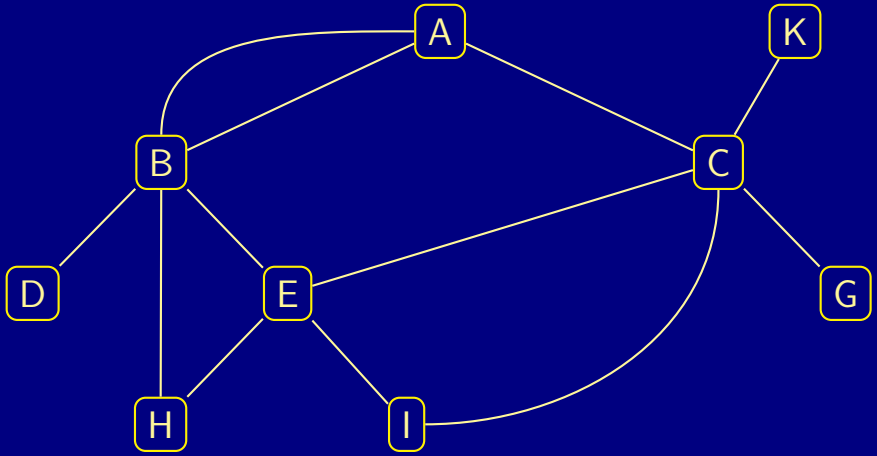
Dijkstra

Johan Montelius

KTH

HT22

undirectional graph



trains in Sweden

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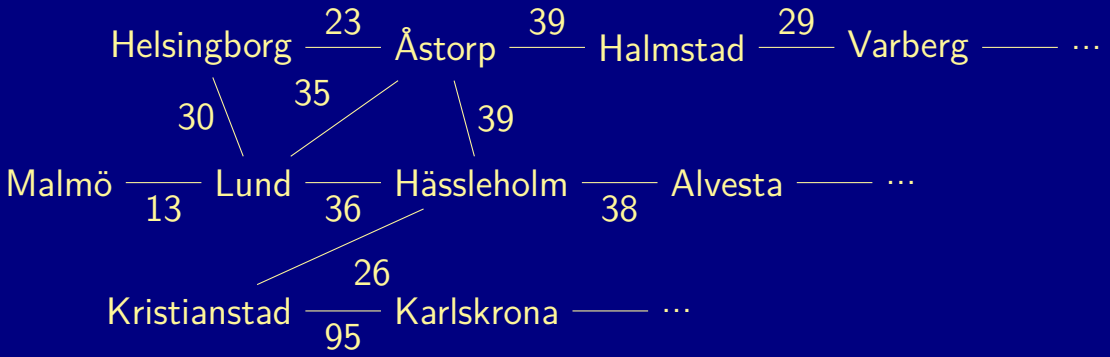

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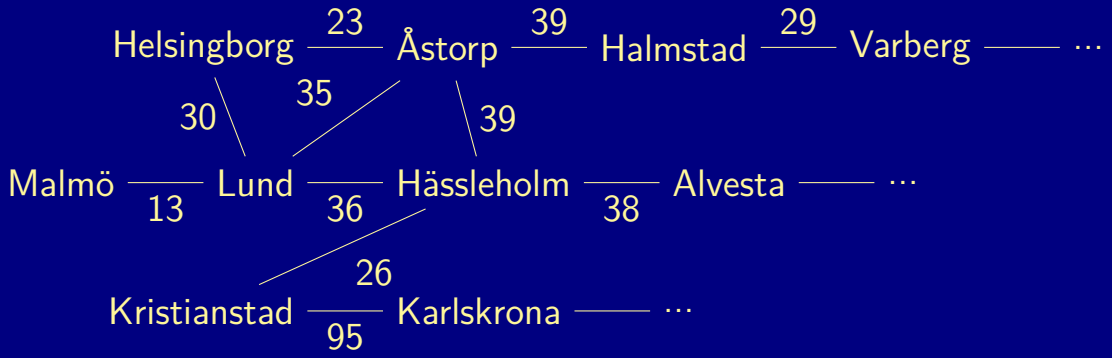
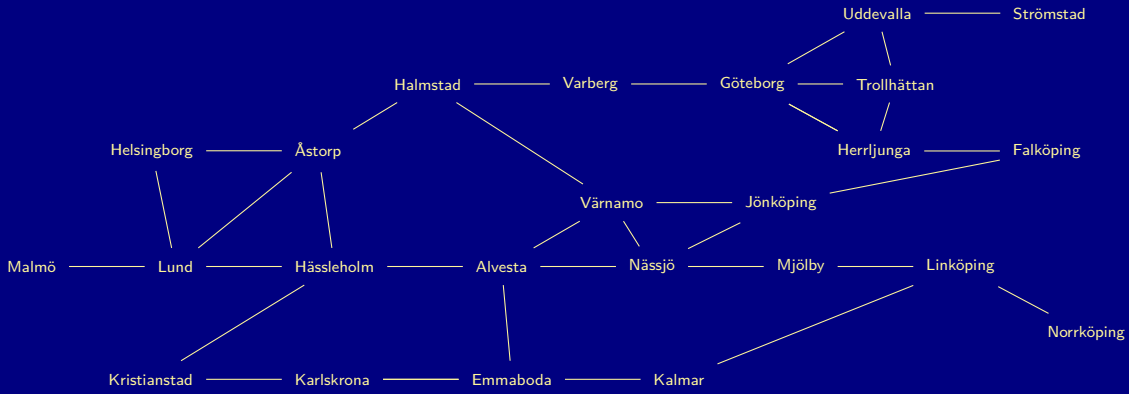


A weighted graph.

trains in Sweden

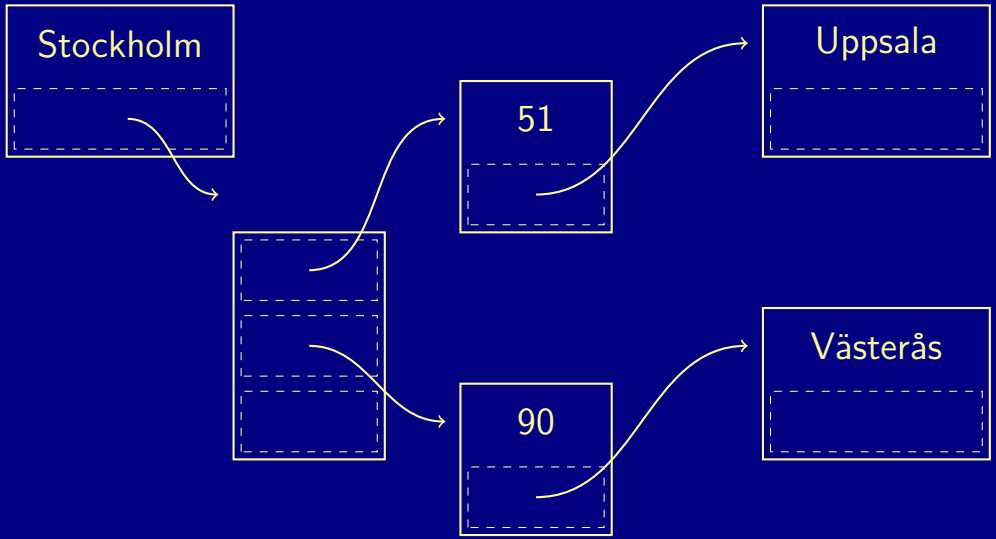


represent the graph

```
public class City {
  String name;
  Connection[] neigbours;
     :
}
```

```
public class Connection {
  City city;
  Integer distance;
     :
}
```

the graph



the naive solution

What is the shortest path from Malmö to Stockholm?

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Set a maximum distance of the path.

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  :

improvement

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improvement

If you have found a path with a distance d,
then any other path should be shorter than the found.

bounded depth first search

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Set a maximum "time traveled".

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Works for railroads - does it work for all weighted graphs?

iterative deepening

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If the minimum path is found at 300 min, how much time have we wasted.

avoid circular paths

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More bookkeeping - keep a trail of cities and don't go back, or

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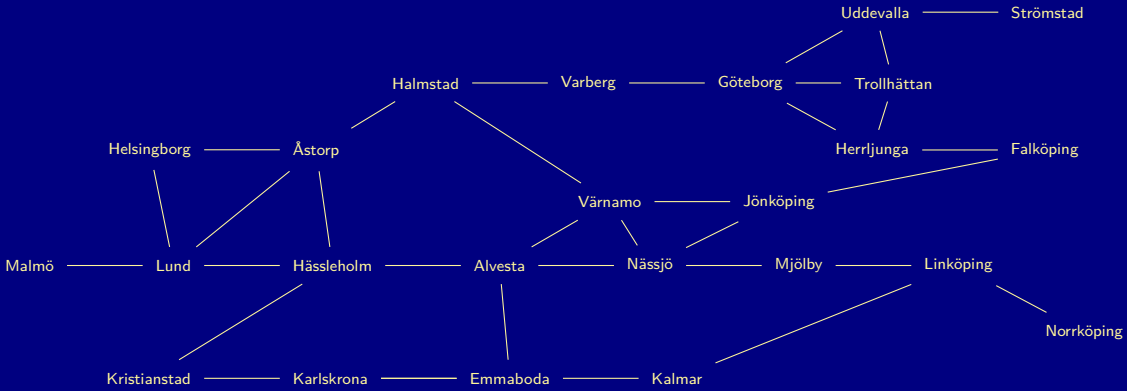
More bookkeeping - keep a trail of cities and don't go back, or

... hava a party!

paint the town red



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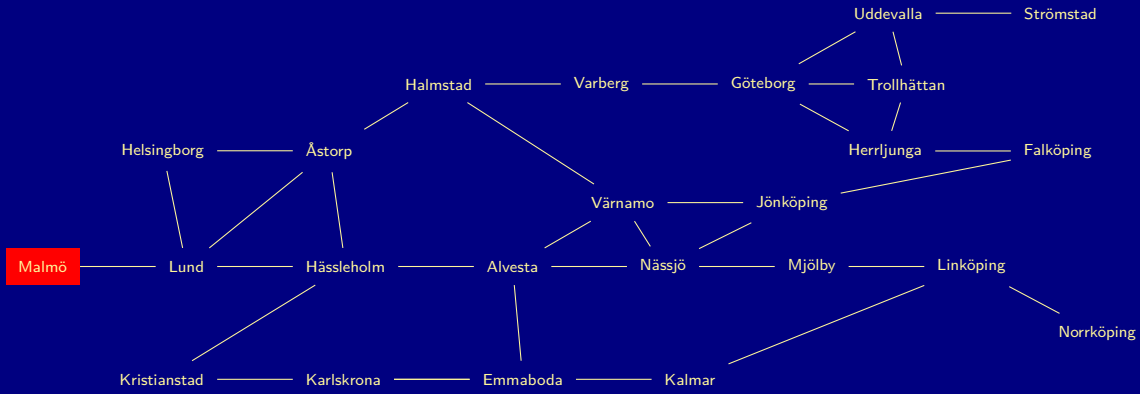

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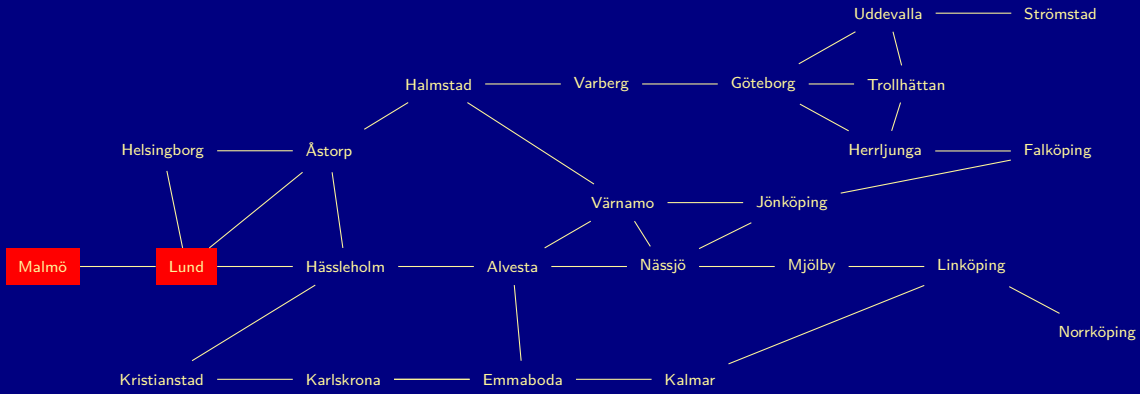

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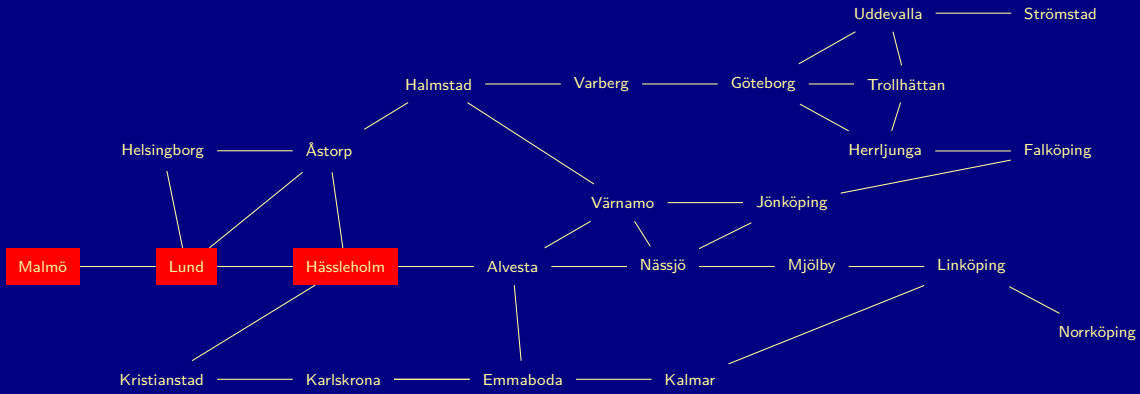

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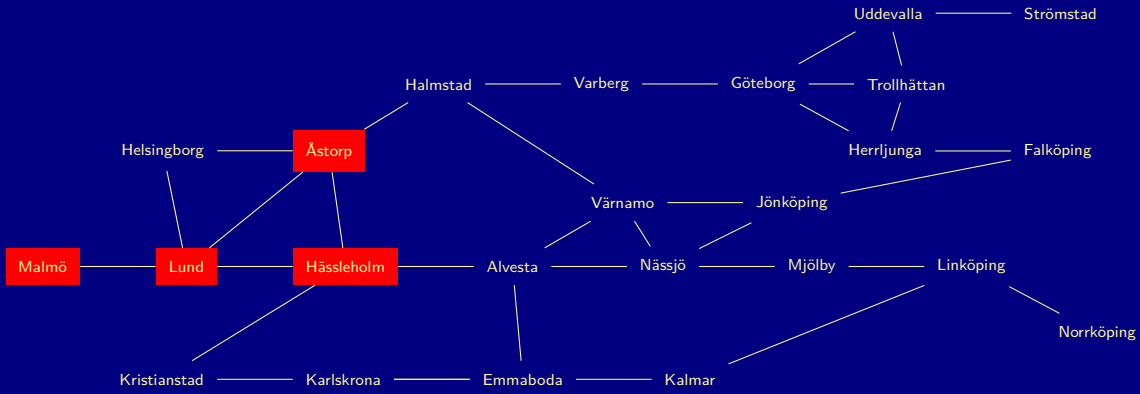

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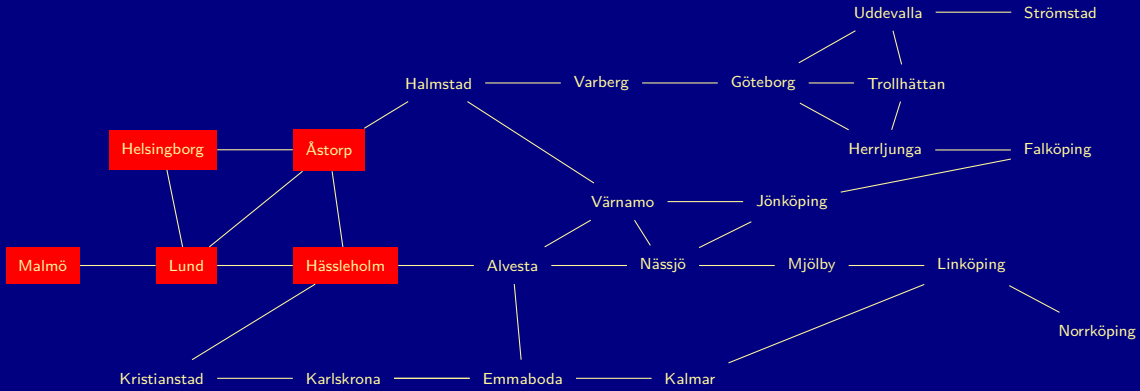

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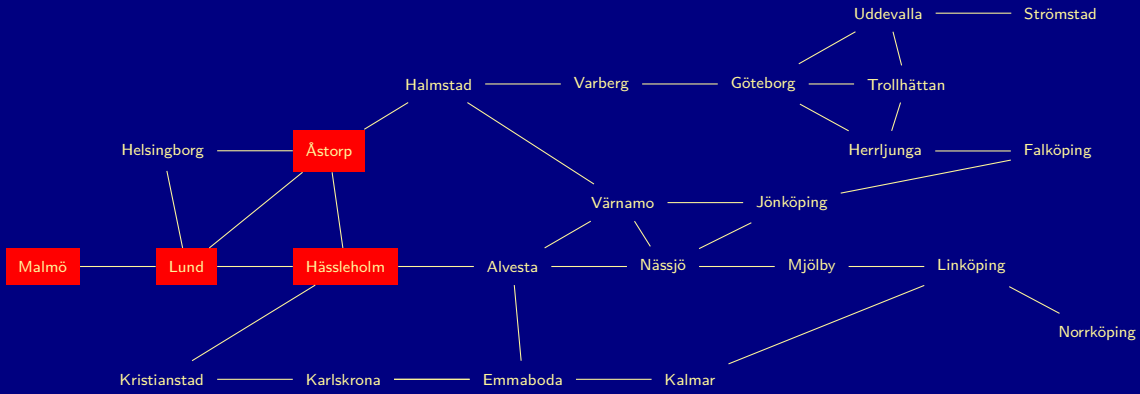

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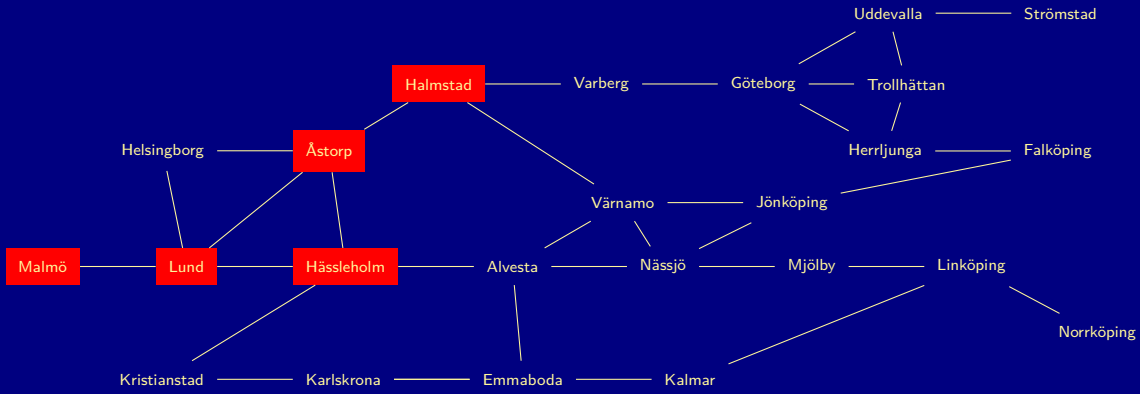

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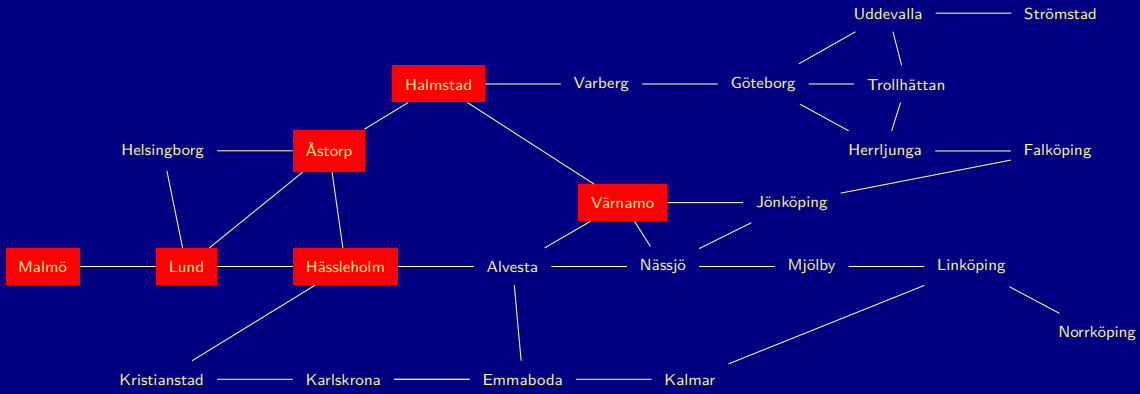

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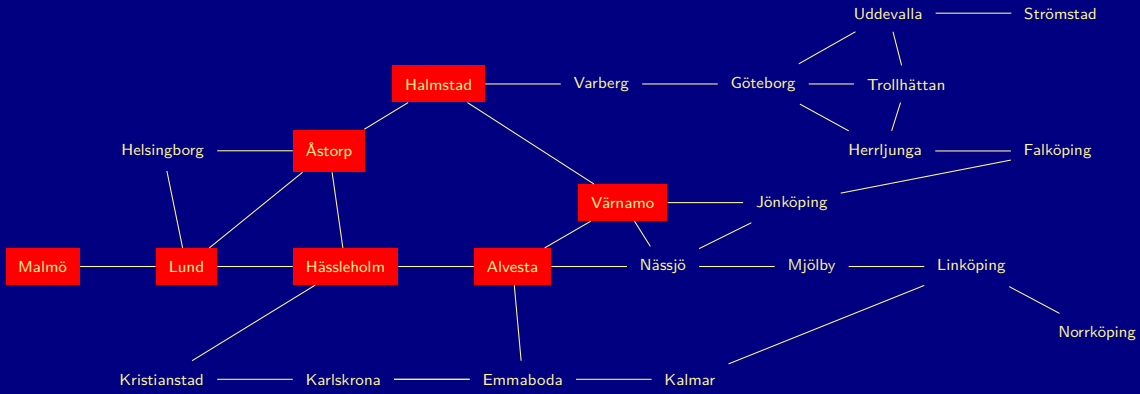
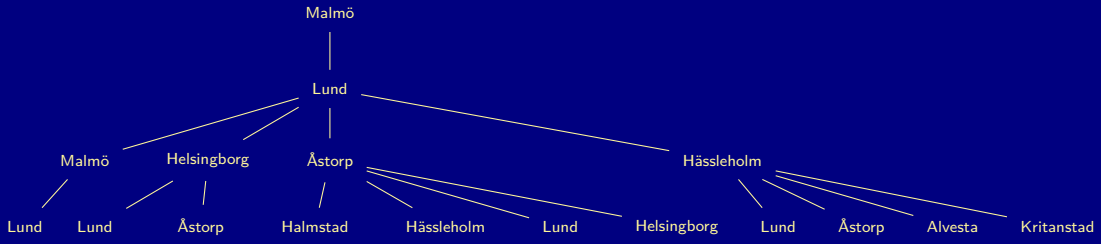
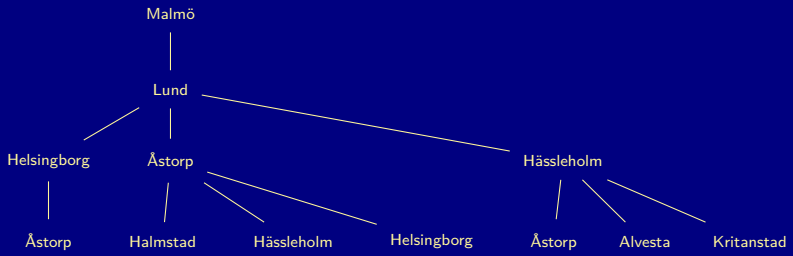


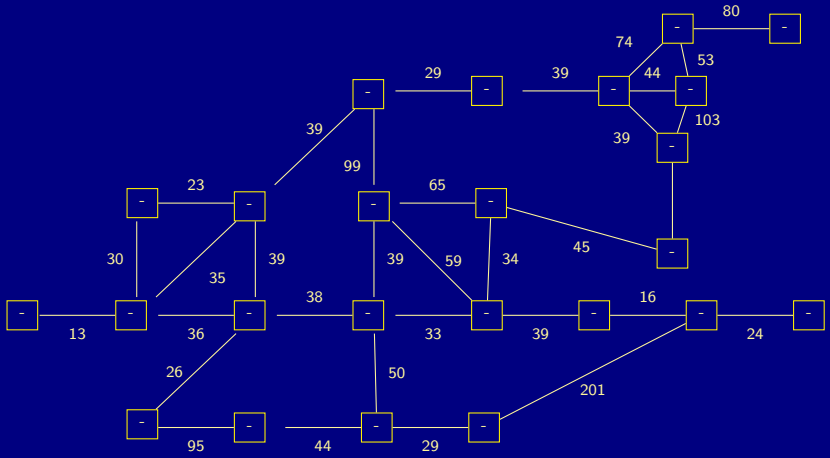
a search tree



avoiding circular paths



move slowly forward



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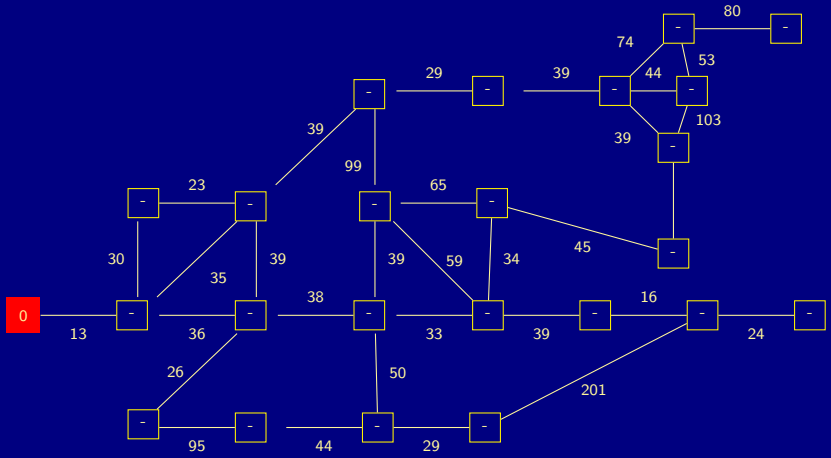

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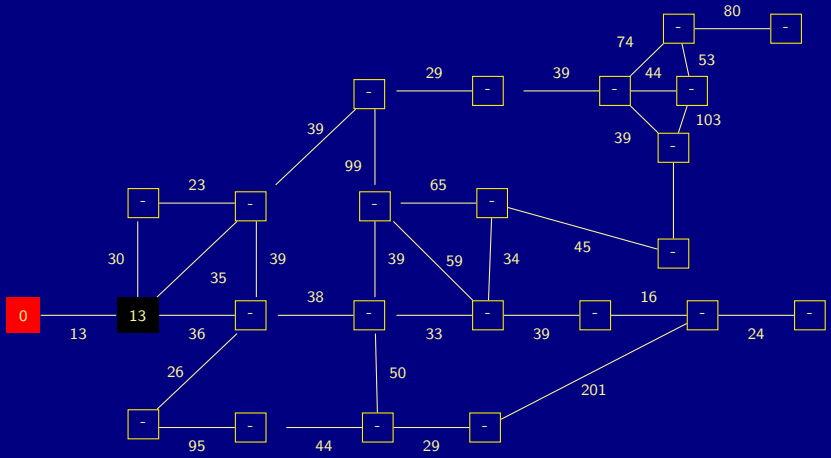

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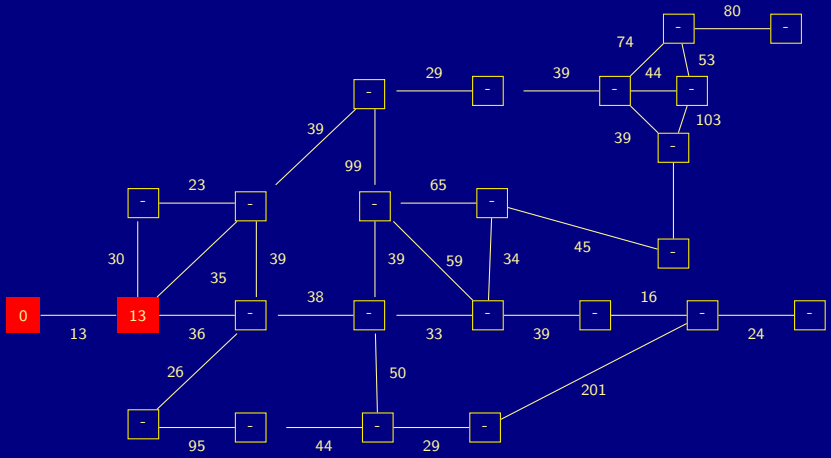

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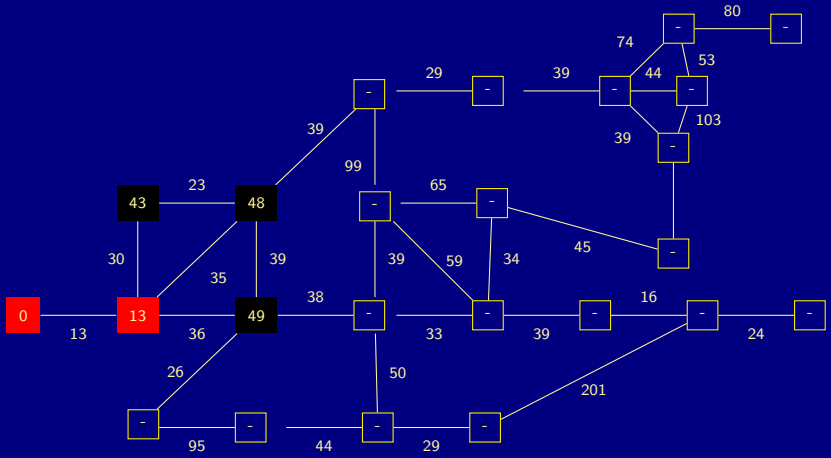

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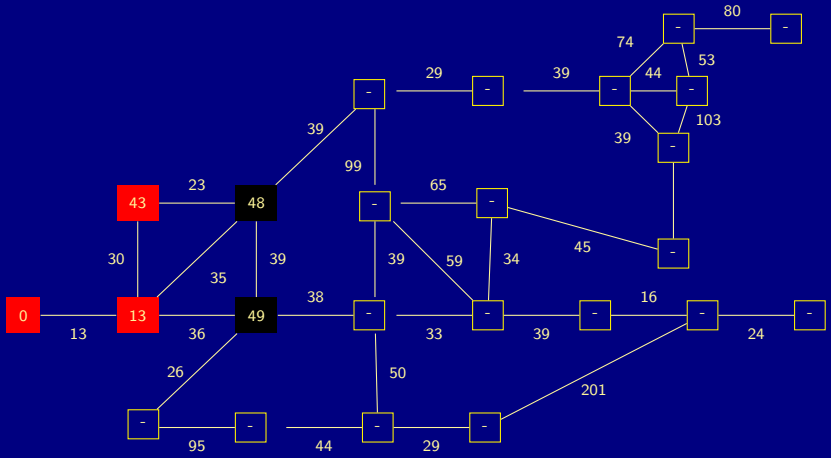

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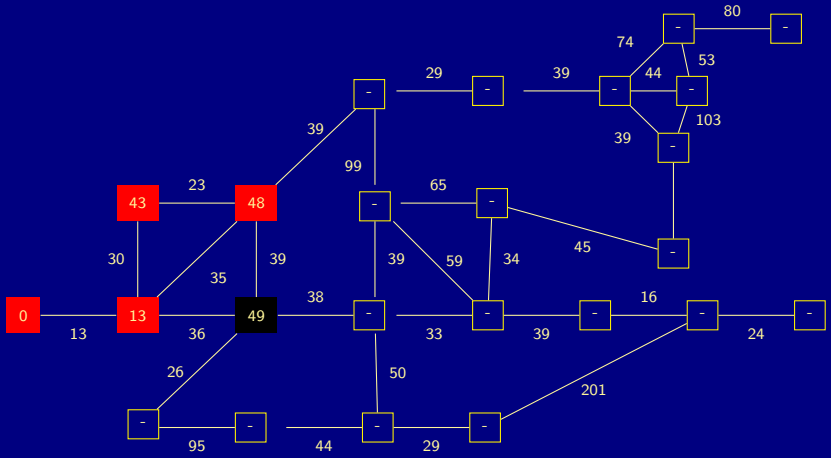

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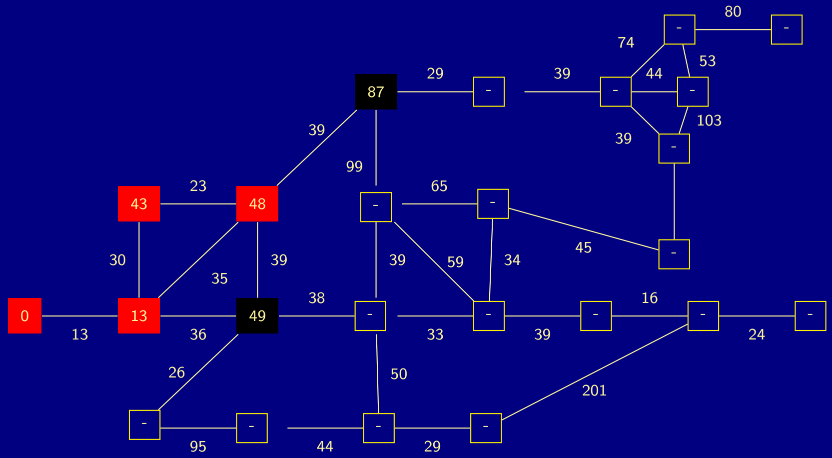

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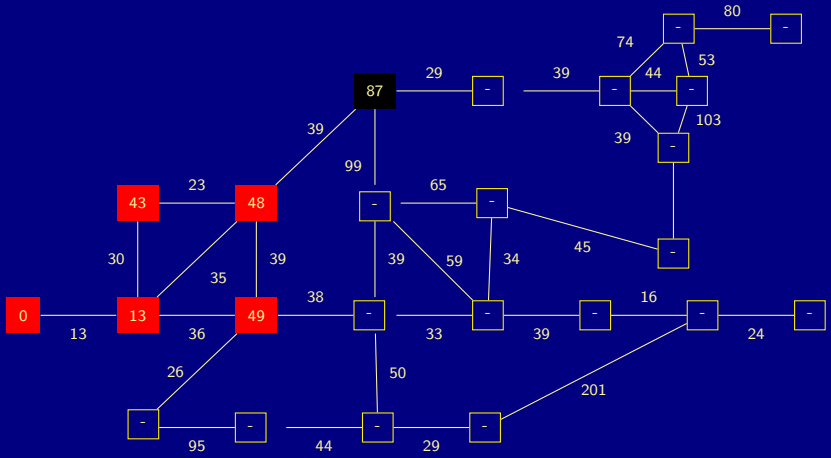

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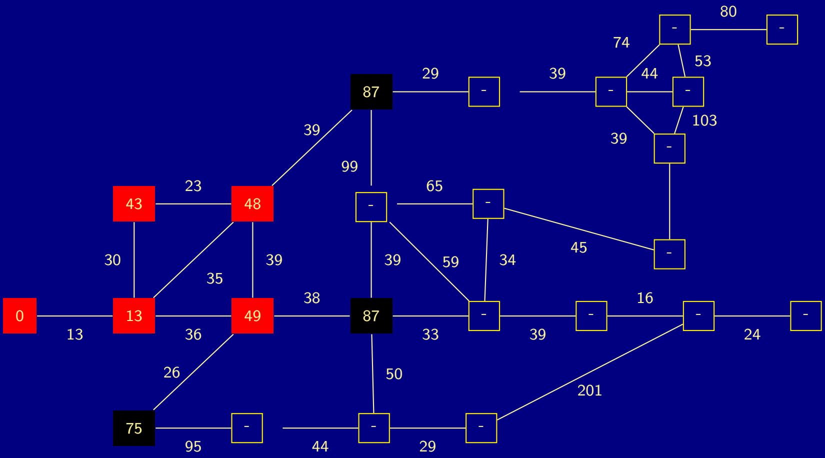

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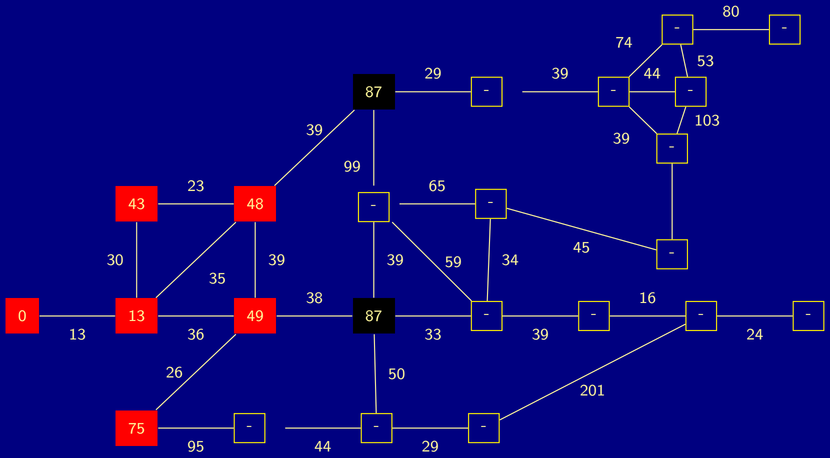

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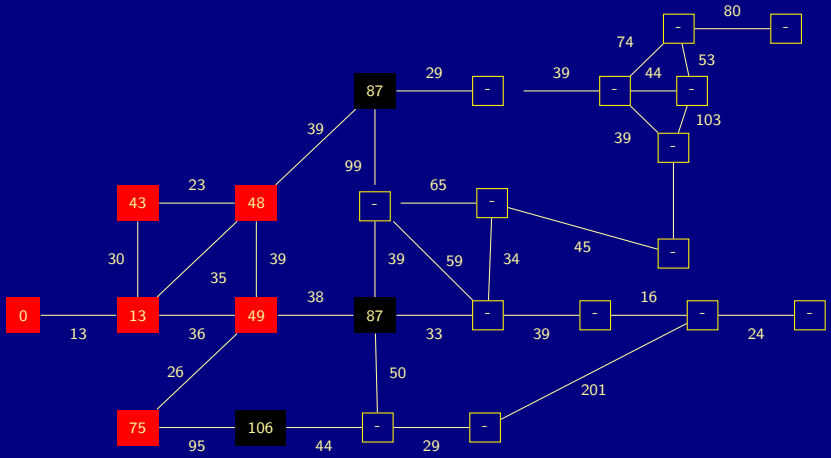

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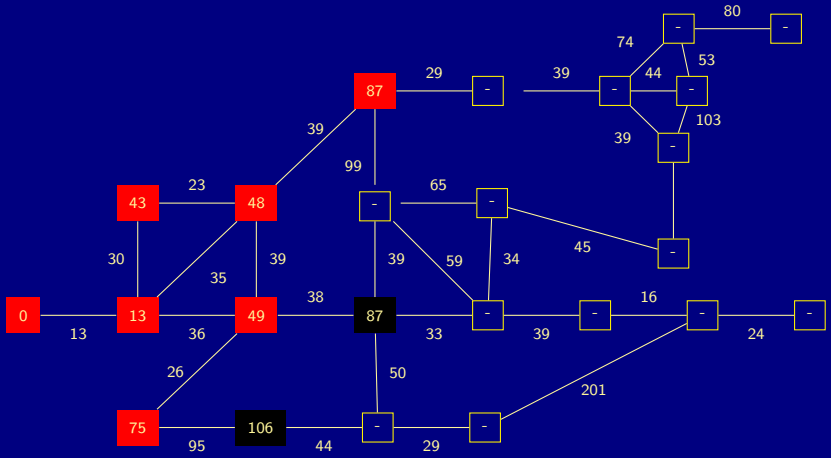

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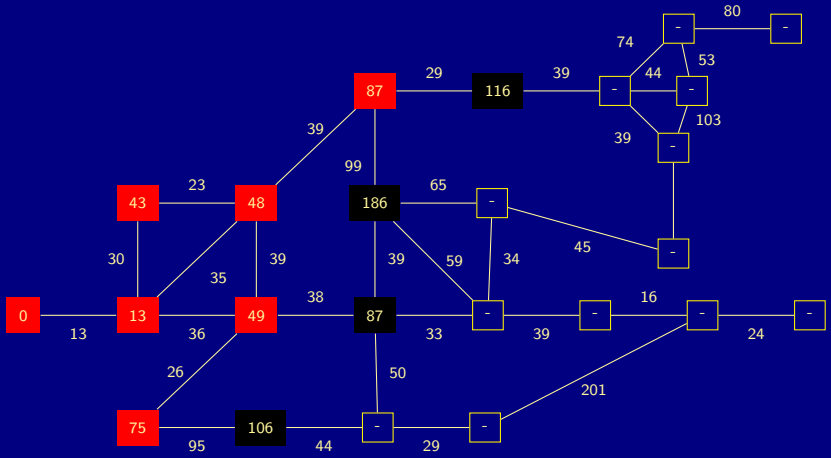
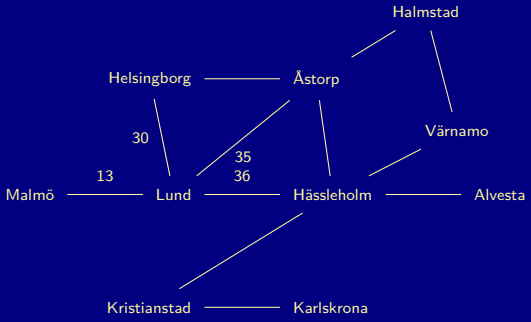


Dijkstra's shortest path

Shortest:



Queue

Dijkstra's shortest path

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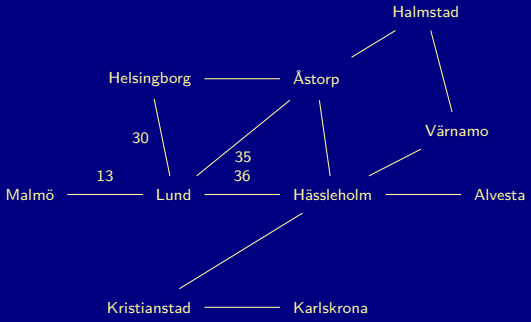
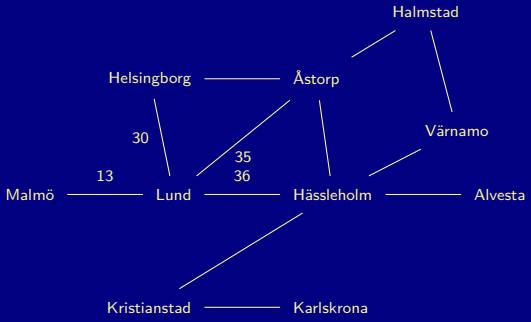


Queue

Malmö-0

Dijkstra's shortest path

Shortest: Malmö-0



Queue

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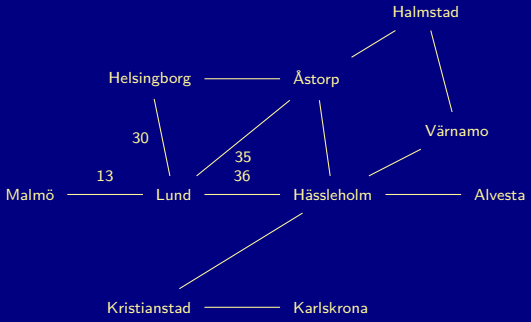
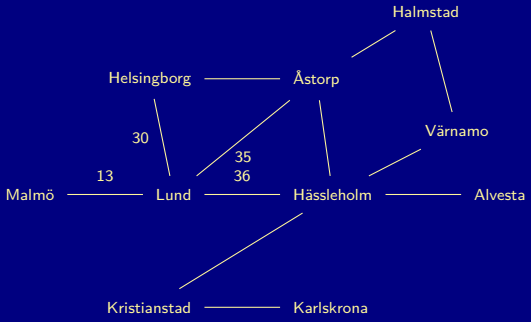


Queue

Lund-13

Dijkstra's shortest path

Shortest: Malmö-0 Lund-13



Queue

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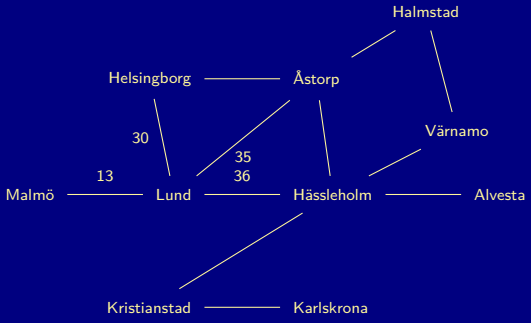


Queue

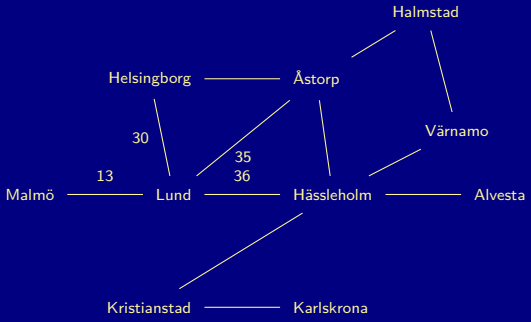
Helsingborg-43

Åstorp-48

Hässleholm-49

Dijkstra's shortest path

Shortest: Malmö-0 Lund-13 Helsingborg-43



Queue

Åstorp-48

Hässleholm-49

the algorithm - invariants

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- Cities in the "shortest found" list are done, there is no shorter path.

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- If a city is in the front of the queue we have found the shortes distance to the city.

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Could it be that we vilolate the invariants?

shortest path

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So you found the shortest distanc, where is the path?

runtime complexity

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