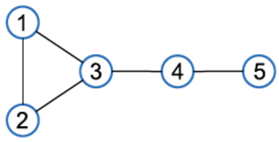
Anti-windup for event-based control

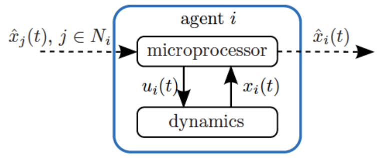




Lehmann et al., CDC, 2012, ThB01.1

Event-based Control of Multi-Agent System





$\dot{x}_i(t) = u_i(t)$

$$u_i(t) = -\sum_{j \in N_i} (\hat{x}_i(t) - \hat{x}_j(t))$$

Event-based broadcasting

$$\hat{x}_i(t) = x_i(t_k^i),\ t \in [t_k^i, t_{k+1}^i[$$

$$0 \le t_0^i \le t_1^i \le t_2^i \le \cdots$$

$$t_{k+1}^i = \inf\{t :\ t > t_k^i, f_i(t) > 0\}$$

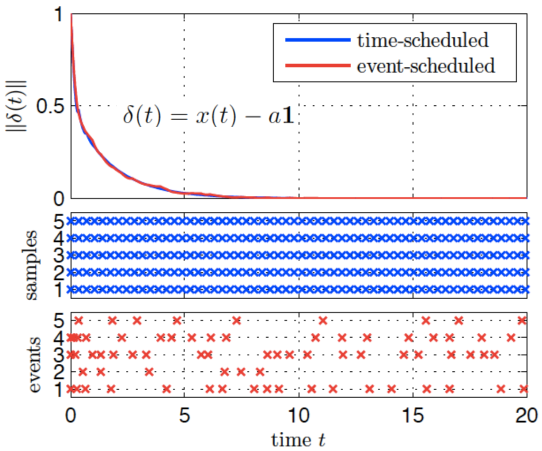
$$f_i(t, e_i(t)) = |e_i(t)| - (c_0 + c_1 e^{-\alpha t})$$

$$e_i(t) = \hat{x}_i(t) - x_i(t)$$

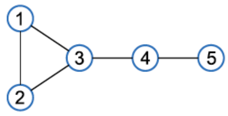
Practical consensus is achieved if $0<\alpha<\lambda_2(L)$

Seyboth et al. (2011)

Event-based vs Periodic Communication



Graph:



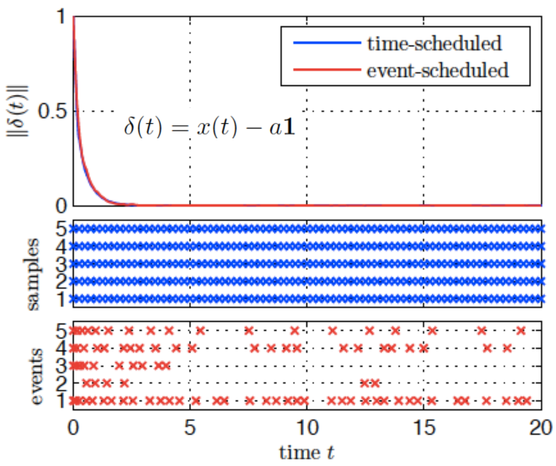
Sampling periods:

- Time-scheduling:
  $\tau_s = 0.350$
  $\tau_{max} = 0.480$
- Event-scheduling:
  $\tau_{mean} = 1.389$

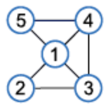
τ_{max} : largest stabilizing sampling period, see *G. Xie et al., ACC2009*

Seyboth et al. (2011)

Event-based vs Periodic Communication



Graph:



Sampling periods:

- Time-scheduling:
  $\tau_s = 0.250$
  $\tau_{max} = 0.400$
- Event-scheduling:
  $\tau_{mean} = 1.053$

τ_{max} : largest stabilizing sampling period, see *G. Xie et al., ACC2009*

Seyboth et al. (2011)

Outline

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Conclusions

- Event-based control is an **enabler for applications** of wireless networked control systems
- Efficient use of **network resources** under control objectives
- **Stochastic control** approach is natural because of probabilistic guarantees for wireless networks
- Many open problems related to **multi-loop** systems and **multi-hop** networks



http://www.ee.kth.se/~kallej

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