REPORT

THE GYRO PROJECT

2002-05-20

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ABSTRACT

The gyro was one of five projects in 2C1241 Project Course in Automatic Control of year 2002. The goals of the project were to stabilize the gyroscope, make it follow pre-specified trajectories and to find limitations in its application. Other aims with the course were to get training in project work and writing reports.

First of all a project plan with a time plan was discussed and created. After system identification, the modeling started. Feedback linearization, $H_8$ and LQG are the controllers that have been tested. The final solution was a combination of several LQG’s based on a reduced system with different working points.

At the demonstration on the group review, we showed that a stabilizing controller successfully has been implemented. Also a prespecified trajectory was loaded and executed with satisfying results.

The course has given a practical knowledge about modeling, design and analysis of a control system. A conclusion that the gyro team has done is that the reality is much more complex than theory describes. It has been a great challenge to make the gyro work!

Group members:
Josefin Johansson Andreas Ragnarstam, Pär Windahl, Denis Jacquet
TABLE OF CONTENT

1 The start up and project planning ................................................................. 3
  1.1 Goal .............................................................................................................. 3
  1.2 Risk analysis ......................................................................................... 3
  1.3 The start-up ......................................................................................... 3
    1.3.1 Work distribution ........................................................................ 4
    1.3.2 Home page ..................................................................................... 4

2 Gyro presentation ....................................................................................... 5
  2.1 Function .................................................................................................. 5
  2.2 Applications .......................................................................................... 5
    2.2.1 Inertial navigation systems ......................................................... 6
    2.2.2 Stabilizer for ships ......................................................................... 6
    2.2.3 Stabilized platforms and gun sights ......................................... 7
    2.2.4 Aircraft instruments .................................................................... 7
  2.3 Front-line development ........................................................................ 7

3 Solution ....................................................................................................... 8
  3.1 System identification ............................................................................ 8
  3.2 Control design ....................................................................................... 10
    3.2.1 Feedback linearization ................................................................. 10
    3.2.2 Optimal control theory ................................................................ 11
    3.2.3 Gain scheduling ........................................................................... 11
  3.3 Trajectory planning ............................................................................... 11
  3.4 Limitations ............................................................................................ 12

4 Course feedback ....................................................................................... 13

5 Conclusions .............................................................................................. 13

6 References .................................................................................................. 14
  Appendix 1 ................................................................................................. 15
  Appendix 2 ................................................................................................. 17
1 THE START UP AND PROJECT PLANNING

On the first meeting of the group goal description, risks of the project, solution and work distribution was discussed. The results were put into a project plan that is shown here and the reversed time plan can be found in Appendix 1.

1.1 Goal

The Customer has ordered a demonstration of the gyroscope process with a control system that stabilizes it and gives the ability of following prespecified trajectories. The limitations of the gyroscope should be highlighted and the presentation will include several features such as an oral presentation, a written report, a poster, a web page and a CD.

1.2 Risk analysis

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<td>Good leadership</td>
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*Table 1: Risk analysis*

As *Table 1* shows, hardware and software problems plus bad communication between group members are the most critical risks of the project. Those risks will be eliminated by keeping sharp attention to them and follow the project plan.

1.3 The start-up

The solution was divided into the following steps:

- Literature study – identification of the dynamics of the gyro
- System identification
- Design of a stabilizing controller
- Trajectory planning
- Limitations
• Written report
• Oral presentation
• CD
• Poster

1.3.1 Work distribution

*Table 2* shows the responsibilities for each member of the team in the different parts of the project.

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</table>

*Table 2: Members and their areas of responsibility*

1.3.2 Home page

From week 12 a home page created by Andreas Ragnerstam was up and running. Weekly progress report, time schedule, group member picture, the poster and all documents produced by the team can be found on the home page (http://www.e.kth.se/~e98_anr/).
2 GYRO PRESENTATION

2.1 Function

A gyroscope is any device consisting of a rapidly spinning wheel set in a framework that permits it to tilt freely in any direction, i.e. to rotate about any axis.

J.-B.-L. Foucault, a 19th-century French scientist is responsible for giving the name gyroscope to a wheel, or rotor, mounted in gimbal rings, i.e. a set of rings that permit it to turn freely in any direction.

The momentum of such a wheel causes it to retain its attitude when the framework is tilted; from this characteristic derive a number of valuable applications.

If the base of a three-frame gyroscope is held in the hand with the rotor spinning and turned about any of the three axes, the rotor axle will continue to point in the original direction in space. This property is known as gyroscope inertia. If the speed of the wheel decreases, the gyroscope inertia gradually disappears, the rotor axle begins to wobble and ultimately takes up any convenient position. Rotors with a high speed and concentration of mass towards the rim of the wheel display the strongest gyroscopic inertia.

It is apparent that gyroscopic inertia depends on the angular velocity and the momentum of inertia of the rotor, or on its angular momentum. The rotor wheel is subject to the laws of rotational motion and inertia in that a freely rotating body will maintain a fixed direction in space, and the rotor tends to preserve its angular momentum, or spinning action, unless acted on by some external force.

The consequence of gyroscopic inertia is that to the observer on Earth the spin axis of a gyroscope makes an apparent movement over a period of time, although this apparent motion merely reflects the revolution of the Earth about its axis. There is one exception to this, that when the spin axis points towards the polar star, there is no movement of the spin axis with respect to the observer's surroundings, as the axis is parallel to the Earth's axis and points toward the Celestial poles.

The gyroscope used in the course is a displacement gyroscope and measure angular displacements between the framework in which it is mounted and the fixed axis.

2.2 Applications

Gyroscopes have a wide field of application and a wide variety of gyroscopic devices have been developed. Inertial navigation systems, ship stabilizer and aircraft instruments are some examples.
2.2.1 Inertial navigation systems
Neither position nor velocity can be sensed directly by an inertial system. Acceleration
can be detected by an accelerometer and this can be used to determine the position of a
ship, aircraft, or space vehicle.

The inertial navigational system comprises three components: the platform, the
gyroscopic frame and the computer. The accelerometers, mounted with their input axes
mutually at right angles, are carried on a platform. Two accelerometers measure
acceleration in the horizontal plane - the requirement for surface navigation.

For space navigation an additional accelerometer measures acceleration in the vertical
plane. Each of the acceleration signals can be converted into distance traveled by
determining, firstly, the total change in velocity which, added to the known initial
velocity, gives the vehicle velocity; and second, the total change in position that, added to
the known initial position, yields the present vehicle position.

The gyroscope frame is responsible for the stabilization of the platform. Three rate
gyroscopes are fitted in the frame with their input axes mutually perpendicular. Two of
the gyroscopes provide the horizontal alignment of the platform - an essential
requirement to eliminate the influence of accelerations due to gravity - while the third is
responsible for the north-south alignment. Pitch, roll and yaw are detected by the three
gyroscope input axes. The gimbal deflection of each of the gyroscopes is converted into a
signal voltage that, when amplified, drives a servomotor via a gear train to rotate the
frame back to its original position.

The computer performs the necessary calculations. Specifically, it applies certain
corrections to the acceleration, integrates acceleration to velocity and velocity to distance,
computes latitude and longitude, and converts geocentric latitudes into geographical
latitudes.

If the inertial system is used for inertial guidance in space navigation, then the computer
also compares the vehicles position with the destination or target position to provide
steering commands and compares the vehicles velocity (both direction and magnitude)
with the programmed velocity vector to provide rocket steering and engine cut-off
commands.

2.2.2 Stabilizer for ships
The main components of a ship stabilizer are a set of fins and the gyroscopes. The fins
protrude from the ship's hull and are so operated that the forward motion of the ship
produces a tilt in one direction on one fin and in the opposite direction on the other fin.

When properly controlled, therefore, these fins oppose the rolling motion. The
gyroscopes sense the vertical angular displacement and the roll velocity and provide the
proper control for the fins.
2.2.3 Stabilized platforms and gun sights
The inertial type platform is extremely small and must be stabilized to an extraordinary degree of precision, but the method of stabilization used for gun platforms is essentially the same.

The gyroscopic gun sight revolutionized aerial gunnery. The sight fitted on the gun contains a gyroscope capable of measuring angular velocities in two lanes at tight angles to each other.

2.2.4 Aircraft instruments
The three primary gyroscope instruments fitted to the flight panel are a rate-of-turn indicator, a directional gyroscope, and an artificial horizon. Such gyroscopes may be driven by electric motors or by air jets. The directional gyroscope forms a standard reference for the pilot and navigator. It is a three-frame gyroscope with its spin axis in the horizontal plane.

An artificial horizon displays the rolling and pitching motion of the aircraft. It consists basically of a three-framed gyroscope with its spin axis vertical and automatic correction devices to counteract the apparent motion of the spin axis around the celestial pole and any other random precessions.

2.3 Front-line development
In space technology the weight of the vehicle is very important. The gyro used during the course is quite heavy and another problem is that spinning mass gyroscopes, which were originally the gyroscopes of choice for space applications, require lubrication and eventually wear out.

Because of the necessity of gyroscopes in spacecraft there is a wide product development. Many gyroscopes designed for use in space use solid-state technology, which means that they are constructed without any moving parts. These gyroscopes provide the required long lifetime but these instruments are very expensive, power-hungry and still quite bulky.

In 1997 the Jet Propulsion Laboratory (JPL) in El Segundo, California, created a new micro gyroscope not bigger than four times four millimeters and a weight under one gram. The long-life and high-performance micro gyro relies on the measurement of vibrations. It notices changes in the vibration of a light piece of micro-machined silicon that has no moving parts. There are several types of micro gyros produced around the world.
3 SOLUTION

3.1 System identification

Throughout this project, a characterization on the plant has been done for identification in order to build a model. The model should mirror the physical reality as accurately as possible.

The equations of motion (Appendix 2) derived from using Lagrange’s method were given in the manual of the gyroscope. To be able to control the system, these equations were linearized through Taylor’s series expansion. By using the neutrally stable equilibrium point as the operating point a state-space model was achieved.

\[ F_i(x_0) + \frac{\partial F_i}{\partial q_4}(x_0) \cdot q_4 + \frac{\partial F_i}{\partial \omega_2}(x_0) \cdot \omega_2 + \frac{\partial F_i}{\partial \omega_3}(x_0) \cdot \omega_3 + \frac{\partial F_i}{\partial \omega_4}(x_0) \cdot \omega_4 \]

\[ x_0 = \{\omega_1 = \Omega, q_2 = q_{20}, q_3 = q_{30}\} \]

where \( F_i \) is the \( i \)’th equation of motion in Appendix 2
The gyro is a complex plant with four degrees of freedom and only two motors as actuators. From this observation, the gyro is deduced to be an under actuated system, which makes trajectory tracking particularly difficult. Besides the non-linear dynamics of the gyro, also the actuators are nonlinear. Both motors have some dead zones and a nonlinear transfer function outside these dead zones.
When deriving the dynamics of the gyroscope, the assumptions that the mass of the different gimbals could be neglected and that they were symmetric were done. Nevertheless, different experiments showed that the structure is not perfectly balanced and that gravity has some influence, which leads to some deviation from what is expected. This point is not a great problem as robust control is supposed to compensate for these errors in the model.

Another deviation from our model, which should be taken into account, is the frictions in the different links. These frictions are quite large in the links with a motor and to get a real good tracking the design should compensate them.

### 3.2 Control design

A 1 degree of freedom (DOF) system was used for identification of the inputs and outputs. This system was created using the two brakes that stop the rotation of two axes \( q_3 \) and \( q_4 \) equals to zero. The equation of the 1 DOF system was easy and a model was created in Matlab, this led to a design of an Internal Model Controller for the 1 DOF system. It worked out well for large angles but problems appeared when having small angles.

Taking the conclusions made in the system identification into account, different designs were tried out and some limitations were found. Feedback linearization, optimal control theory and gain scheduling are here presented step by step towards the final solution.

#### 3.2.1 Feedback linearization.

From the full dynamics of the gyro, it was possible to use feedback linearization and then linearize some states. As there were two actuators, two feedback loops were used in order to linearize two of the states and still have some non-linear states. The problem with this
method was that the non-linear states got more complicated and parts of the intuition of the plant dynamics were lost. This method was tried on the reduced system; obtained by the two brakes on axes 3 and 4.

The feedback linearization worked well but the kind of trajectory possible to follow was very restricted.

3.2.2 Optimal control theory
Optimal control theory was the second method used. The simulations were very promising, but it was not possible to implement these solution on the Simulink and Real Time Workshop (RTW) as some parameters were time varying. A linear quadratic controller, designed with the Jacoby Bellman equations, was tried in order to track a trajectory with the non-linear plant and the feedback loop contained a time varying matrix gain. As the experimental setup did not enable use of time varying components, optimal control cannot be applied as linearization around a trajectory. This led to time varying system equations.

3.2.3 Gain scheduling
The last method used was gain scheduling. In this method the system equations were linearized around different operating points and switched between different controllers according to the current working point. To make this possible the state space was divided into different regions were a controller was designed for each region. For the controllers, different designs as LQR/LQG or $H_2/H_8$ were possible to use. The LQR controller, which was simple to implement, enabled to set the most important states for the tracking as well as control the torque magnitudes feed to the plant but had some limited robust properties. $H_8$ was more robust and usually led to a better controller. One of the problems encountered with the gain scheduling method was that the linearized system was uncontrollable with a controllability matrix of rank five. This was due to the fact that two states depend on two other states in every operating point, which coincide with intuition. To overcome this limitation, the system was reduced in the different operating points and only the states of importance for tracking were kept.

3.3 Trajectory planning
The trajectories were chosen so that movements will be within the limitations of the controllers, which is not to have too big variations from the initial position on the $q_3$ axis. This would only vary at maximum $\pm \pi/4$ radians at the same time that $q_4$ was moving, so $q_3$ is set to follow a sinusoid with a small amplitude of about 0.2 radians. The $q_4$-axis should be able to vary very much if there weren’t to big steps, this should be shown, so $q_4$ is set to follow different ramps and some smaller steps. It is rotated several rotations round the axis at various speeds. Another thing that is shown is positioning the gimbals in a certain position given by the user. This should not be set outside the controllers limitations and too big steps should be avoided.
3.4 Limitations

To mount this gyroscope on spacecraft wouldn’t be recommended due to its large weight and inability to follow trajectories in more than two dimensions.

The LQG design based on a reduced system gives limitations on the controllable area, because two states always have to be removed from the equations. If states $w_1$ and $q_2$ are removed then the system is unable to control angles of $q_2$ and the angular velocity of $w_1$. This leads to a controllability of just two dimensions and could lead to heavy vibrations.

The plant also has built in limitations as dead zones of the motors and upper velocity limits. These facts give limitations in the amplitude of input signals and make the system hard to respond to small angels or fast changes.
4 COURSE FEEDBACK

This project course differs from other courses at KTH in many ways. The course is problem based; students can plan their work very freely and have great possibilities of deciding their schedule with a certain goal in their mind. The gyro team has a mostly positive experience of the course but some small changes would make it even better.

One good experience has been the group composition. No one in the group knew each other before the course started and this is a much greater challenge than working with old friends. An idea is therefore that all groups should be formed randomly to make the project more similar to a future situation in an industrial project.

A problem in the gyro group was the slow course start. The course kick-off was really good but because of the long holiday around Easter, the work didn’t start properly until the week of the midterm exam. If the groups had been formed the first week of the course or maybe even before the course started, a better project start would have been possible.

The greatest advantage with a project course is its practical character in comparison with many other courses. The hands-on experience is very developing and gives a deeper understanding for the subject. The course administration with a customer and a consultant worked very well. Until next year the institution should try to find a larger lab room, the bad air quality (too many people) and the noise (helicopter) in the lab made it sometimes very hard to work efficiently.

One goal with the course has been to practice in writing reports. Maybe a good example of a written report should have been showed to make it easier for the students to develop there writing.

5 CONCLUSIONS

The project plan and the time plan that was decided the first week of the project have been followed quite well. Before having an overview of the problem and the group, it was difficult to plan the work properly and the group is satisfied with the results.

A knowledge that the group has got is that even though a controller looks nice in simulations it does not mean that it works in reality.
6 REFERENCES


Skogestad, Sigurd & Postlethwaite, Ian, (1997), Multivariable Feedback
Appendix 1

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Appendix 2

The equations of motion

\[ T_1 + JD \times (\sin(q_2) \times w_2 \times w_3 \times \sin(q_2) \times \sin(q_3) \times w_3 \times w_4 - \cos(q_2) \times \cos(q_3) \times w_2 \times w_4 - w_{1p} \times \cos(q_2) \times w_3 \times \sin(q_2) \times \cos(q_3) \times w_4) = 0 \]

\[ T_2 + (IC + JC + JD \times \cos(q_3) \times w_3 \times w_4 + JD \times \cos(q_2) \times \cos(q_3) \times w_1 \times w_4 - JD \times w_1 \times w_3 + (ID + JD + JC) \times \sin(q_2) \times (\cos(q_2) \times w_3)^2 + 2 \times w_2 \times \sin(q_2) \times \cos(q_3) \times w_3 \times w_4 - \cos(q_2) \times \cos(q_3) \times (w_3 \times w_4) + (IC + ID) \times (\sin(q_3) \times w_4 - w_{2p}) = 0 \]

\[ (IB - KB) \times \sin(q_3) \times \cos(q_3) \times (w_4)^2 + JC \times \cos(q_2) \times (\sin(q_2) \times w_2 \times w_3 + \sin(q_3) \times w_3 \times w_4 - \cos(q_2) \times \cos(q_3) \times w_2 \times w_4) + ID \times \cos(q_2) \times (\sin(q_2) \times w_3 \times w_4 + \cos(q_2) \times \cos(q_3) \times w_2 \times w_4 - \sin(q_2) \times \cos(q_3) \times (w_3 \times w_4 - w_{1p} \times \sin(q_2) \times \cos(q_3) \times w_2 \times w_4) + JD \times \sin(q_2) \times \cos(q_3) \times (\sin(q_2) \times w_2 \times w_3 + \sin(q_2) \times \sin(q_3) \times w_3 \times w_4 - \cos(q_2) \times \cos(q_3) \times w_2 \times w_4) + JC \times \sin(q_2) \times (\cos(q_2) \times \sin(q_3) \times \cos(q_3) \times w_3 \times w_4 - w_{2p} \times \sin(q_2) \times \cos(q_3) \times w_2 \times w_4 - \cos(q_2) \times \cos(q_3) \times (w_3 \times w_4 - w_{1p} \times \sin(q_2) \times \cos(q_3) \times w_2 \times w_4) + JD \times \sin(q_2) \times \cos(q_3) \times (\sin(q_2) \times w_2 \times w_3 + \sin(q_2) \times \sin(q_3) \times w_3 \times w_4 - \cos(q_2) \times \cos(q_3) \times w_2 \times w_4) + IC \times \sin(q_3) \times (w_4)^2 + JC \times \sin(q_2) \times \cos(q_3) \times (\sin(q_2) \times w_2 \times w_3 + \sin(q_2) \times \sin(q_3) \times w_3 \times w_4 - \cos(q_2) \times \cos(q_3) \times w_2 \times w_4) + JD \times \sin(q_2) \times \cos(q_3) \times (\sin(q_2) \times w_2 \times w_3 + \sin(q_2) \times \sin(q_3) \times w_3 \times w_4 - \cos(q_2) \times \cos(q_3) \times w_2 \times w_4) + IC \times \sin(q_3) \times (w_4)^2 = 0 \]

\[ (IC + JC + JD \times \cos(q_3) \times w_3 \times w_4 + JD \times \cos(q_2) \times \cos(q_3) \times w_1 \times w_4 - JD \times w_1 \times w_3 + (ID + JD + JC) \times \sin(q_2) \times (\cos(q_2) \times w_3)^2 + 2 \times w_2 \times \sin(q_2) \times \cos(q_3) \times w_3 \times w_4 - \cos(q_2) \times \cos(q_3) \times (w_3 \times w_4) + (IC + ID) \times (\sin(q_3) \times w_4 - w_{2p}) = 0 \]

\[ (IB - KB) \times \sin(q_3) \times \cos(q_3) \times (w_4)^2 + JC \times \cos(q_2) \times (\sin(q_2) \times w_2 \times w_3 + \sin(q_2) \times \sin(q_3) \times w_3 \times w_4 - \cos(q_2) \times \cos(q_3) \times w_2 \times w_4) + ID \times \cos(q_2) \times (\sin(q_2) \times w_2 \times w_3 + \sin(q_2) \times \sin(q_3) \times w_3 \times w_4 - \cos(q_2) \times \cos(q_3) \times w_2 \times w_4) + JD \times \sin(q_2) \times \cos(q_3) \times (\sin(q_2) \times w_2 \times w_3 + \sin(q_2) \times \sin(q_3) \times w_3 \times w_4 - \cos(q_2) \times \cos(q_3) \times w_2 \times w_4) + JC \times \sin(q_2) \times \cos(q_3) \times (\sin(q_2) \times w_2 \times w_3 + \sin(q_2) \times \sin(q_3) \times w_3 \times w_4 - \cos(q_2) \times \cos(q_3) \times w_2 \times w_4) + JD \times \sin(q_2) \times \cos(q_3) \times (\sin(q_2) \times w_2 \times w_3 + \sin(q_2) \times \sin(q_3) \times w_3 \times w_4 - \cos(q_2) \times \cos(q_3) \times w_2 \times w_4) + IC \times \sin(q_3) \times (w_4)^2 = 0 \]