## Hash tables

Johan Montelius

KTH
HT23

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- A linked list: ok, but $O(n)$ lookup operation.
- A sorted tree : much better, $O(\lg (n))$ operations (add and lookup).
- A sorted array : binary search gives us $O(\lg (n))$ lookup, but ...
zip code data base


## zip code data base

A file containing: zip code, name and population.

111 15,STOCKHOLM<br>, 3<br>111 20,STOCKHOLM<br>, 50<br>111 21,STOCKHOLM<br>, 344<br>111 22,STOCKHOLM<br>, 149

## An array of nodes

```
pubic class Zip {
    Node[] data;
    private class Node {
    String code;
    String name;
    Integer pop;
    }
```


## zip codes are ordered

```
public Zip(String file) {
    data = new Node[10000];
        :
    data[i++] = new Node(row[0], row[1], Integer.valueOf(row
}
```


## binary search

```
public String binary(String zip) {
    int mn = 0;
    int mx = max;
    while (true) {
        int index = (mn + mx)/2;
        int cmp zip.compareTo(data[index].code);
        if (cmp == 0) {
                return data[index].name;
        }
    }
    return null;
```


## use zip code as index

```
public Zip(String file) {
    data = new Node[100000];
    :
    Integer key = Integer.valueOf(row[O].replaceAll("\\s",
    data[key] = new Node(key, row[1], Integer.valueOf(row[2
}
```

perfect ....
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- O(1) lookup
perfect ....
- $O(1)$ lookup
- what's the problem?


## using key as index



## the hash function



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- Keys are evenly distributed over the range of indices.
- The range $0 . . k$ is resonable (?) small.
- Few (?) keys map to the same index.


## example of hash function

```
public static int hash(Integer key, int M) {
    return key % M;
}
```


## how about a string

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```
int R = 31;
public static Integer hash(String key, int M) {
    char[] chars = key.toCharArray();
    int value = 0;
    for (int i = 0; i < chars.length; i++) {
        value = (R * value + chars[i]) % M;
    }
    return value;
}
```


## collisions

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What should we do if we have collisions?

## buckets

key $\rightarrow h($ key $)$


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We need to search for the item.
linear probing
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## clustering



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what about remove


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- buckets: not a problem
- open adressing: .... problem
R.I.P

Replace removed key/values with a tombstone.

## increase the size

In a dynamic array we simply copied everything to a larger array ... problem?

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Computing a chryptographic hash is more expensive.

