



## Introduction to Model Order Reduction

Ph.D. course, 7.0 credits

In engineering and science it is often desirable to use the simplest possible mathematical model that “does the job”. First-principles modeling or system identification commonly result in unnecessarily high-dimensional mathematical models. Model (order) reduction concerns systematic approximation of such models. There are many advantages of working with models with a low-dimensional state space. For example, low-dimensional models are easier to analyze and much faster to simulate. Model reduction methods have successfully been used to solve large-scale problems in areas such as control engineering, signal processing, image compression, fluid mechanics, and power systems. In this Ph.D. course, we give an introduction to some powerful available reduction techniques, as well as to the required underlying mathematics.

**Course webpage:** <http://people.kth.se/~hsan/modred.htm>

### Course goals

After finishing the course, the Ph.D. student will

- be able to distinguish between difficult and simple model reduction problems;
- have a thorough understanding of Principle Component Analysis (PCA)/Singular Value Decomposition (SVD);
- understand the interplay between linear operators on Hilbert spaces, controllability, observability, and model reduction;
- know the theory behind balanced truncation and Hankel norm approximation;
- know how convex optimization and Linear Matrix Inequalities (LMIs) can be used to solve model reduction problems;
- know the basics of the behavioral modeling approach and dissipative systems theory; and
- be able to reduce systems while preserving dissipativity properties, and reduce linear feedback controllers while taking the overall system performance into account.

### Instructors

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## Prerequisites

Basic linear systems theory (state-space realizations, Laplace transforms etc.), matrix theory, and basic MATLAB-programming.

## Grading

Grade (pass/fail) is based turn-in exercises (50%) and written exam (50%).

### *Exercises*

- Exercises handed out with each lecture.
- At the end of the course, at least 75% of the exercises should have been solved and turned in on time.
- Exercises for Lectures 1-4 due **April 17**.
- Exercises for Lectures 5-8 due **May 12**.

### *Exam*

- A 24h take-home exam.
- You decide when to take it, but it should be completed at the latest 3 months after course ends.
- No cooperation allowed.
- Problems similar to exercises.

## Course literature

Research papers and parts of the books

- Antoulas, A.C., "Approximation of Large-Scale Dynamical Systems", Society for Industrial and Applied Mathematics, 2005.
- Obinata, G. and Anderson, B.D.O., "Model Reduction for Control System Design", Springer-Verlag, London, 2001.
- Luenberger, D.G., "Optimization by Vector Space Methods", Wiley, 1969.
- Green, M. and Limebeer, D.J.N., "Linear Robust Control", Dover, 2012.
- Doyle, J.C., Francis, B., and Tannenbaum, A., "Feedback Control Theory", Macmillan Publishing Co., 1990.

**Program (dates and times subject to change)**

Date	Time	Event	Room	Lecturer	Topic
2014-03-31	13:15-15:00	Lecture 1	Q13	HS	Introduction. The model reduction problem.
2014-04-02	13:15-15:00	Lecture 2	L41	HS	Model truncation, singular perturbation.
2014-04-04	10:15-12:00	Exercise 1	Q24	HS	Review of linear systems and Hilbert spaces etc.
2014-04-07	13:15-15:00	Lecture 3	Q24	BB	POD/PCA/SVD-based reduction.
2014-04-09	13:15-15:00	Lecture 4	Q24	BB	Gramians and balanced realizations.
2014-04-24	13:15-15:00	Exercise 2	Q24	HS/BB	<b>Exercises due April 17</b>
2014-04-25	13:15-15:00	Lecture 5	Q24	BB	Balanced truncation and weighted extensions.
2014-04-28	10:15-12:00	Lecture 6	Q24	BB	Controller reduction and linear matrix inequalities (LMIs).
2014-05-05	10:15-12:00	Lecture 7	Q13	HS	Optimal model order reduction in the Hankel norm.
2014-05-07	15:15-17:00	Lecture 8	Q24	HS	Optimal model order reduction in the H2/H-infinity norms.
2014-05-09	10:15-12:00	Lecture 9	Q13	MB	Behavioral approach, kernel representations
2014-05-12	13:15-15:00	Lecture 10	Q13	MB	Controllability, passivity, dissipativity, storage functions
2014-05-14	10:15-12:00	Lecture 11	Q24	MB	Maximum and minimum storage functions
2014-05-16	10:15-12:00	Exercise 3	Q13	HS/BB	<b>Exercises due May 12</b>
2014-05-19	13:15-15:00	Lecture 12	Q13	MB	The LQ problem, H-infinity-norm as dissipativity
2014-05-21	10:15-12:00	Lecture 13	L41	MB	Passivity preserving model reduction
2014-05-26	10:15-12:00	Lecture 14	Q13	MB	Dissipativity preserving model reduction
2014-05-28	10:15-12:00	Exercise 4	M24	MB	