Complexity

Johan Montelius

KTH

HT23

big-O

An estimate of the change in execution time...
when the data set grows large.

big-O

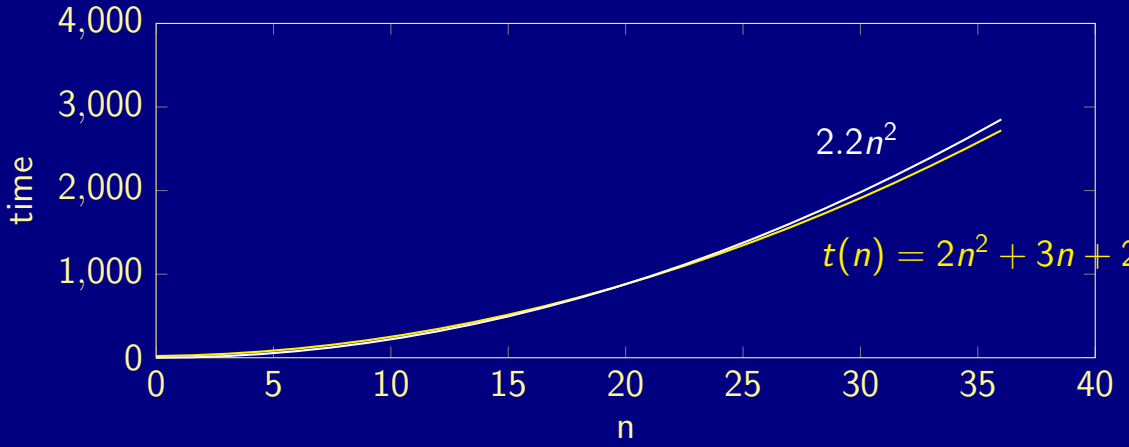
An estimate of the change in execution time...

when the data set grows large.

big-O

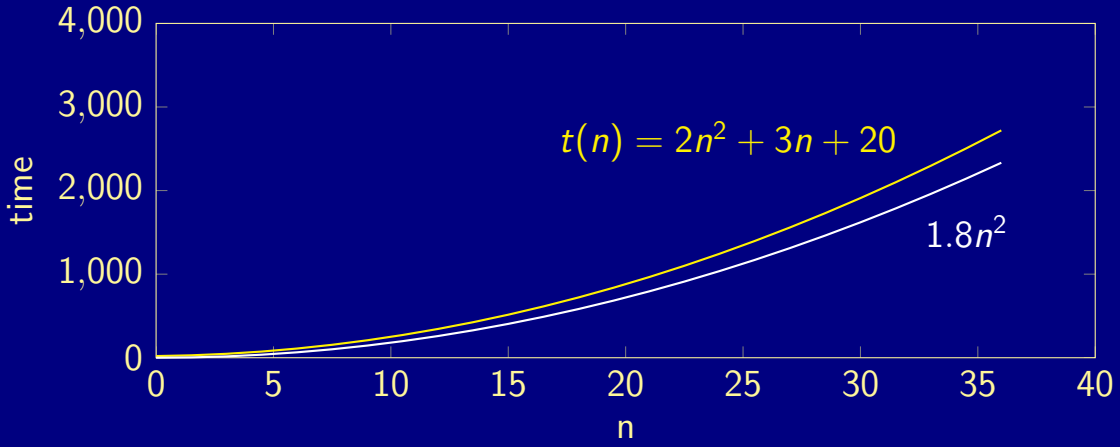
An estimate of the change in execution time...
when the data set grows large.

an upper bound



Ergo: $t(n)$ is in $O(n^2)$ since there is a k such that $k \times n^2 > t(n)$ above some n.

a lower bound



Ergo: $t(n)$ is in $\Omega(n^2)$ since there is a k such that $k \times n^2 < t(n)$ above some n.

Big-O, Ω and *Theta*

- A functions upper bound is limited by $O(g(n))$.
- Its lower bound is limited by $\Omega(g(n))$.
- If a function is limited by $O(g(n))$ and $\Omega(g(n))$ then it is limited by $\Theta(g(n))$.

Big-O, Ω and *Theta*

- A functions upper bound is limited by $O(g(n))$.
- Its lower bound is limited by $\Omega(g(n))$.
- If a function is limited by $O(g(n))$ and $\Omega(g(n))$ then it is limited by $\Theta(g(n))$.

Big-O, Ω and *Theta*

- A functions upper bound is limited by $O(g(n))$.
- Its lower bound is limited by $\Omega(g(n))$.
- If a function is limited by $O(g(n))$ and $\Omega(g(n))$ then it is limited by $\Theta(g(n))$.

Big-O, Ω and *Theta*

- A functions upper bound is limited by $O(g(n))$.
- Its lower bound is limited by $\Omega(g(n))$.
- If a function is limited by $O(g(n))$ and $\Omega(g(n))$ then it is limited by $\Theta(g(n))$.

order of ordo

Let's order the following functions:

order of ordo

Let's order the following functions:

$0.1 \times n + log_{10}(n)$ $5 \times \sqrt{n} + 34$

$ln(n)^2 + 34$ $log_2(n) + 30$

$n^4 + 20 \times n + 32$ $50 \times n + 100$

$20 \times n^2 + 100$ $n \times log_{10}(n)$

order of ordo

Let's order the following functions:

$O(n)$ $\quad 5 \times \sqrt{n} + 34$

$ln(n)^2 + 34$ $\quad log_2(n) + 30$

$n^4 + 20 \times n + 32$ $\quad 50 \times n + 100$

$20 \times n^2 + 100$ $\quad n \times log_{10}(n)$

order of ordo

Let's order the following functions:

$O(n)$ $O(\sqrt{n})$

$ln(n)^2 + 34$ $log_2(n) + 30$

$n^4 + 20 \times n + 32$ $50 \times n + 100$

$20 \times n^2 + 100$ $n \times log_{10}(n)$

order of ordo

Let's order the following functions:

$O(n)$ $O(\sqrt{n})$

$O(lg(n)^2)$ $log_2(n) + 30$

$n^4 + 20 \times n + 32$ $50 \times n + 100$

$20 \times n^2 + 100$ $n \times log_{10}(n)$

order of ordo

Let's order the following functions:

$O(n)$ $O(\sqrt{n})$

$O(lg(n)^2)$ $O(lg(n))$

$n^4 + 20 \times n + 32$ $50 \times n + 100$

$20 \times n^2 + 100$ $n \times log_{10}(n)$

order of ordo

Let's order the following functions:

$O(n)$ $O(\sqrt{n})$

$O(lg(n)^2)$ $O(lg(n))$

$O(n^4)$ $50 \times n + 100$

$20 \times n^2 + 100$ $n \times log_{10}(n)$

order of ordo

Let's order the following functions:

$O(n)$ $O(\sqrt{n})$

$O(lg(n)^2)$ $O(lg(n))$

$O(n^4)$ $O(n)$

$20 \times n^2 + 100$ $n \times log_{10}(n)$

order of ordo

Let's order the following functions:

$O(n)$ $O(\sqrt{n})$

$O(lg(n)^2)$ $O(lg(n))$

$O(n^4)$ $O(n)$

$O(n^2)$ $n \times log_{10}(n)$

order of ordo

Let's order the following functions:

$O(n)$ $O(\sqrt{n})$

$O(lg(n)^2)$ $O(lg(n))$

$O(n^4)$ $O(n)$

$O(n^2)$ $O(n \times lg(n))$

order of ordo

Let's order the following functions:

$O(n)$ $O(\sqrt{n})$

$O(lg(n)^2)$ $O(lg(n))$

$O(n^4)$ $O(n)$

$O(n^2)$ $O(n \times lg(n))$

order of ordo

Let's order the following functions:

$O(n)$ $O(\sqrt{n})$

$O(lg(n)^2)$ $O(lg(n))$

$O(n^4)$ $O(n^2)$ $O(n)$

$O(n \times lg(n))$

order of ordo

Let's order the following functions:

$O(n)$ $O(\sqrt{n})$

$O(lg(n)^2)$ $O(lg(n))$

$O(n^4)$ $O(n^2)$ $O(n \times lg(n))$ $O(n)$

order of ordo

Let's order the following functions:

$$O(\sqrt{n})$$

$$O(lg(n)^2) \qquad O(lg(n))$$

$$O(n^4) \quad O(n^2) \quad O(n \times lg(n)) \quad O(n)$$

order of ordo

Let's order the following functions:

$$O(lg(n)^2) \qquad O(lg(n))$$

$$O(n^4) \quad O(n^2) \quad O(n \times lg(n)) \quad O(n) \quad O(\sqrt{n})$$

order of ordo

Let's order the following functions:

$$O(n^4) \quad O(n^2) \quad O(n \times lg(n)) \quad O(n) \quad O(\sqrt{n}) \quad O(lg(n)) \quad O(lg(n)^2)$$

order of ordo

Let's order the following functions:

$$O(n^4) \quad O(n^2) \quad O(n \times lg(n)) \quad O(n) \quad O(\sqrt{n}) \quad O(lg(n)^2) \quad O(lg(n))$$

order of ordo

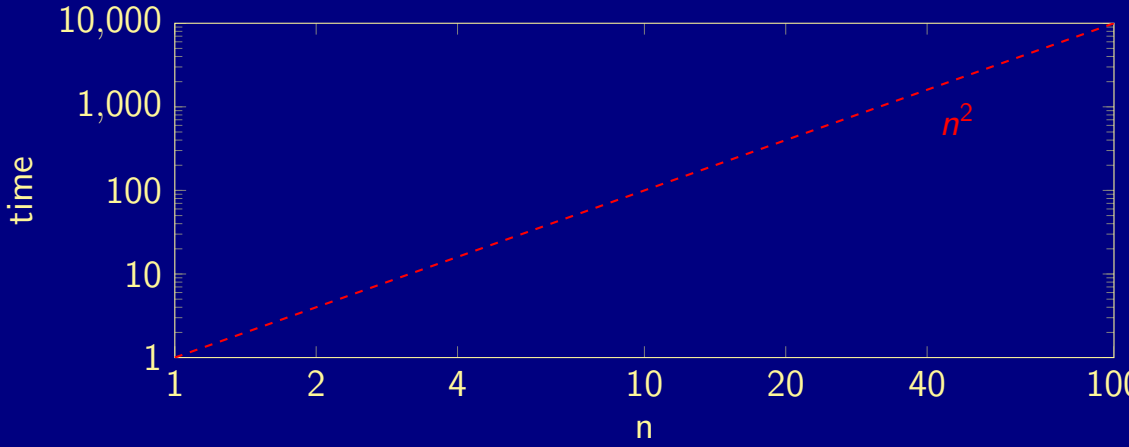
Let's order the following functions:

$$O(n^4) \quad O(n^2) \quad O(n \times lg(n)) \quad O(n) \quad O(\sqrt{n}) \quad O(lg(n)^2) \quad O(lg(n))$$

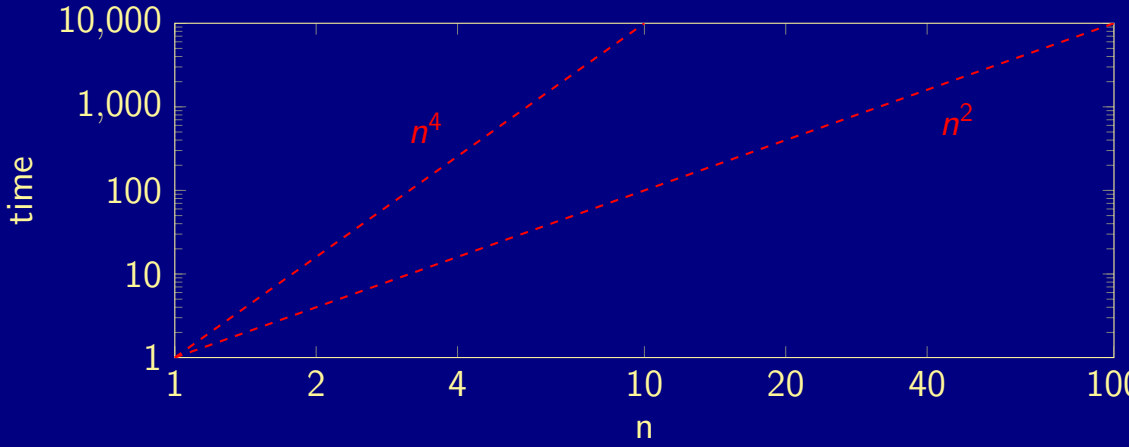
orderd by complexity - not execution time given a specific n

the cost of chess

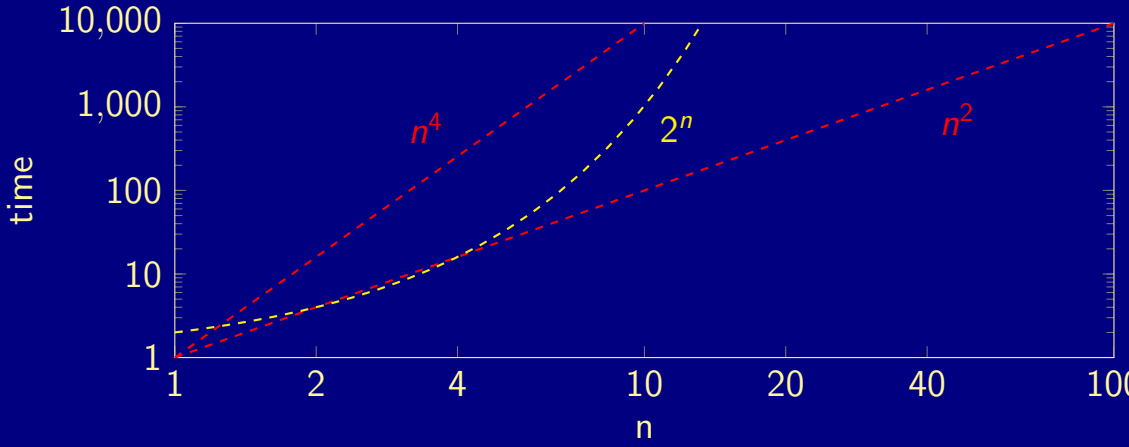
exponential time



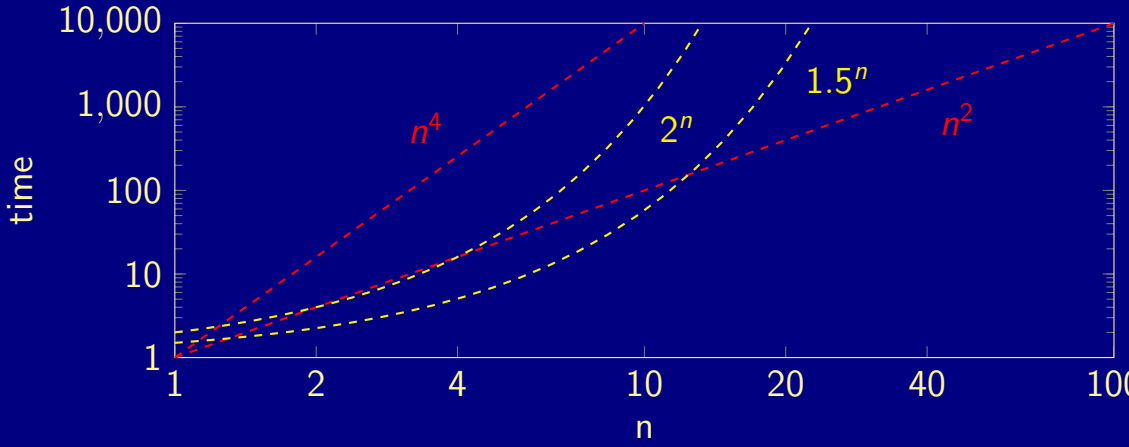
exponential time



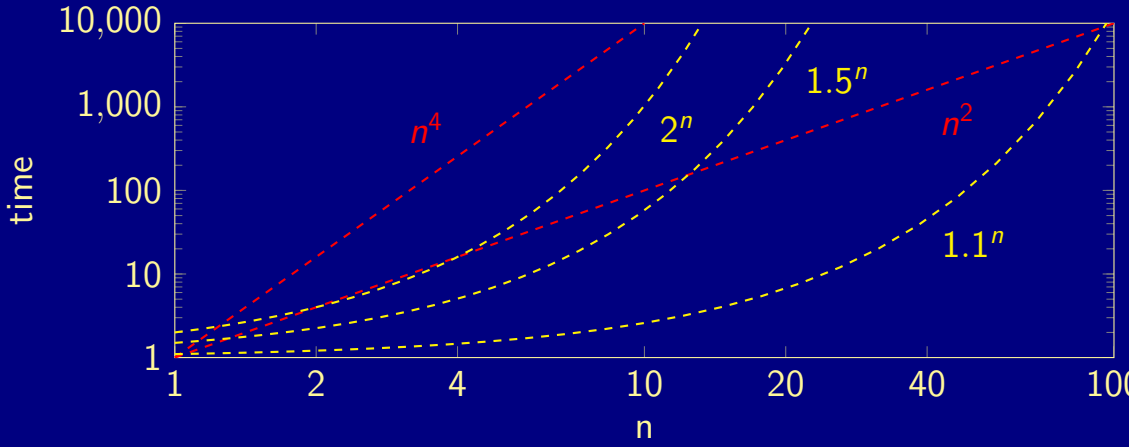
exponential time



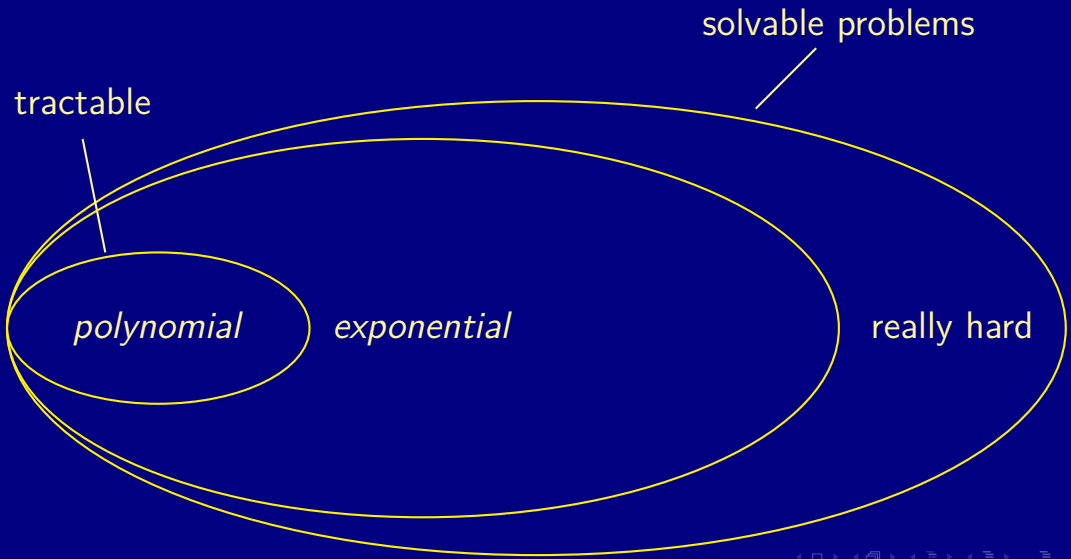
exponential time



exponential time



tractable, intractable and even worse



this sentence is false

this sentence is false

Kurt Gödel

There are things that can not be decided.

this sentence is false

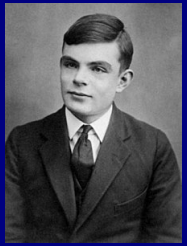
Alonso Church
lambda calculus

Kurt Gödel
There are things that can not be decided.

this sentence is false

Alonso Church
lambda calculus

Kurt Gödel
There are things that can not be decided.

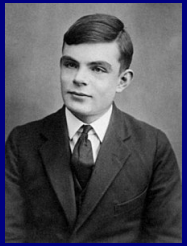


Alan Turing
the Turing machine

this sentence is false

Alonso Church
lambda calculus

Kurt Gödel
There are things that can not be decided.



Alan Turing
the Turing machine

Apart from things that can not be computed, we can compute everything :-)

The Imitation Game

problem complexity

problem complexity

It's not easy to determine the classification of a problem.

problem complexity

It's not easy to determine the classification of a problem.

If we have an algorithm at least we know the upper limit.

problem complexity

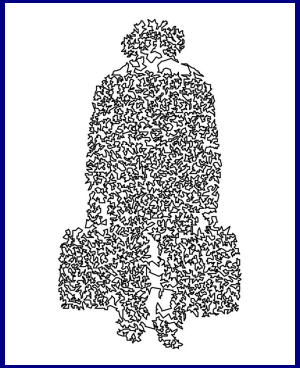
It's not easy to determine the classification of a problem.

If we have an algorithm at least we know the upper limit.

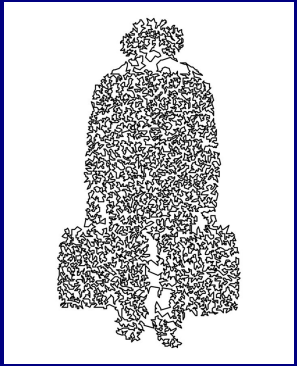
We have problems where it's easy to find an exponential algorithm, but no proof that there is no polynomial algorithm.

traveling salesman

traveling salesman

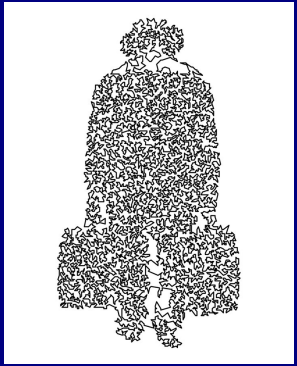


traveling salesman



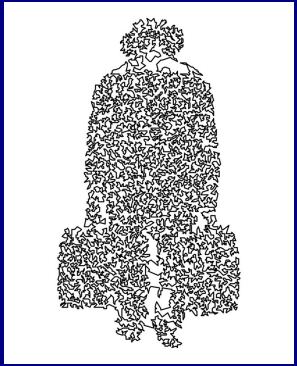
- Find the shortest road that visits all cities.

traveling salesman



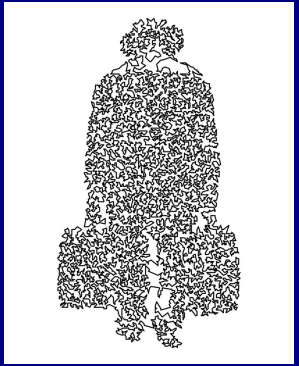
- Find the shortest road that visits all cities.
- Let's try them all ...

traveling salesman



- Find the shortest road that visits all cities.
- Let's try them all ...
- .... exponential solution :-(

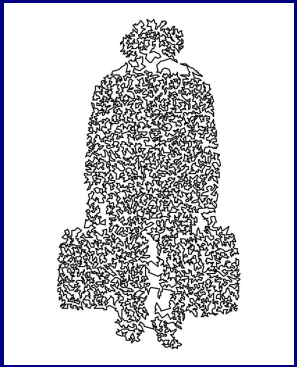
traveling salesman



- Find the shortest road that visits all cities.
- Let's try them all ...
- .... exponential solution :-(
- If I give you the path - how can you verify that it is the shortest?

traveling salesman - almost the same

traveling salesman - almost the same



traveling salesman - almost the same

- Find a road visiting all cities but that is less than x km.

traveling salesman - almost the same

- Find a road visiting all cities but that is less than x km.
- Let's try them all ...

traveling salesman - almost the same

- Find a road visiting all cities but that is less than x km.
- Let's try them all ...
- .... exponential solution :-(

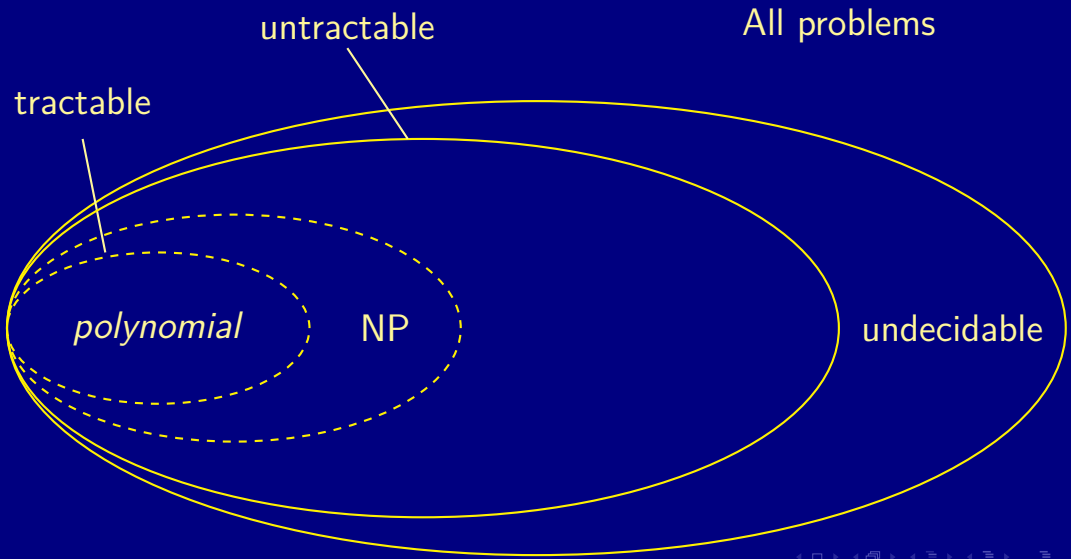
traveling salesman - almost the same

- Find a road visiting all cities but that is less than x km.
- Let's try them all ...
- .... exponential solution :-(
- A solution is easy to verify in polynomial time.

traveling salesman - almost the same

- Find a road visiting all cities but that is less than x km.
- Let's try them all ...
- .... exponential solution :-(
- A solution is easy to verify in polynomial time.
- A Non-deterministic Polynomial problem - NP.

non-deterministic polynomial



extra income

extra income

$$P = NP$$

extra income

$$P = NP$$

If you can prove it, or prove that it does not holds,

extra income

$$P = NP$$

If you can prove it, or prove that it does not holds, then you can claim a million dollar.

The Millennium Prize Problems

there is still hope

there is still hope