



# Event-based Control for Distributed Systems

Karl H. Johansson and Maben Rabi

ACCESS Linnaeus Centre

Electrical Engineering, Royal Institute of Technology  
Stockholm, Sweden

DISC Summer School on Distributed Control and Estimation

June 2-5, 2009, Noordwijkerhout



## ACCESS Linnaeus Centre

- One of Europe's largest **university research center** in networks
  - 35 senior researchers and 80 PhD students
  - Total research budget about 6 MEUR per year
- Cross-disciplinary research on the convergence of **computing, communication and control**
- Strong **industrial collaborations** through an industrial partnership program
- Extensive **mobility program**



# Outline

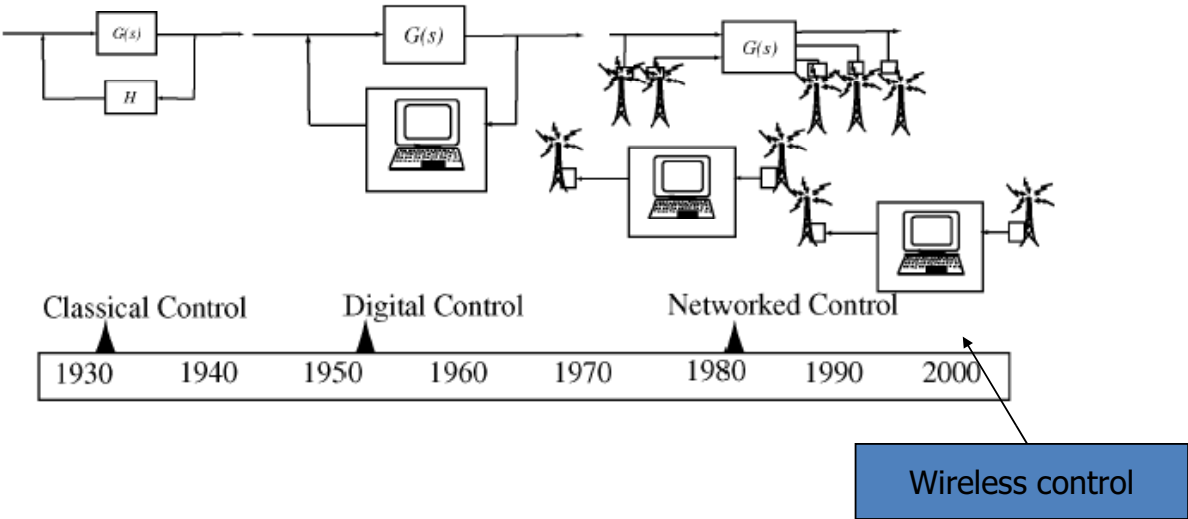
- **Introduction**
- **Motivation:** Control over wireless networks
- **Architecture** for event-based control
- **Design** of event detector and control generator
- **Extensions**
- **Conclusions**

---

# Outline

- **Introduction**
- **Motivation:** Control over wireless networks
- **Architecture** for event-based control
- **Design** of event detector and control generator
- **Extensions**
  - Multiple control loops
  - Influence of communication losses
  - Event-based PID control
- **Conclusions**

# A history of control



[Baillieul & Antsaklis, 2007]

# Today's wireless control systems

**Industrial automation**

**Home automation**

**Transportation networks**

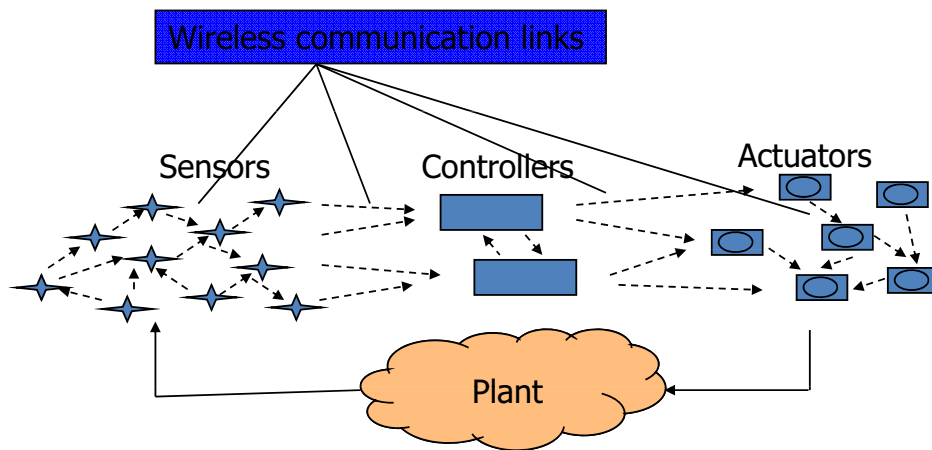
**Marine habitat mapping**

**Surveillance**

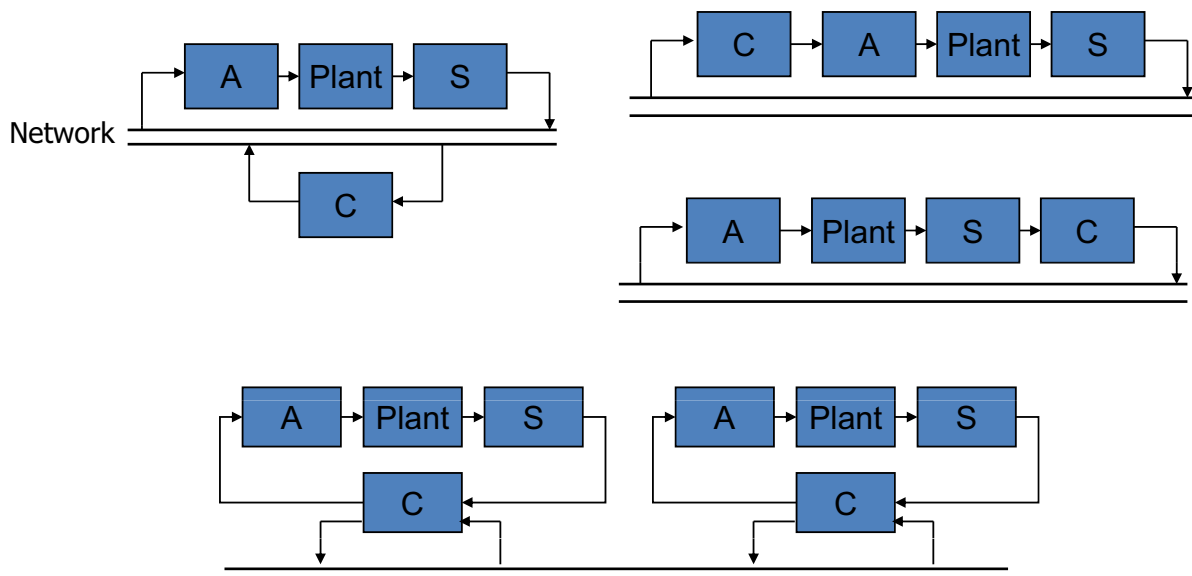
**Environmental monitoring**

# Control over wireless networks

How to control a plant when sensor, actuator and controller nodes are wireless network devices?



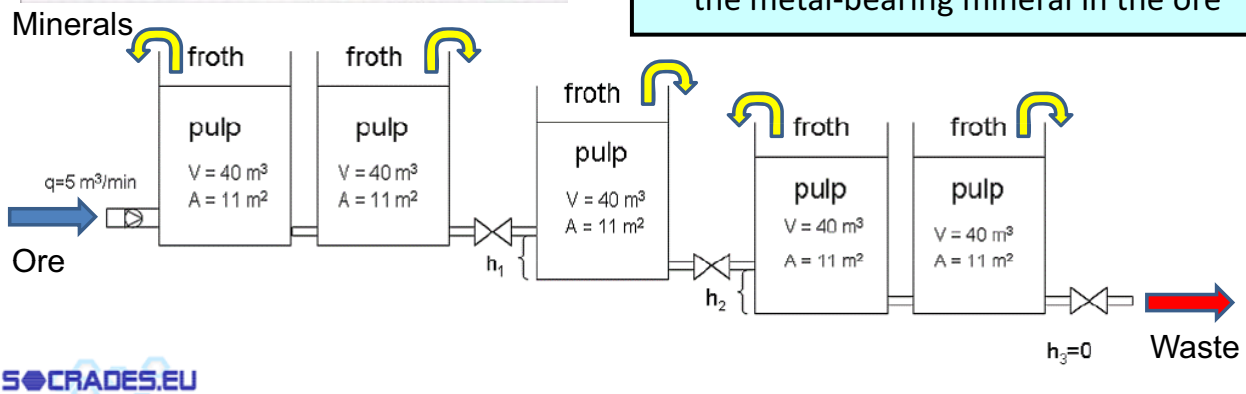
## Networked control architectures



# Example: Froth flotation process

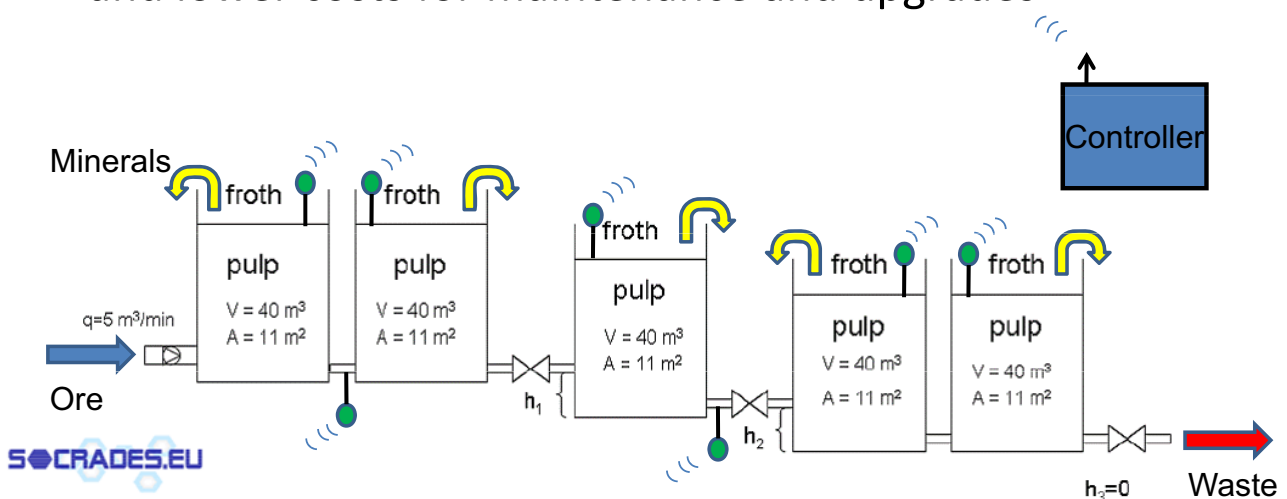


- Froth flotation process concentrates the metal-bearing mineral in the ore

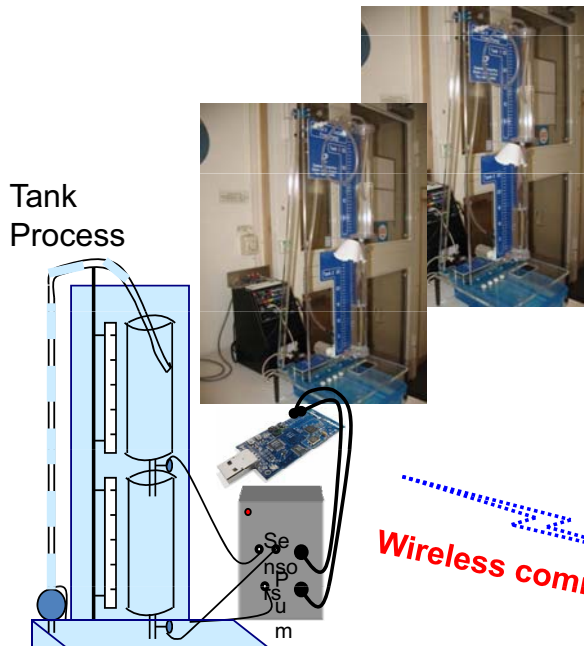


## Wireless control of flotation process

- Level and flow sensors are used for regulating flotation process using SISO PID control
- Wireless sensors enable more flexible control strategies and lower costs for maintenance and upgrades

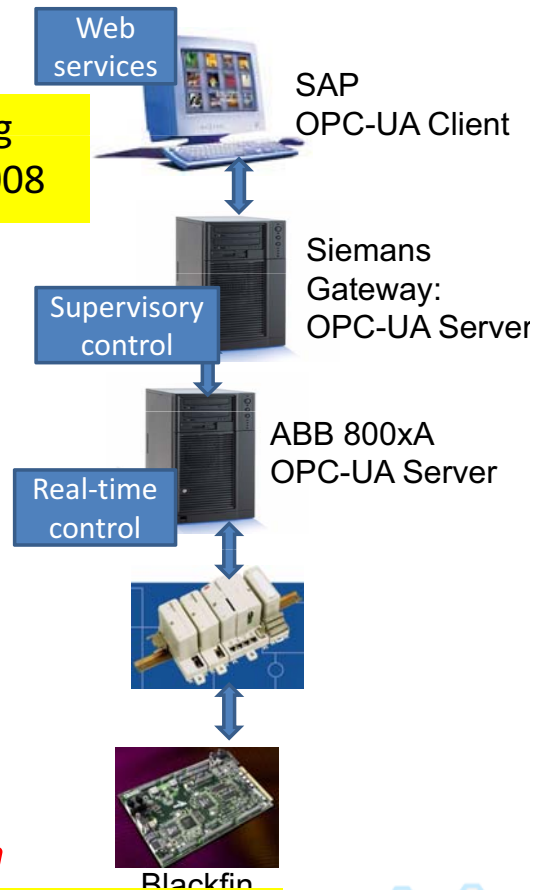


**Wireless control demonstration integrating ABB-Siemens-SAP systems 17-18 Jun 2008**

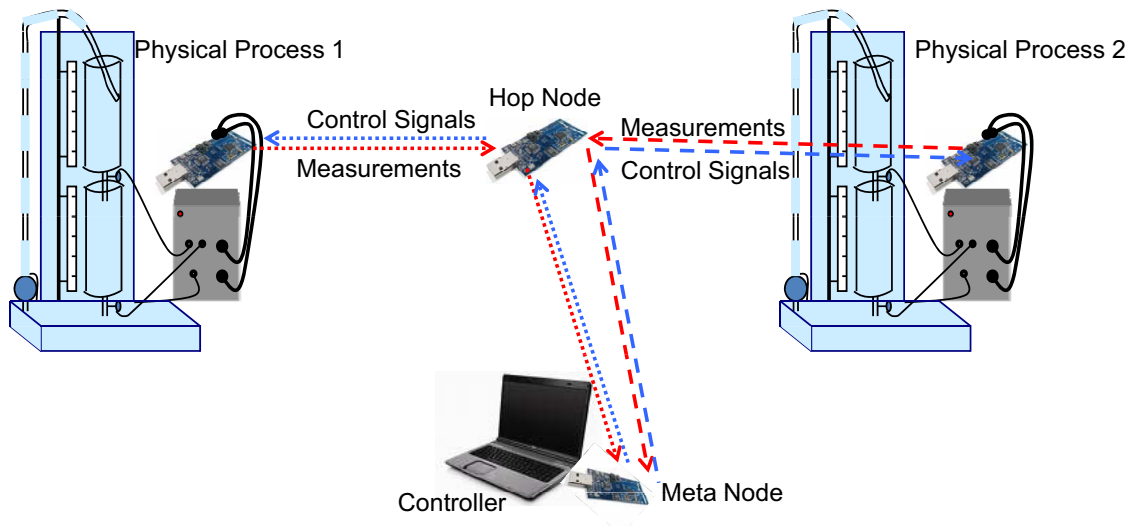


**Froth flotation process demo at Boliden in Aug 2009**

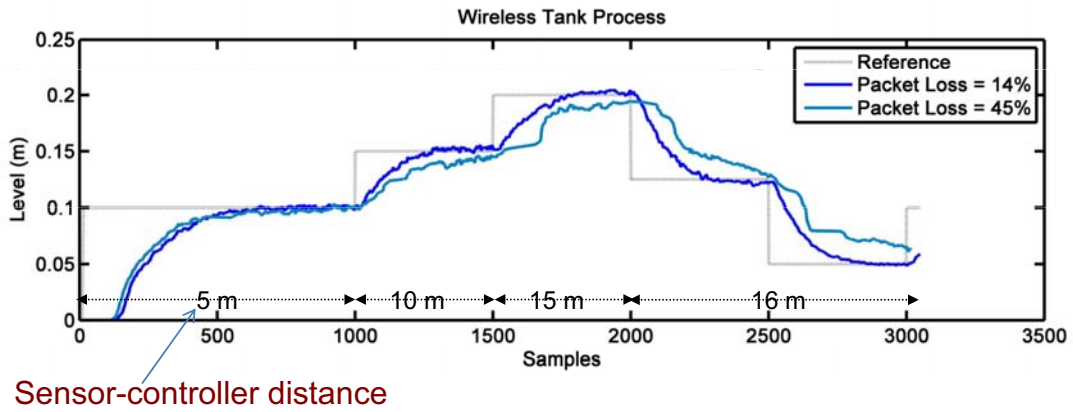
*Wireless communication*



# Experimental setup for demo on control over multi-hop network



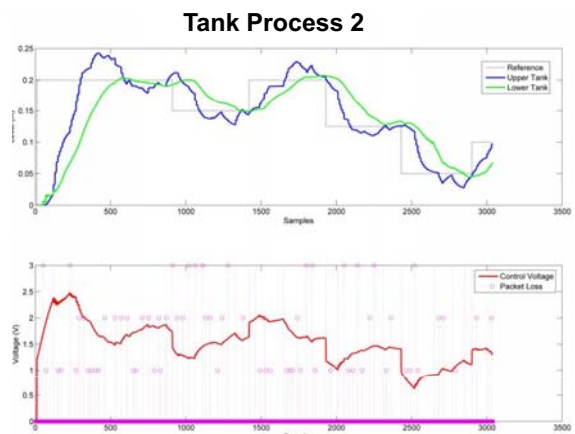
# Effect of packet loss



# Effect of radio interference

Interference from multiple motes transmitting and receiving simultaneously to tank processes 1 and 2

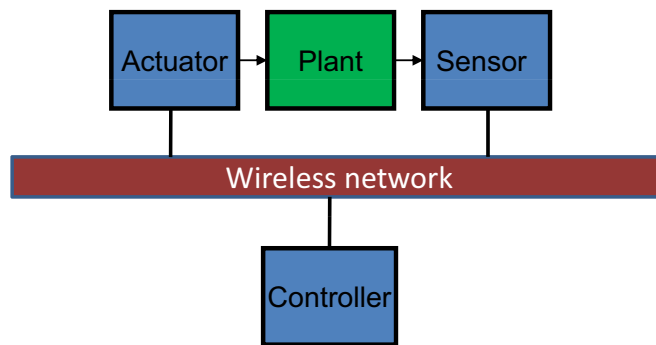
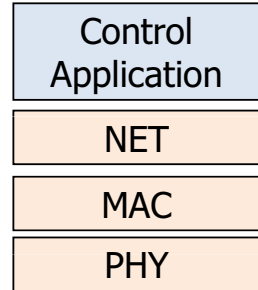
Packet loss = 45%, Sampling time = 1 s



# A communication or a control problem?

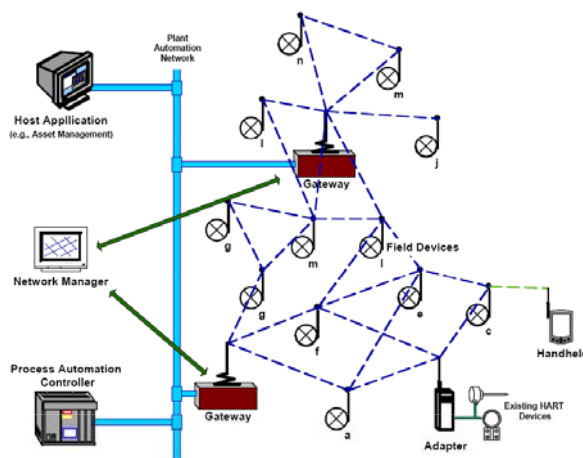
## Approaches to control over wireless networks:

1. Communication protocol suitable for control
2. Control application that compensates for communication imperfections
3. Integrated design of control application and communication layers

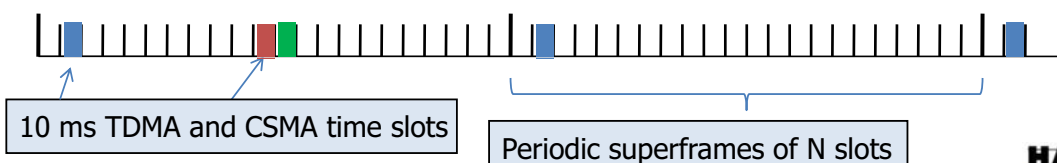


## WirelessHART

Wireless networking protocol standard (2007)  
designed for sensing and control applications

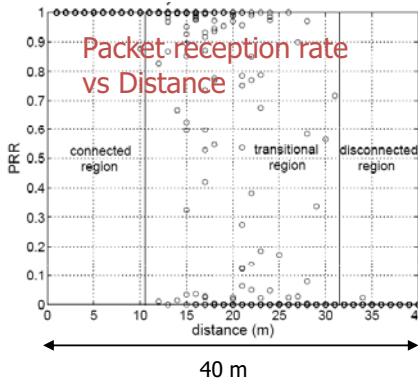


	Standard HART	WirelessHART
Layer 7 Application	Command oriented, predefined data types and application procedures	
Layer 6 Presentation		
Layer 5 Session		
Layer 4 Transport	Auto-segmented transfer of large data sets, reliable stream transport, and negotiated segment sizes	
Layer 3 Network		Power-optimized, redundant path mesh network
Layer 2 Data Link	A token passing master/slave protocol	Time-synchronized, frequency hopping protocol
Layer 1 Physical	Simultaneous analog & digital signalling (4-20mA wire)	IEEE 802.15.4-2006, 2.4GHz

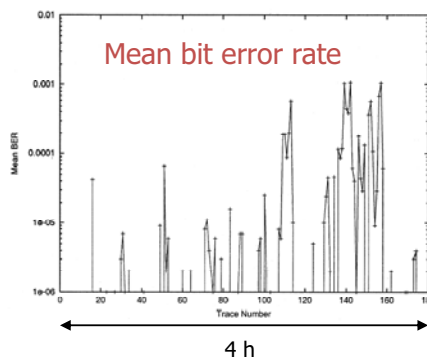




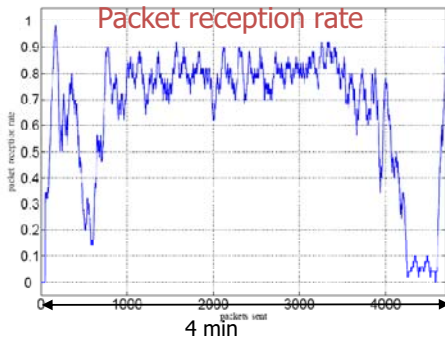
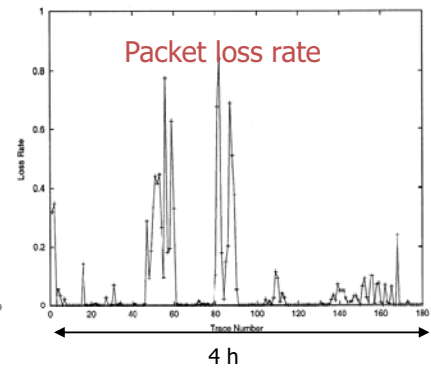
# Wireless channels may deteriorate control performance



Zuniga & Krishnamachari, *SECON*, 2004

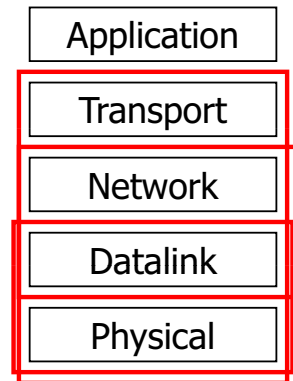


Willig et al., *IEEE Trans. Ind. Electron.*, 49, 6, 2002



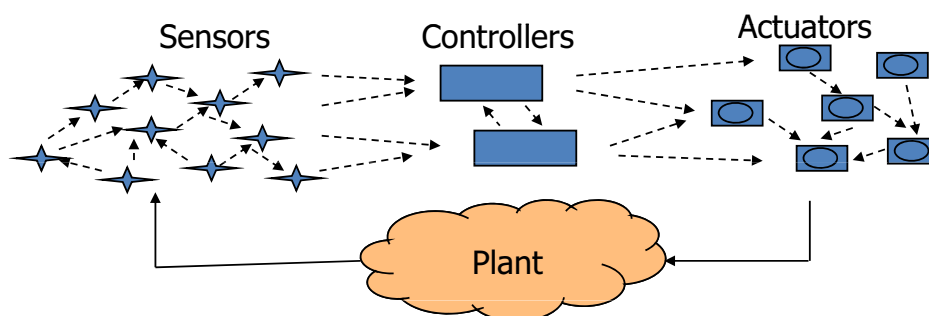
Park et al., *KTH*, 2007

- Large variations in
- Connectivity
  - Bit and packet delivery
  - End-to-end delivery



How trade-off network resources and control performance?

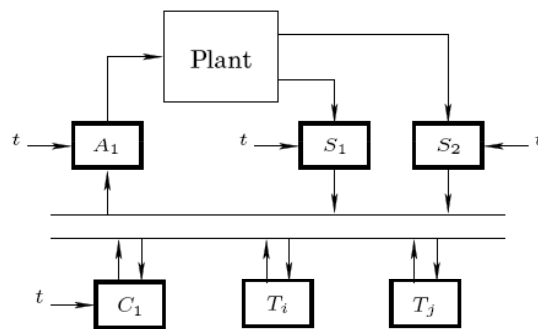
How move intelligence from central units to local devices?



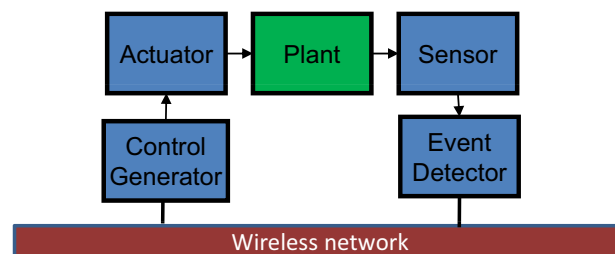
# A fundamental challenge in wireless control

A traditional conflict between

- time-driven, synchronous, sampled data **control engineering** and
- event-driven, asynchronous, ad hoc **wireless networking**




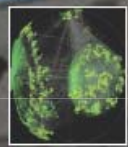


## A new architecture for wireless control



**1st Workshop on Distributed Estimation and Control in Networked Systems**  
(NecSys'09) [www.necsys.org](http://www.necsys.org)  
24-26 September, 2009, Venice, Italy





<p><b>TECHNICAL PROGRAM COMMITTEE</b></p> <p><b>A. Bemporad (U. Rome)</b>  <b>A. Bicchi (U. Pisa)</b>  <b>P.-K. Brauer (MIT)</b>  <b>V. Braslavski (U. Toronto)</b>  <b>C. Casadevall-Wu (CMU)</b>  <b>C. Corradini (U. Aquila)</b>  <b>M. Csiszar (U. Southampton)</b>  <b>C. Desbache (U. Chicago)</b>  <b>M. Dianati (Gallarate)</b>  <b>I. Durrant-Whyte (UCSD)</b>  <b>B. Duerschler (Delft U.)</b>  <b>D. Eragincin (U. Twente)</b>  <b>M. Ferdjani (Ecole Normale Sup.)</b>  <b>M. Franceschetti (INRIA)</b>  <b>D. Francis (U. Toronto)</b>  <b>C. Frazho (MIT)</b>  <b>M. Gaifor (UC Berkeley)</b>  <b>L. Georghiades (U. Warwick)</b>  <b>V. Gupta (U. Texas at Austin)</b>  <b>J. Hespanha (MIT)</b>  <b>D. Hill (AMS)</b>  <b>M. Johansson (KTH)</b>  <b>C. Langbort (USC)</b>  <b>J. Lyapunov (U. Rochester)</b>  <b>A. Maimon (Tel Aviv)</b>  <b>G. Mair (U. Melbourne)</b>  <b>A. Mudd (MIT)</b>  <b>R. Murray (U. California)</b>  <b>M. Oishi (U. Melbourne)</b>  <b>H. Olf (U. Stuttgart)</b>  <b>D. Palmer (Hong Kong U.)</b>  <b>A. Papachristodoulou (U. Oxford)</b>  <b>A. Paschke (U. of London)</b>  <b>W. Ren (UTAS)</b>  <b>A. Ribeiro (U. Paris)</b>  <b>A.P. Richard (UCL)</b>  <b>J. Shamma (Georgia Tech)</b>  <b>D. Stokich (New Jersey IT)</b>  <b>B. Stoumpos (CMU)</b>  <b>M. Stokich (CMU)</b>  <b>A. Spong (UTRC)</b>  <b>A.K. Spong (Georgia Tech)</b>  <b>H. Tamer (U. Delaware)</b>  <b>L. Wang (Harvard)</b>  <b>L. Xiao (Microsoft)</b></p>	<p style="text-align: center;"><b>CONTEXT</b></p> <p>Networked systems are complex dynamical systems composed of a large number of diverse systems interacting through a communication medium. These systems arise as natural models in many areas of engineering and sciences, such as power networks, distributed control systems, biological networks, and animal cooperative aggregation and flocking.</p> <p>There are both features that are common to all these systems. First, they deal with complex dynamics, second designing globally optimal behavior for these systems requires the resolution of large-scale optimization problems, which typically necessitate a prohibitive amount of computational effort. Desirable features of the operation of these systems include robustness to uncertainties and disturbances, and adaptability to unknown and changing conditions.</p> <p style="text-align: center;"><b>SCOPE</b></p> <p>The 1st Workshop on Distributed Estimation and Control in Networked Systems (NecSys'09) will focus on the most innovative mathematical methods proposed in the last few years for the analysis and design of networked systems.</p> <p>The aim of the workshop is to bring together researchers from control, computer science, communication, game theory, statistics, mathematics and other areas to discuss emerging topics in networked systems of common interest.</p> <p style="text-align: center;"><b>THEMATIC AREAS</b></p> <ul style="list-style-type: none"> <li>• Coordinated control and estimation over networks.</li> <li>• Consensus and distributed averaging.</li> <li>• Multi-agent systems and flocking.</li> <li>• Control with communication constraints and quantization.</li> <li>• Decentralized algorithms for computation over sensor networks.</li> <li>• Resilient algorithms and game algorithms.</li> <li>• Message passing, queueing and local propagation.</li> <li>• Graph models for networks: Percolation, Bethe's coding.</li> <li>• Distributed and decentralized signal processing.</li> <li>• Decentralized and cooperative optimization.</li> </ul> <p style="text-align: center;"><b>WORKSHOP ORGANIZATION</b></p> <p>The workshop will consist of two days of research presentations (24-25 Sep.) and one day of tutorials (26 Sep.).</p> <p>The research presentations will be 12 invited talks by international experts and 4 interactive sessions of contributed papers. The number of contributions per session will be limited to provide interaction with all participants.</p> <p>The tutorial day will consist of 4 double lectures conveying the most recent results related to the conference topics, specifically tailored to help students and scholars interested in an introduction to the area of networked systems.</p> <p style="text-align: center;"><b>VENUE</b></p> <p>The workshop will be hosted in an environment suitable for research interaction and spontaneous discussions, namely, the Cultural Center "Dors Ducale Artigianeri", <a href="http://www.dorsducale-artigianeri.it/">http://www.dorsducale-artigianeri.it/</a></p>	<p><b>ORGANIZING COMMITTEE</b></p> <p><b>General Chair:</b>  <b>Stefano Zampieri (UCL)</b></p> <p><b>Vice-General Chair:</b>  <b>Paolo Bolzerni (UCL)</b>  <b>Andrius Dolezal (MIT)</b></p> <p><b>Tech. Program Co-Chair:</b>  <b>Karl H. Johansson (KTH)</b>  <b>Andrius Dolezal (MIT)</b></p> <p><b>Advisory Committee:</b>  <b>Stephen Boyd (Stanford)</b>  <b>Richard Marden (MIT)</b>  <b>P.K. Spong (UTRC)</b>  <b>David Stokich (CMU)</b>  <b>Michael Murray (Oxford)</b>  <b>John Lyapunov (MIT)</b></p> <p><b>Publicity Chair:</b>  <b>Angela Cenedella (UCL)</b></p> <p><b>Local Arrangements Chair:</b>  <b>Alessandra Ciochi (UCL)</b></p>
--	---	--

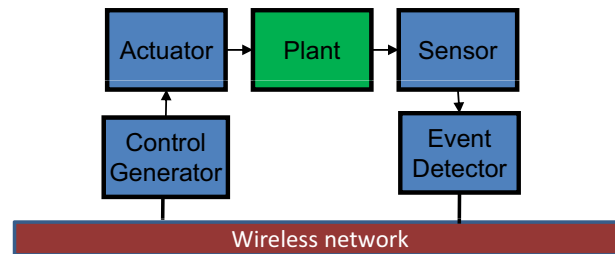
**Important Dates**  
 Submitted full paper due: March 27, 2009  
 Notification of acceptance: May 31, 2009  
 Final version due: June 23, 2009

**Submission Instructions**  
 Authors are invited to submit full papers of up to 8 pages and 6 pages at final submission.  
 Please visit workshop website [www.necsys.org](http://www.necsys.org) for more information.

## Outline

- Introduction
- Motivation: control over wireless networks
- Architecture for event-based control
- Design of event detector and control generator
- Extensions
  - Multiple control loops
  - Influence of communication losses
  - Event-based PID control
- Conclusions

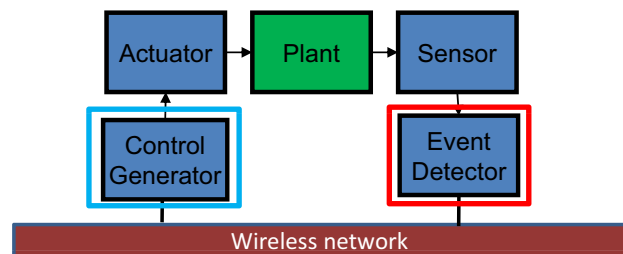
# Architecture for event-based control



Åström, 2007, Rabi and J., WICON, 2008

## When to transmit?

- Medium access control-like mechanism at sensor
  - E.g., threshold crossing



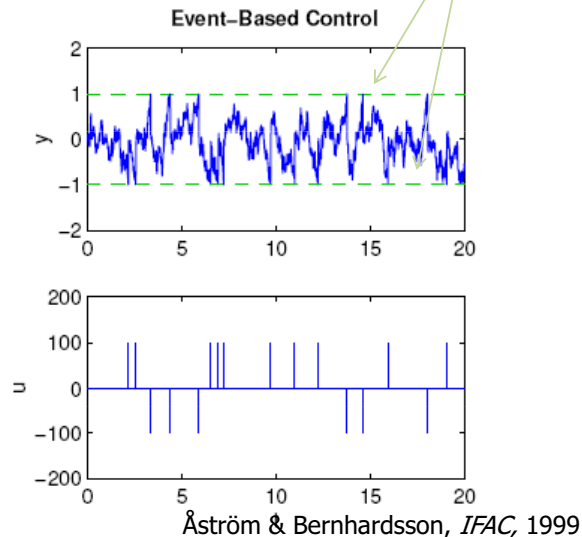
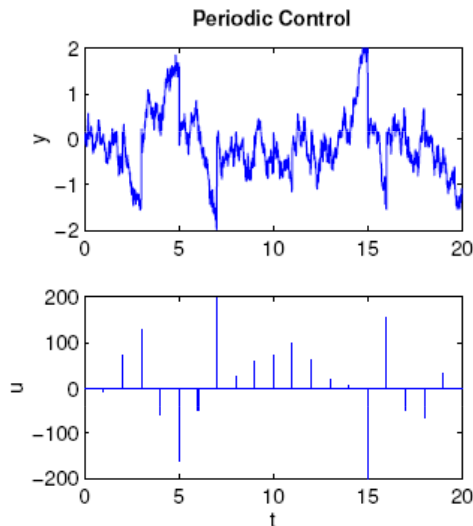
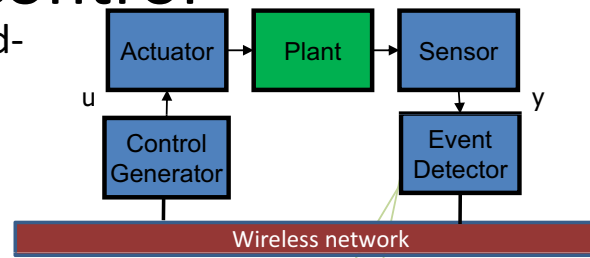
## How to control?

- Execute control law over fixed control alphabet
  - E.g., piecewise constant controls, impulse control

Rabi et al., 2008

# Example: Fixed threshold with impulse control

- Event-detector implemented as fixed-level threshold at sensor
- Event-based impulse control better than periodic impulse control



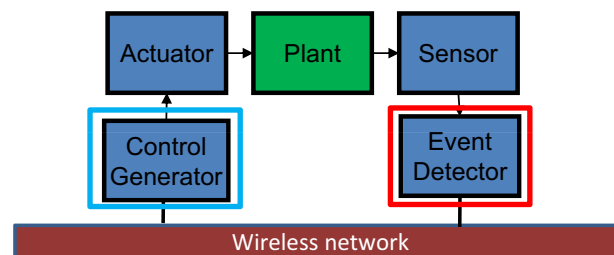
## Outline

- **Introduction**
- **Motivation:** control over wireless networks
- **Architecture** for event-based control
- **Design** of event detector and control generator
- **Extensions**
  - Multiple control loops
  - Influence of communication losses
  - Event-based PID control
- **Conclusions**

# Design of control generator and event detector

1. Impulse
2. Zero order hold
3. Higher order hold

1. Fixed threshold
2. Adaptive sampling



## Plant model and control cost

**Plant**  $dx = udt + dv,$

$v$  is a Wiener process:  $E(v(t+s) - v(t))^2 = |s|$

**Cost function**  $V = \frac{1}{T} E \int_0^T x^2(t) dt.$

Discussion later on how to treat general dynamics, sensor noise etc

# Periodic impulse control

Impulse applied at events  $t_k$

$$u(t) = -x(t_k)\delta(t - t_k),$$

**Periodic** reset of state every event.

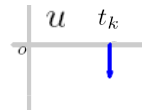
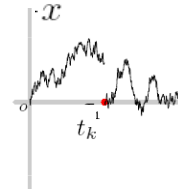
State grows linearly as

$$E(v(t+s) - v(t))^2 = |s|$$

between sample instances, because  $dx = udt + dv$ ,

Average variance over sampling period  $h$  is  $\frac{1}{2}h$  so the cost is

$$V_{PIH} = \frac{1}{2}h.$$



Åström, 2007

# Periodic ZoH control

Traditional sampled-data control theory gives that

$V = \frac{1}{h} \int_0^h E x^2(t) dt$  is minimized for the sampled system

$$x(t+h) = x(t) + hu(t) + e(t),$$

with

$$u = -Lx = \frac{1}{h} \frac{3 + \sqrt{3}}{2 + \sqrt{3}} x$$

derived from

$$S = \Phi^T S \Phi + Q_1 - L^T R L, \quad L = R^{-1}(\Gamma^T S \Phi + Q_{12}^T), \quad R = Q_2 + \Gamma^T S \Gamma,$$

The minimum gives the cost

$$V_{PZOH} = \frac{3 + \sqrt{3}}{6} h$$

Åström, 2007

# Event-based impulse control with fixed threshold

Suppose an event is generated whenever

$$|x(t_k)| = a$$

generating impulse control

$$u(t) = -x(t_k)\delta(t - t_k),$$

One can show that the average time  
between two events is

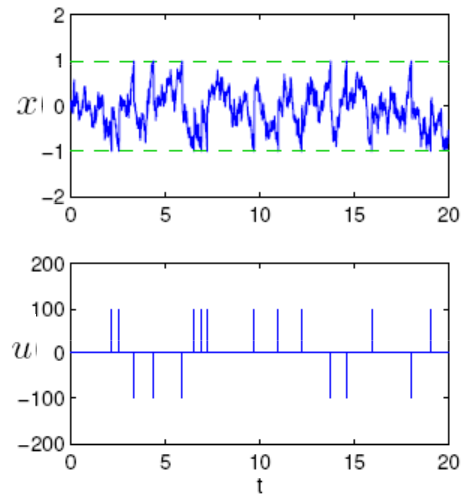
$$h_E := E(T_{\pm d}) = E(x_{T_{\pm d}}^2) = a^2$$

and that the pdf of  $x$  is triangular:

$$f(x) = (a - |x|)/a^2$$

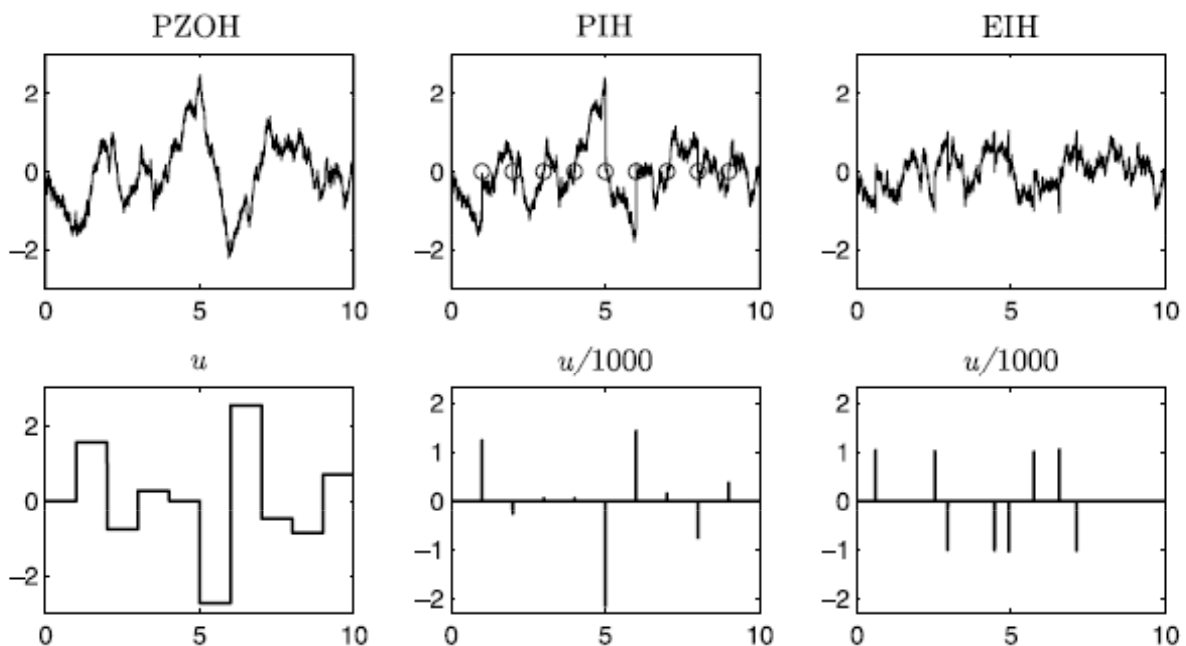
The cost is

$$V_{EIH} = \frac{a^2}{6} = \frac{h_E}{6}$$



Åström, 2007

## Comparison



Åström, 2007



CALL FOR PAPERS

IEEE Transactions on Industrial Informatics

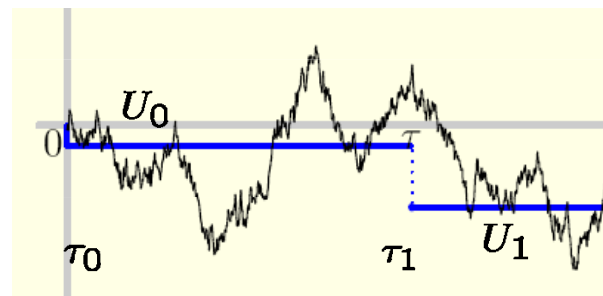
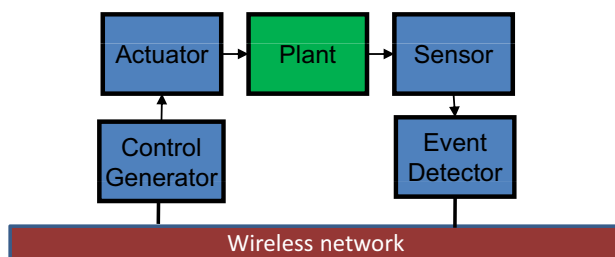
Special Section on:



“Control of Wireless Networks:  
Foundations, Networking, Applications”

<p><b>Special Section Guest Editors</b></p> <p><b>Carlo Fischione</b> ACCESS Linnaeus Centre Electrical Engineering Royal Institute of Technology SE-100 44 Stockholm, Sweden Phone: +46 8 7907466 Fax: +46 8 7907329 E-mail: carlofi@ee.kth.se</p> <p><b>Karl Henrik Johansson</b> ACCESS Linnaeus Centre Electrical Engineering Royal Institute of Technology SE-100 44 Stockholm, Sweden Phone: +46 8 7907321 Fax: +46 8 7907329 E-mail: kallej@ee.kth.se</p> <p><b>Mikael Johansson</b> ACCESS Linnaeus Centre Electrical Engineering Royal Institute of Technology SE-100 44 Stockholm, Sweden Phone: +46 8 7907436 Fax: +46 8 7907329 E-mail: mikaelj@ee.kth.se</p> <p><b>Andreas Willig</b> Telecommunication Networks Group Technical University Berlin Sekt. FT-5 Einsteinufer 25 10587 Berlin Germany Phone: +49 30 314 23836 Fax: +49 30 314 23818 E-mail: awillig@ieee.org</p>	<p><b>Background:</b> Wireless technologies, nowadays a commodity in personal and data communication, have the potential of providing significant benefits in factory and industrial automation systems. The wireless way of communicating makes plant setup and modification easier, more flexible and cost-efficient. It provides a natural approach for communication with mobile machines and robots, where fixed cables are in constant danger of breaking. The industrial interest in wireless solutions is growing rapidly: standardization efforts such as Wireless HART and ISA 100 are underway, and hardware for embedded wireless is dropping in price. It is expected that wireless technologies will be integrated into distributed control systems on a broad scale. However, there are still open issues about reliability, performance, and data security of the wireless control loop that may limit the rate of adoption.</p> <p>The goal of the special section is to attract theoretical and practical papers attacking the main issues and problems regarding the adoption of wireless technologies for networked control, ranging from theoretical foundations to the reporting of implementation experiences and applications.</p> <p>Topics for the special section include, but are not limited to, the following:</p> <ul style="list-style-type: none"> <li>- Industrial wireless networking for efficient, reliable and timely data transmission: low-layer (MAC/link/physical-layer) protocols and multi-hop (routing, transport) protocols</li> <li>- Theory and methodology for reliable and robust wireless networked control</li> <li>- Novel solutions for re-configurable, resilient and fault tolerant wireless control</li> <li>- Security for industrial wireless systems</li> <li>- Middleware and higher-layer support for wireless networked control systems</li> <li>- Hybrid wired/wireless networked control systems</li> <li>- Experiences from industrial deployments of wireless control</li> <li>- Innovative wireless networked control applications</li> </ul> <p>Submissions must represent original material that have been neither submitted to, nor published in, any other journal. Extended versions of papers previously published in conference proceedings may be eligible for consideration, provided that the authors inform the Special Section Guest Editors at the time of submission.</p> <p><b>Manuscript preparation and submission:</b> Follow the guidelines in “Information for Authors” in <a href="http://ieee-ies.org/it/">http://ieee-ies.org/it/</a> Submit using Manuscript Central only <a href="http://mc.manuscriptcentral.com/it">http://mc.manuscriptcentral.com/it</a></p> <p><b>Paper submission deadline:</b> October 31, 2009 <b>Expected publication date:</b> November 2010 (tentative)</p> <p>Note: The recommended papers for the section are subject to final approval by the Editor in Chief. Some papers may be published outside the special section, at his discretion.</p>
--	--

# Event-based ZoH control with adaptive sampling



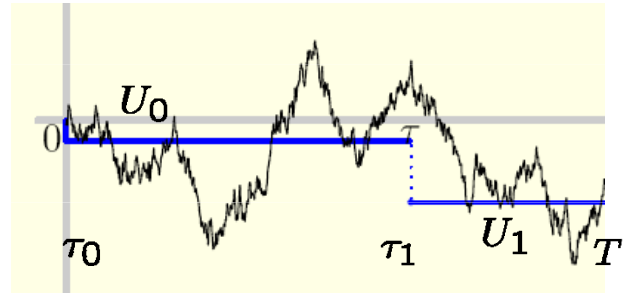
How choose  $\{U_i\}$  and  $\{\tau_i\}$  to minimize  $V = \frac{1}{T} E \int_0^T x^2(t) dt$ .

# Controlled Brownian motion with one sampling event

$$dx_t = u_t dt + dB_t$$

$$\min_{U_0, U_1, \tau} J = \min_{U_0, U_1, \tau} \mathbf{E} \int_0^T x_s^2 ds$$

$$= \min_{U_0, U_1, \tau} \left[ \mathbf{E} \int_0^\tau x_s^2 ds + \mathbf{E} \int_\tau^T x_s^2 ds \right]$$

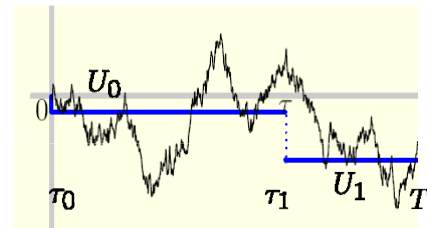


A joint optimal control and optimal stopping problem

Rabi et al., 2008

$$dx_t = u_t dt + dB_t$$

$$\min_{U_0, U_1, \tau} J = \min_{U_0, U_1, \tau} \mathbf{E} \int_0^T x_s^2 ds$$



If  $\tau$  chosen deterministically (**not depending on  $x_t$** )  
and  $x_0 = 0$ :

$$U_0^* = 0 \quad U_1^* = -\frac{3x_{T/2}}{T} \quad \tau^* = T/2$$

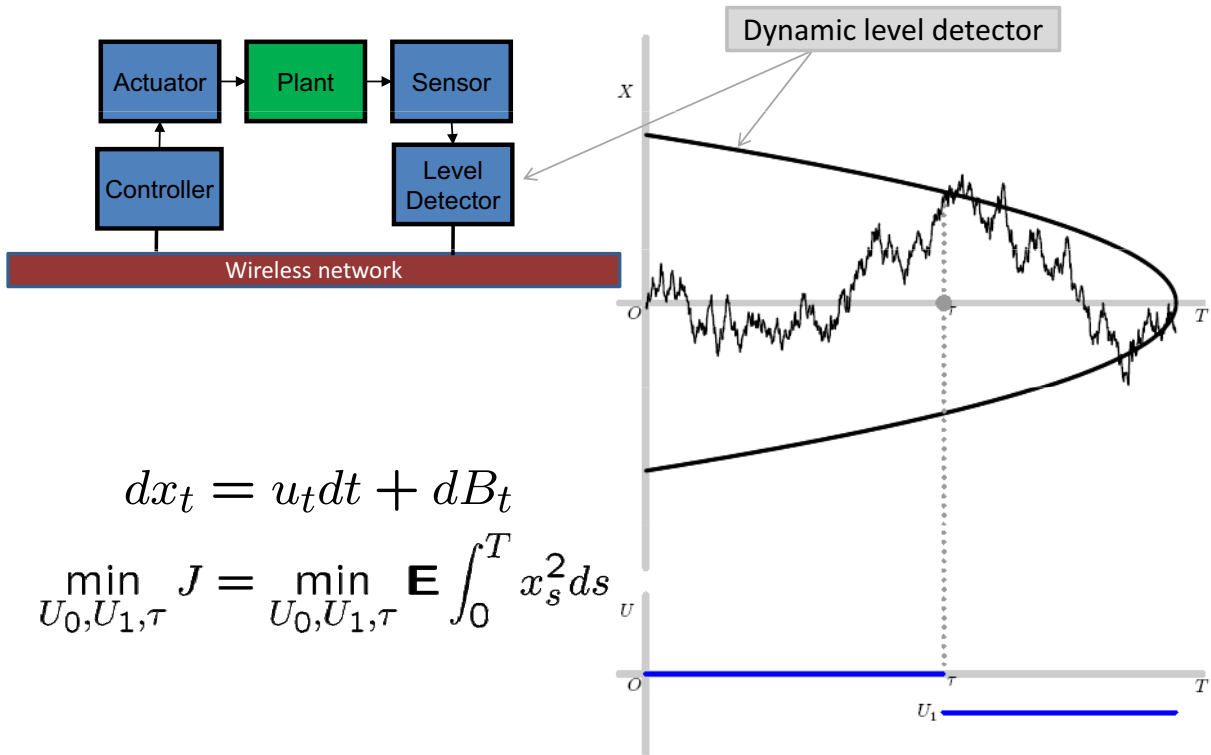
If  $\tau$  is event-driven (depending on  $x_t$ ) and  $x_0 = 0$ :

$$U_0^* = 0 \quad U_1^* = -\frac{3x_{\tau^*}}{2(T - \tau^*)}$$

$$\tau^* = \inf \left\{ t : \underbrace{x_t^2}_{\text{Envelope}} \geq \sqrt{3}(T - t) \right\}$$

Envelope defines optimal level detector

# Optimal level detector



$$dx_t = u_t dt + dB_t$$

$$\min_{U_0, U_1, \tau} J = \min_{U_0, U_1, \tau} \mathbf{E} \int_0^T x_s^2 ds$$

## Proof

$$\min_{U_0, U_1, \tau} J = \min_{U_0, U_1, \tau} \mathbf{E} \int_0^T x_s^2 ds = \min_{U_0, U_1, \tau} \left[ \mathbf{E} \int_0^\tau x_s^2 ds + \mathbf{E} \int_\tau^T x_s^2 ds \right]$$

$$\mathbf{E} \left\{ \int_\tau^T x_s^2 ds \mid \tau, x_\tau, U_1 \right\} = \left[ x_t = x_\tau + \int_\tau^t U_1 ds + \int_\tau^t dB_s \right]$$

$$= \int_\tau^T \mathbf{E} \left\{ \left[ x_\tau^2 + U_1^2 (t - \tau)^2 + (B_t - B_\tau)^2 + 2x_\tau U_1 (t - \tau) \right. \right.$$

$$\left. \left. + 2x_\tau (B_t - B_\tau) + 2U_1 (t - \tau) (B_t - B_\tau) \right] \right\} dt$$

$$= \left[ \mathbf{E} B_t = 0, \mathbf{E} B_t^2 = t, \delta := T - \tau \right] = \delta x_\tau^2 + \frac{\delta^3}{3} U_1 + \frac{\delta^2}{2} + \delta^2 x_\tau U_1$$

$$= \frac{\delta}{4} x_\tau^2 + \delta \left( \frac{x_\tau \sqrt{3}}{2} + \frac{\delta U_1}{\sqrt{3}} \right) + \frac{\delta^2}{2}$$

Hence, optimal control  $U_1^* = U_1^*(x_\tau, T - \tau) = -\frac{3x_\tau}{2(T - \tau)}$

$$J(U_0, U_1^*, \tau) = \mathbf{E} \int_0^\tau x_s^2 ds + \mathbf{E} \left\{ \frac{T-\tau}{4} x_\tau^2 + \frac{(T-\tau)^2}{2} \right\}$$

If  $\tau$  chosen deterministically (not depending on  $x_t$ ) and  $x_0 = 0$ :

$$J(U_0, U_1^*, \theta) = \frac{\theta^3}{3} U_0^2 + \frac{\theta^2}{2} + \frac{T-\theta}{4} (U_0^2 \theta^2 + \theta) + \frac{(T-\theta)^2}{2}$$

Hence,

$$U_0^* = 0 \quad U_1^* = -\frac{3x_{T/2}}{T} \quad \tau^* = T/2$$

which gives

$$J(U_0^*, U_1^*, \tau^*) = \frac{5T^2}{16}$$

If  $\tau$  is event-driven (depending on  $x_t$ ) and  $x_0 = 0$ :

$$\begin{aligned} J(U_0, U_1^*, \tau) &= \mathbf{E} \int_0^\tau x_s^2 ds + \mathbf{E} \left\{ \frac{T-\tau}{4} x_\tau^2 + \frac{(T-\tau)^2}{2} \right\} = \dots \\ &= \frac{T^2}{2} + \frac{U_0^2 T^3}{3} - \mathbf{E} \left\{ \left( \frac{x_\tau \sqrt{3}}{2} + \frac{(T-\tau)U_0}{\sqrt{3}} \right)^2 (T-\tau) \right\} \\ &= \frac{T^2}{2} - \frac{3}{4} \mathbf{E} \{ x_\tau^2 (T-\tau) \} \end{aligned}$$

because from symmetry  $U^* = 0$ .

Find  $\tau$  that maximizes  $f(x_\tau, \tau) = \mathbf{E} \{ x_\tau^2 (T-\tau) \}$

Find  $\tau$  that maximizes  $f(x_\tau, \tau) = \mathbf{E}\{x_\tau^2(T - \tau)\}$

Suppose there exists smooth  $g(x, t)$  such that

$$\begin{aligned}g(x, t) &\geq x^2(T - t) \\ \frac{1}{2}g_{xx}(x, t) + g_t(x, t) &= 0\end{aligned}$$

Then, for  $0 \leq t \leq \tau \leq T$ ,

$$\begin{aligned}f(x_\tau, \tau) &= \mathbf{E}\{x_\tau^2(T - \tau)\} \leq \mathbf{E}\{g(x_\tau, \tau)\} = g(x_t, t) + \mathbf{E} \int_t^\tau dg(x_\tau, \tau) \\ &= [\text{Ito formula}] = g(x_t, t) + \mathbf{E} \int_t^\tau \left(\frac{1}{2}g_{xx} + g_t\right) dt \\ &= g(x_t, t)\end{aligned}$$

Hence,  $g$  is an upper bound for the expected reward.

We next show that equality can be achieved.

$$g(x_t, t) = \frac{\sqrt{3}}{1 + \sqrt{3}} \left( \frac{x_t^4}{6} + x_t(T - t) + \frac{(T - t)^2}{2} \right)$$

is a solution to

$$\frac{1}{2}g_{xx}(x, t) + g_t(x, t) = 0$$

Moreover,

$$\begin{aligned}g(x_t, t) - x_t^2(T - t) &= \frac{1}{2(1 + \sqrt{3})} \left( \frac{x_t^4}{3} - \frac{2}{\sqrt{3}}x_t^2(T - t) + (T - t)^2 \right) \\ &= \frac{1}{2(1 + \sqrt{3})} \left( \frac{x_t^4}{\sqrt{3}} - (T - t)^2 \right) = 0\end{aligned}$$

If  $x_t^2 = \sqrt{3}(T - t)$ .

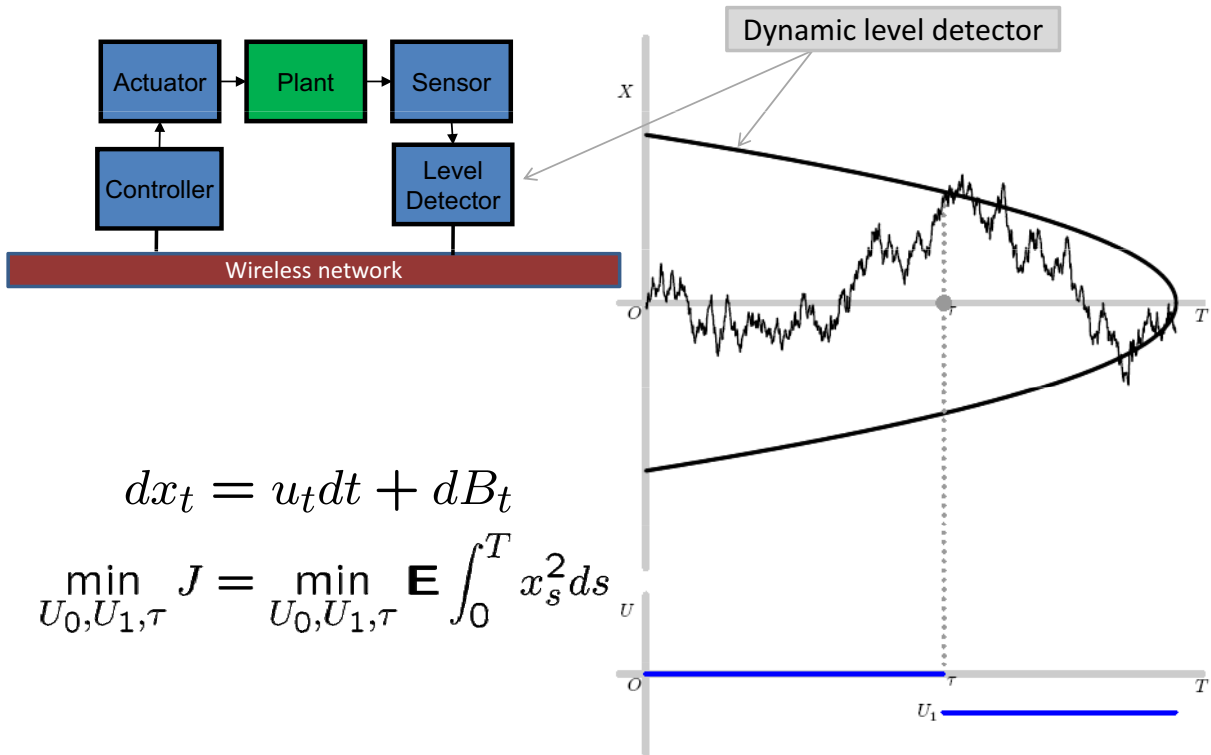
Hence, the optimal sampling time is

$$\tau^* = \inf\{t : x_t^2 \geq \sqrt{3}(T - t)\}$$

which gives

$$J(U_0^*, U_1^*, \tau^*) = \frac{T^2}{8}$$

# Optimal level detector



Stockholm City of Innovation

APRIL 12–16, 2010

## CPSWEEK 2010 IN STOCKHOLM

—the place to meet next year!

Stockholm has a long history, but it is also a city of change and innovation. Sweden's capital is an acknowledged leader in Information and Communication Technologies, Life Sciences and Medicine. The Cyber-Physical Systems Week in Stockholm will be able to benefit from Sweden's unique position in the areas of cyber-physical systems, bringing together academia and industry at KTH Royal Institute of Technology.

Five co-located conferences: RTAS, IPSN, HSCC, CPSC, LCTES

Stockholm provides an excellent innovative center with several well known universities (KTH, Karolinska Institutet, Uppsala University etc.), research organizations and institutes (Nobel Foundation, Royal Academy of Sciences, Swedish institute of Computer Science etc), and industries (ABB, AstraZeneca, Ericsson, Scania, Volvo etc.) In order to promote industrial interest and participation, we are planning for an exhibition and workshops specifically targeting industry.

Welcome to the CPSWeek 2010,  
The Organizing Committee



↑ CPSWeek 2010 will be hosted by KTH —the largest and oldest technical university in Sweden—located on a charming campus in downtown Stockholm.

↗ The banquet will be a spectacular event held at Stockholm City Hall which is the venue of the annual Nobel Banquet, held every year after the Nobel Prize Award Ceremony to celebrate great scientific and cultural achievements.

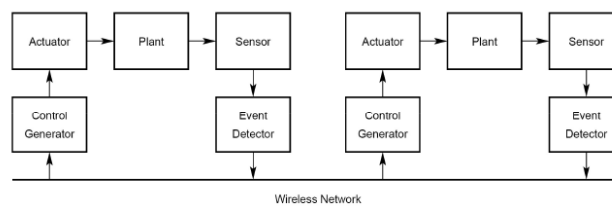
MORE INFORMATION ABOUT CPSWEEK 2010:  
[www.cpsweek2010.se](http://www.cpsweek2010.se)

# Outline

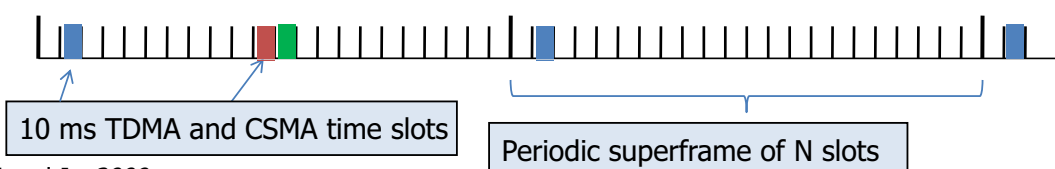
- **Introduction**
- **Motivation:** control over wireless networks
- **Architecture** for event-based control
- **Design** of event detector and control generator
- **Extensions**
  - Multiple control loops
  - Influence of communication losses
  - Event-based PID control
- **Conclusions**

## Multiple control loops

- N control loops sharing the same wireless network



- Time Division Multiple Access or contention-based medium access



# System model and performance measures

Plant  $dx_t = dW_t + u_t dt, x(0) = x_0,$

Sampling events  $\mathcal{T} = \{\tau_0, \tau_1, \tau_2, \dots\},$

Impulse control  $u_t = \sum_{n=0}^{\infty} x_{\tau_n} \delta(\tau_n)$

Average sampling rate  $R_\tau = \limsup_{M \rightarrow \infty} \frac{1}{M} \mathbb{E} \left[ \int_0^M \sum_{n=0}^{\infty} \mathbf{1}_{\{\tau_n \leq M\}} \delta(s - \tau_n) ds \right]$

Average cost  $J = \limsup_{M \rightarrow \infty} \frac{1}{M} \mathbb{E} \left[ \int_0^M x_s^2 ds \right]$

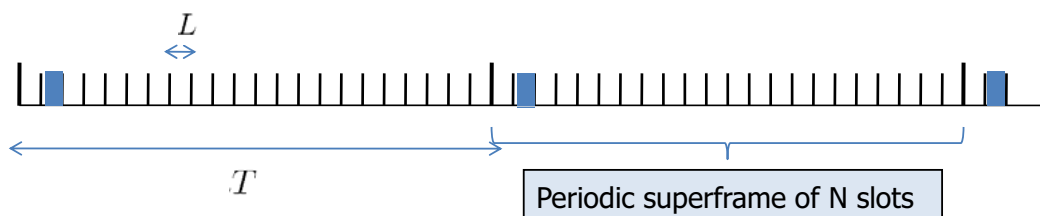
## Periodic sampling of multiple loops

Sampling events  $\tau_n = nT$  for  $n \geq 0$

Slot length L gives  $T = NL$

Average sampling rate  $R_{\text{Periodic}} = \frac{1}{T}$

Average cost  $J_{\text{Periodic}} = \frac{T}{2}$



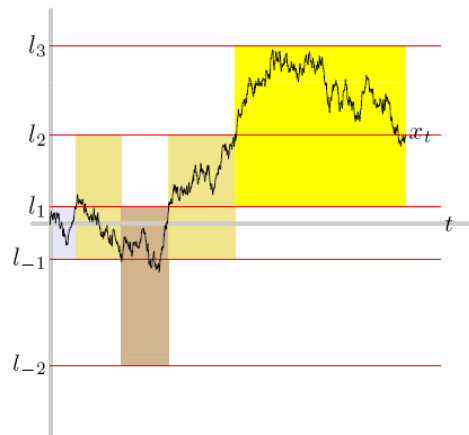


# Level-triggered control

Ordered set of levels  $\mathcal{L} = \{\dots, l_{-2}, l_{-1}, l_0, l_1, l_2, \dots\}$   $l_0 = 0$

Multiple levels needed because we allow packet loss

Lebesgue sampling  $\tau = \inf \{ \tau | \tau > \tau_i, x_\tau \in \mathcal{L}, x_\tau \notin x_{\tau_i} \}$



# Level-triggered control

For Brownian motion, equidistant sampling is optimal

$$\mathcal{L}^* = \{k\Delta | k \in \mathbb{Z}\}$$

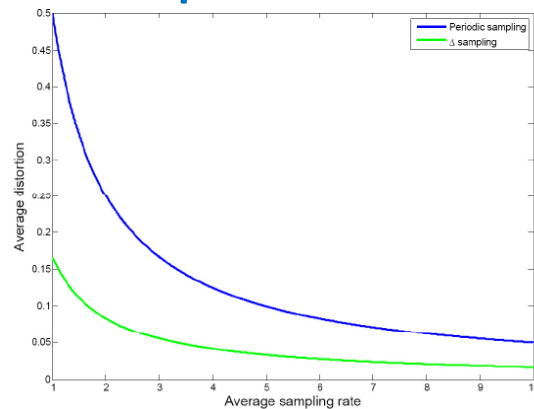
First exit time

$$\tau_\Delta = \inf \{ \tau | \tau \geq 0, x_\tau \notin (\xi - \Delta, \xi + \Delta), x_0 = \xi \}$$

Average sampling rate  $R_\Delta = \frac{1}{\mathbb{E}[\tau_\Delta]} = \frac{1}{\Delta^2}$ ,

Average cost  $J_\Delta = \frac{\mathbb{E}[\int_0^{\tau_\Delta} x_s^2 ds]}{\mathbb{E}[\tau_\Delta]} = \frac{\Delta^2}{6}$ .

## Comparison between **periodic** and **event-based** control



$T = \Delta^2$  gives equal average sampling rate for periodic control and event-based control

Event-based impulse control is 3 times better than periodic impulse control

What about the influence of communication losses?  
When is event-based better and vice versa?

## Influence of communication losses

Times when packets are successfully received  $\rho_i \in \{\tau_0 = 0, \tau_1, \tau_2, \dots\}$ ,

$$\{\rho_0 = 0, \rho_1, \rho_2, \dots\} \cdot \rho_i \geq \tau_i,$$

Average rate of packet reception

$$R_\rho = \limsup_{M \rightarrow \infty} \frac{1}{M} \mathbb{E} \left[ \int_0^M \sum_{n=0}^{\infty} \mathbf{1}_{\{\rho_n \leq M\}} \delta(s - \rho_n) ds \right] = p \cdot R_\tau$$

Define the times between successful packet receptions  $\rho_{(p, \Delta)}$

$$\text{Average cost } J_p = \limsup_{T \rightarrow \infty} \frac{1}{T} \mathbb{E} \left[ \int_0^T x_s^2 ds \right] = \frac{\mathbb{E} \left[ \int_0^{\rho_{(p, \Delta)}} x_s^2 ds \right]}{\mathbb{E} [\rho_{(p, \Delta)}]}$$

# IID losses

## Proposition

Suppose packet losses are IID. Then,

$$J_p = \frac{\Delta^2 (5p + 1)}{6(1 - p)}$$

## Remark

Event-based control is better than period control under IID losses if

$$\frac{(1 + 5p)}{3(1 - p)} \geq 1$$

So if the loss probability

$$p \geq 0.25$$

then traditional periodically sampled control is preferable.

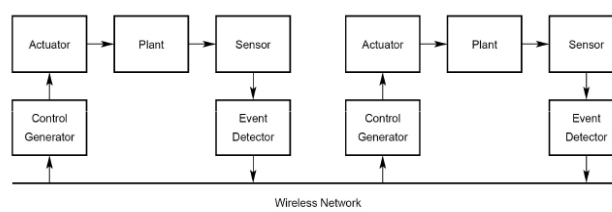
Rabi and J., 2009

## Losses depending on the other loops

Suppose the loss processes across the different loops are independent, so that the sample streams of the other sensors only matter through their average behaviour (cf., *Poisson arrivals see time averages*, PASTA)

The likelihood that a sample generated in one loop faces at least one competing transmission is then

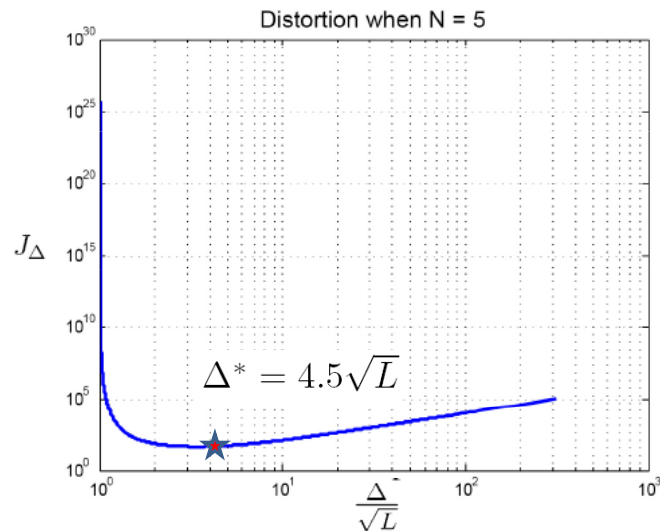
$$p = 1 - \left(1 - \frac{L}{\Delta^2}\right)^{N-1}$$



# Losses depending on the other loops

Average cost  $J_{\Delta} = \frac{L(6 - 5\beta^{N-1})}{6\beta^{N-1}(1 - \beta)}$   $\beta = 1 - \frac{L}{\Delta^2}$

gives trade-off between control performance and network resources

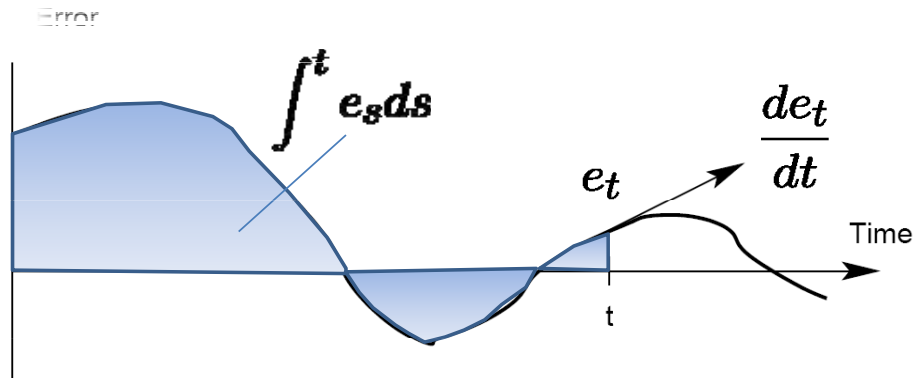
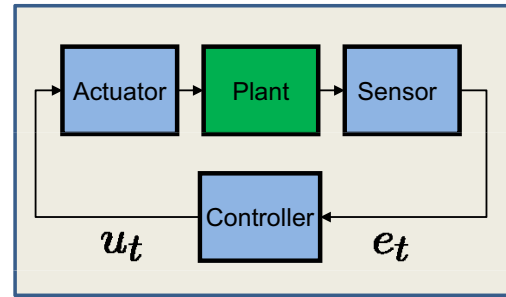


## Outline

- **Introduction**
- **Motivation:** control over wireless networks
- **Architecture** for event-based control
- **Design** of event detector and control generator
- **Extensions**
  - Multiple control loops
  - Influence of communication losses
  - Event-based PID control
- **Conclusions**

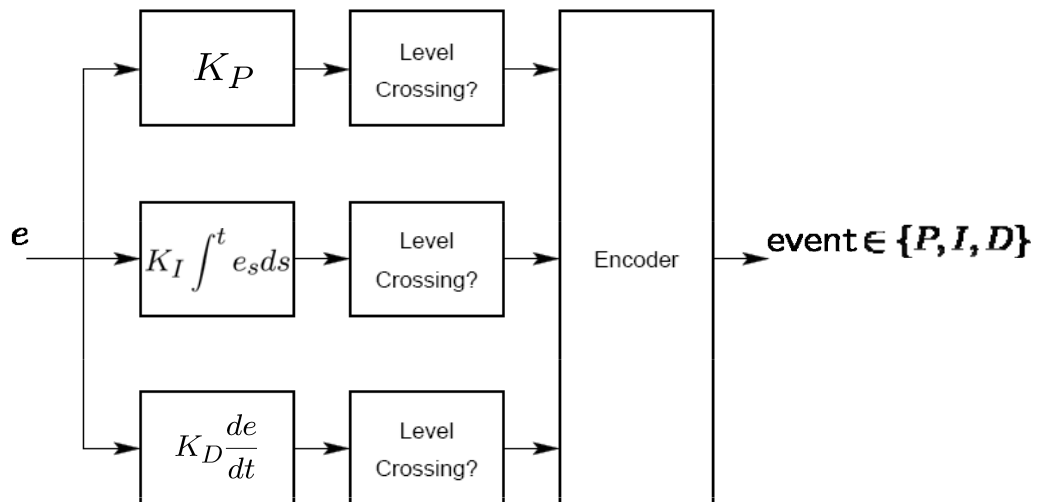
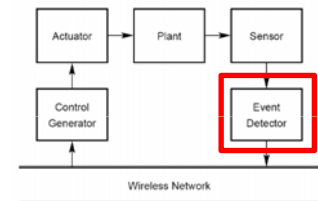
# Proportion-Integral-Derivative control

$$u_t = K_P e_t + K_I \int^t e_s ds + K_D \frac{de_t}{dt}$$

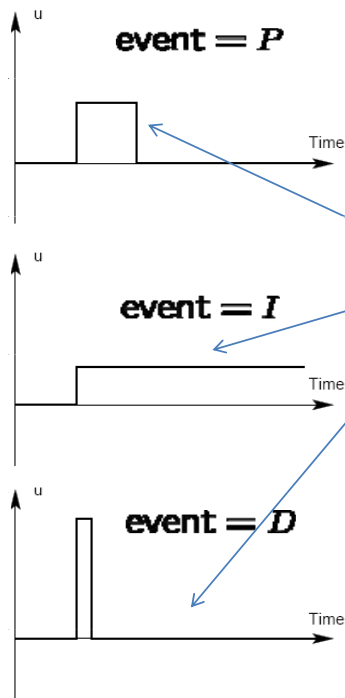
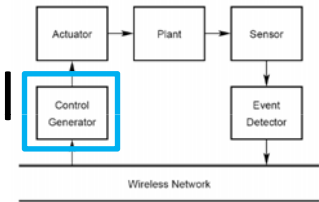


**How extend PID control to event-based control?**

## Event-detector for PID control



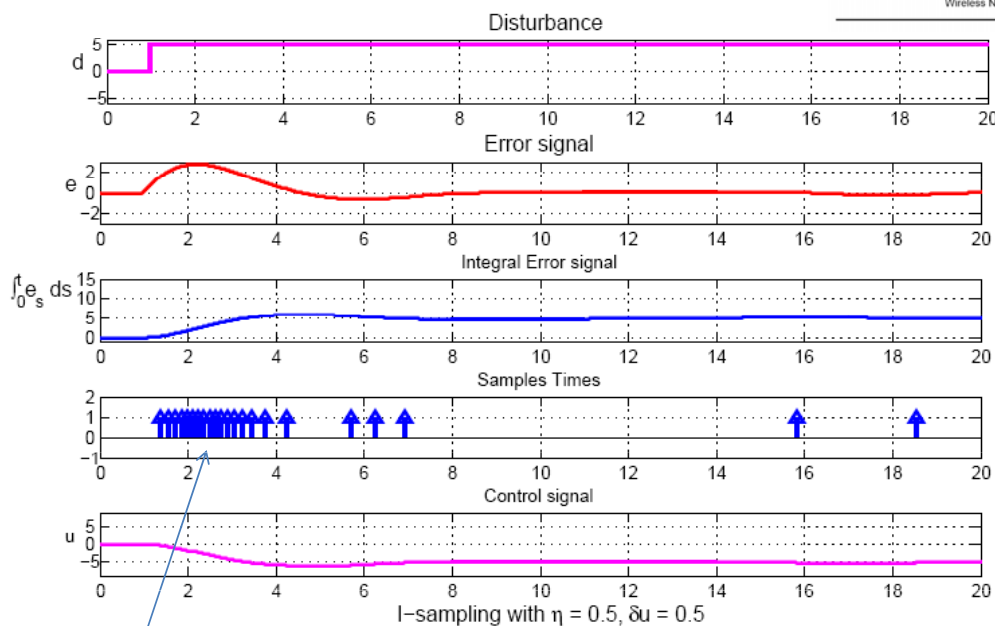
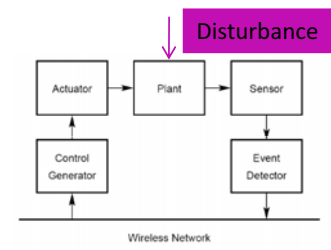
# Control generator for PID control



Control alphabet consists of three symbols, which are activated depending on the event

Rabi and J., WICON, 2008

## Example: Integral control



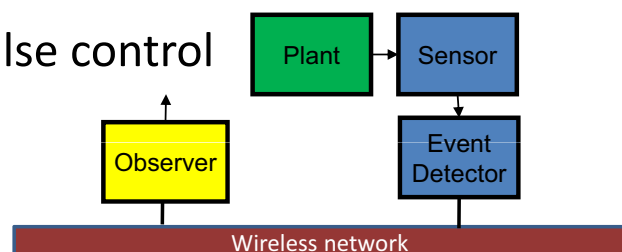
Communicate only when integral error triggers events

# Outline

- **Introduction**
- **Motivation:** control over wireless networks
- **Architecture** for event-based control
- **Design** of event detector and control generator
- **Extensions**
  - Multiple control loops
  - Influence of communication losses
  - Event-based PID control
- **Conclusions**

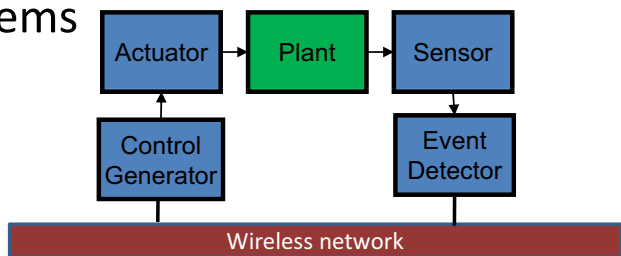
## What was not covered?

- Observations with sensor noise
- Linear and nonlinear plant dynamics
  - Adaptive sampling based on policy iterations
- Deterministic setting: Lyapunov stability etc
  - See Reading recommendations
- Event-based estimation
  - Similarities with impulse control



# Conclusions

- Wide range of emerging wireless control applications
- Event-based control to support asynchronous networking
  - “If it ain’t broken, don’t fix it” [Åström]
- Event-based control architecture allows network nodes to take local decisions
- Event detector and control generator choice leads to interesting theoretical problems



<http://www.ee.kth.se/~kallej>

## Reading recommendations

Lectures are mainly based on the following material:

- K. J. Åström and B. Bernhardsson, “Comparison of periodic and event based sampling for first-order stochastic systems”, IFAC World Congress, 1999.
- K. J. Åström and B. Bernhardsson, “Comparison of Riemann and Lebesgue sampling for first order stochastic systems”, IEEE CDC, 2002.
- M. Rabi, “Packet based Inference and Control”, PhD thesis, University of Maryland, 2006
- K. J. Åström, “Event based control”, In Analysis and Design of Nonlinear Control Systems: In Honor of Alberto Isidori. Springer Verlag. 2007.
- T. Henningsson, “Event-Based Control and Estimation with Stochastic Disturbances”, Lic Thesis, Lund University, 2008.
- M. Rabi and K. H. Johansson, “Event-triggered strategies for industrial control over wireless networks”, WICON, 2008.
- M. Rabi, K. H. Johansson, and M. Johansson, “Optimal stopping for event-triggered sensing and actuation”, IEEE CDC, 2008.
- M. Rabi and K. H. Johansson, “Optimal stopping for updating controls”, International Workshop on Sequential Methods, 2009.
- M. Rabi and K. H. Johansson, “Scheduling packets for event-triggered control”, ECC, 2009.



# Reading recommendations

## **Stochastic control and optimal stopping:**

- Bernt K. Øksendal. Stochastic Differential Equations: An Introduction with Applications. Berlin: Springer, 2003
- Karl Johan Åström, Introduction to Stochastic Control Theory, Dover, New York, 2006. Reprint. Originally published by Academic Press 1970
- Harold J. Kushner, On the optimum timing of observations for linear control systems with unknown initial state, IEEE Trans. Automatic Control, AC-9 (1964), pp. 144–150.
- Goran Peskir and Albert Shiryaev, Optimal stopping and free-boundary problems, Lectures in Mathematics ETH Zürich, Birkhäuser Verlag, Basel, 2006.

## **Related recent work but in deterministic settings:**

- Dimos Dimarogonas, MIT; Maurice Heemels, TUE ; Michael Lemmon, UND; Paulo Tabuada, UCLA; etc.