Higher Order Barrier Certificates for Leader-Follower Multi-Agent Systems

Maryam Sharifi and Dimos V. Dimarogonas

***Abstract*—This paper presents control strategies based on time-varying convergent higher order control barrier functions for a class of leader-follower multi-agent systems under signal temporal logic (STL) tasks. Each agent is assigned a local STL task which may be dependent on the behavior of agents involved in other tasks. We consider one or more than one leader in the multi-agent system. The leader has knowledge on the associated tasks and controls the performance of the subgroup involved agents. The followers are not aware of the tasks, and do not have any control authority to reach them. They follow the leader commands indirectly, according to their dynamics interconnections, for the task satisfaction. We further assume that the input-to-state stability (ISS) property for the multi-agent system is fulfilled. First, robust solutions for the task satisfaction, based on the leader's accessibility to the follower agents' states are suggested. In addition, using the notion of higher order barrier functions, individual barrier certificates for each agent evolving in a formation dynamic structure are proposed. For the case of presence of more leaders in the subgroups, we provide decentralized barrier certificates. Our approach finds solutions to guarantee the satisfaction of STL tasks independent of the agents' initial conditions.**

I. Introduction

The improved capabilities of coordination in a group of systems over single-agent systems to handle task complexity and robustness to agent failures, makes the field of multi-agent systems a popular research topic. The design of multi-agent coordination strategies typically deals with group behaviors such as achieving and maintaining consensus [1], formations [2], covering areas of interest [3], environmental exploration [4], connectivity maintenance [5], and collision avoidance [6]. However, many complex tasks may not be defined as stand-alone traditional control objectives and need employing some tools from computer science such as formal verification in order to define general specifications in temporal logic formulations that induce a sequence of control actions [7], [8]. Among those formulations, signal temporal logic (STL) is beneficial as it is interpreted over continuous-time signals [9], allows for imposing tasks with strict deadlines and introduces quantitative robust semantics [10].

While assigning the same distributed control strategy to all robots may be suitable for simpler and more traditional control objectives, our aim here is to tackle high-level and more complex task specifications in the form of STL. We choose to consider a heterogeneous, leader-follower approach to the problem. In leader-follower networks, a subset of agents with advanced actuation, computation and communication capabilities, namely the leaders, are responsible for guiding the whole group to satisfy STL tasks in a decentralized and cooperative way while fulfilling the transient constraints. While this approach will include the homogeneous (leaderless) decentralized case as a subcase, it will guarantee the significant improvement of several important attributes of the system, including scalability, robustness with respect to failures, and resource usability, since only a subset of the team (the leaders) need to be actuated for the specification fulfilment. In [11], several measures of controllability for leader-follower networks are defined and utilized as the performance metric. The authors in [12] investigate the problem of assigning a predetermined number of leaders formulated as a convex objective function, to minimize the overall variance in the network subject to stochastic disturbances. Leader selection to achieve the stabilization and tracking via the notion of manipulability is considered in [13]. However, these approaches don't take into account complex tasks with space and time constraints prescribed by STL.

We present control strategies for first and second order leader-follower multi-agent systems under local STL tasks. For this aim, we present a notion of *time-varying convergent higher order control barrier functions (TCHCBF)* to address the high relative degree constraints and provide individual barrier certificates for each follower. Control barrier functions [14] guarantee the existence of a control law that renders a desired set forward invariant. Nonsmooth, higher order and time-varying control barrier functions are provided in [15], [16] and [17], respectively. In addition, decentralized barrier certificates are provided in [18]. Nevertheless, appropriate control barrier functions to maintain the desired behavior of leader-follower multi-agent systems under STL tasks haven't been introduced yet, to the best of our knowledge. We consider connected graph topologies where each local STL task is defined on a subset of connected agents containing one or more leaders. The subsystems are not fully decoupled. Hence, each task may be dependent on the behavior of other group agents. The leader agent has the knowledge of the associated local task and is responsible for its satisfaction. The followers are not aware of the prescribed tasks and don't have any control authority to meet them.

We first consider the case of first order dynamics leader-follower networks. Due to the deficiencies in the rank of the input matrix, there exist singularities in the associated constraints. This issue results from the under-actuated property of the system caused by the follower agents which are not

This work was supported by the ERC CoG LEAFHOUND, the Swedish Research Council (VR) and the Knut & Alice Wallenberg Foundation (KAW).
The authors are with Division of Decision and Control Systems, School of Electrical Engineering and Computer Science, KTH Royal Institute of Technology, Stockholm, Sweden. {msharifi, dimos}@kth.se.

influenced by direct actuation. We tackle the singularities by providing novel barrier function certificates for specific graph topologies to guarantee fixed-time convergence to the specified safe sets and remaining there onwards. We call these sets *fixed-time convergent and forward invariant*. We then consider higher order leader-follower networks, where the relative-degree of each agent is more than one. Moreover, there exist again singularities which cause infeasibilties in the satisfaction of required constraints due to the existence of follower agents. We provide higher order convergent control barrier functions and singularity avoidance solutions to satisfy specifications. Then, we focus on second order dynamic leader-follower networks and provide higher order barrier certificates in order to tackle the higher-order constraints, (e.g., position dependent constraints) and the under-actuated property of the network.

In this paper, we consider (first and higher order dynamics) leader-follower networks. The control barrier certificates are provided based on the knowledge of the leader from the followers and a connected network topology assumption, with the aim to guarantee convergence and forward invariance of the desired sets. We provide relaxed barrier certificates for the input signal under the assumption that the leader has only partial knowledge of the followers' states and network topology (according to Assumption 3), while there is no need for the leader to know the upper bound of the norm corresponding to the dynamic terms of its non-neighbor agents. This upper-bound determines the ultimate convergent set for the network under the specified tasks. Furthermore, in order to improve scalability of the network control solution and account for more general STL formulas, we have proposed individual barrier certificates for each agent. Utilizing the higher order barrier functions for the followers according to the formation dynamic structure of the network, we are able to maintain the constraints using the leader's control input. In addition, we consider subgroups consisting of more than one leader and provide decentralized constraints as functions of the leaders' control input signal.

The rest of the paper is organized as follows. Section II gives some preliminaries on STL, leader-follower multi-agent systems and time-varying barrier functions. First order systems are considered in Sections III. Higher order leader-follower networks are considered in Section IV and higher order, individual and decentralized barrier certificates with some illustrative examples are introduced. Finally, some concluding points are presented in Sections V.

II. Preliminaries and problem formulation

A. Signal temporal logic (STL)

Signal temporal logic (STL) [9] is based on predicates ν which are obtained by evaluation of a continuously differentiable predicate function $h : \mathbb{R}^d \to \mathbb{R}$ as $\nu := \top$ (True) if $h(\mathbf{x}) \geq 0$ and $\nu := \bot$ (False) if $h(\mathbf{x}) < 0$ for $\mathbf{x} \in \mathbb{R}^d$. The STL syntax is then given by

$$\phi ::= \top|\nu|\neg\phi|\phi' \wedge \phi''|\phi' U_{[a,b]}\phi'',$$

where $\neg$ and $\wedge$ denote negation and conjunction, respectively and ϕ', ϕ'' are STL formulas, and $U_{[a,b]}$ is the until operator with $a \leq b < \infty$. In addition, define $F_{[a,b]}\phi := \top U_{[a,b]}\phi$ (eventually operator) and $G_{[a,b]}\phi := \neg F_{[a,b]}\neg\phi$ (always operator). Note that $\neg\mu$ can be encoded in the STL syntax above by defining $\bar{\mu} := \neg\mu$ and $\bar{h}(\nu) := -h(\nu)$. Let $(\mathbf{x}, t) \models \phi$ denote the satisfaction relation, i.e., a formula ϕ is satisfiable if $\exists \mathbf{x} : \mathbb{R}_{\geq 0} \to \mathbb{R}^d$ such that $(\mathbf{x}, t) \models \phi$.

Definition 1. *[9] (STL Semantics): For a signal $\mathbf{x} : \mathbb{R}_{\geq 0} \to \mathbb{R}^d$, the STL semantics are recursively given by:*

$$\begin{aligned}
(\mathbf{x}, t) \models \nu \quad & \Leftrightarrow h(\mathbf{x}) \geq 0, \\
(\mathbf{x}, t) \models \neg\phi \quad & \Leftrightarrow \neg((\mathbf{x}, t) \models \phi), \\
(\mathbf{x}, t) \models \phi' \wedge \phi'' \quad & \Leftrightarrow (\mathbf{x}, t) \models \phi' \wedge (\mathbf{x}, t) \models \phi'', \\
(\mathbf{x}, t) \models \phi' U_{[a,b]}\phi'' & \Leftrightarrow \exists t_1 \in [t+a, t+b]\ s.t. (\mathbf{x}, t_1) \models \phi'' \\
& \quad \wedge \forall t_2 \in [t, t_1], (\mathbf{x}, t_2) \models \phi', \\
(\mathbf{x}, t) \models F_{[a,b]}\phi \quad & \Leftrightarrow \exists t_1 \in [t+a, t+b]\ s.t. (\mathbf{x}, t_1) \models \phi, \\
(\mathbf{x}, t) \models G_{[a,b]}\phi \quad & \Leftrightarrow \forall t_1 \in [t+a, t+b]\ s.t. (\mathbf{x}, t_1) \models \phi.
\end{aligned}$$

B. Leader-follower multi-agent systems

Consider a connected undirected graph $\mathcal{G} := (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} := \{1, \cdots, n\}$ indicates the set consisting of n agents and $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ represents communication links. Without loss of generality, we suppose the first n_f agents as followers and the last n_l agents as leaders, with corresponding vertices, sets denoted as $\mathcal{V}_f := \{1, \cdots, n_f\}$ and $\mathcal{V}_l := \{n_f + 1, \cdots, n_f + n_l\}$, respectively, with $n_f + n_l = n$. Let $p_i \in \mathbb{R}$, $v_i \in \mathbb{R}$ and $u_i \in \mathbb{R}$ denote the position, velocity and control input of agent $i \in \mathcal{V}$, respectively. Moreover, $\mathcal{N}_i$ denotes the set of neighbors of agent i and $|\mathcal{N}_i|$ determines the cardinality of the set $\mathcal{N}_i$. In addition, we define the stacked vector of all elements in the set $\mathcal{X}$ with cardinality $|\mathcal{X}|$, as $[x_i]_{i\in\mathcal{X}} := [x_{i_1}^\top, \cdots, x_{i_{|\mathcal{X}|}}^\top]^\top$, $i_1, \cdots, i_{|\mathcal{X}|} \in \mathcal{X}$. Then, the 1^{st} order dynamics of agent i can be described as

$$\dot{p}_i = \mathfrak{f}_i^s(p_i, [p_j]_{j\in\mathcal{N}_i}) + b_i \mathfrak{g}_i^s(p_i) u_i, \tag{1}$$

where $b_i = 0$, $i \in \{1, \cdots, n_f\}$, indicates the followers and $b_i = 1$, $i \in \{n_f + 1, \cdots, n_f + n_l\}$, denotes the leaders. In addition, $\mathfrak{f}_i^s : \mathbb{R}^{1+|\mathcal{N}_i|} \to \mathbb{R}$, $\mathfrak{g}_i^s : \mathbb{R} \to \mathbb{R}$ are assumed to be locally Lipschitz continuous functions.

We also introduce the 2^{nd} order dynamics for the followers for $b_i = 0$ and the leaders for $b_i = 1$ as follows.

$$\begin{bmatrix} p_i \\ v_i \end{bmatrix} = \underbrace{\begin{bmatrix} v_i \\ f_i^d(p_i, [p_j]_{j\in\mathcal{N}_i}, v_i, [v_j]_{j\in\mathcal{N}_i}) \end{bmatrix}}_{\mathfrak{f}_i^d(p_i, [p_j]_{j\in\mathcal{N}_i}, v_i, [v_j]_{j\in\mathcal{N}_i})} + b_i \begin{bmatrix} 0 \\ \mathfrak{g}_i^d(v_i) u_i \end{bmatrix}, \tag{2}$$

in which $\mathfrak{f}_i^d : \mathbb{R}^{2+2|\mathcal{N}_i|} \to \mathbb{R}$, $\mathfrak{g}_i^d : \mathbb{R} \to \mathbb{R}$ are locally Lipschitz continuous functions.

We consider the STL fragment

$$\psi ::= \top|\nu|\psi' \wedge \psi'', \tag{3a}$$

$$\phi ::= G_{[a,b]}\psi|F_{[a,b]}\psi|\psi' U_{[a,b]}\psi''|\phi' \wedge \phi'', \tag{3b}$$

where ψ', ψ'' are formulas of class ψ in (3a) and ϕ', ϕ'' are formulas of class ϕ in (3b). It is worth mentioning that these formulas can be extended to consider disjunctions ($\vee$) using automata based approaches [19].

Consider formulas ϕ^s and ϕ^d of the form (3b), corresponding to the 1^{st} and 2^{nd} order leader-follower multi-agent systems, respectively. The formula ϕ^s (resp. ϕ^d) consists of a number of temporal operators and its satisfaction depends on the behavior of the set of agents $\mathcal{V} = \{1, \cdots, n\}$. By behavior of an agent i, we mean the state trajectories that evolve according to (1) (resp. (2)).

Assumption 1. *Predicate functions in ϕ^s (resp. ϕ^d) are concave.*

Concave predicate functions contain linear functions as well as functions corresponding to reachability tasks ($\|x-p\|^2 \leq \epsilon$, $p \in \mathbb{R}^n$, $\epsilon \geq 0$). As the minimum of concave predicate functions is again concave, they are useful in constructing valid control barrier functions [20, Lemmas 3, 4].

Based on (1) and (2), we write the stacked dynamics for the set of agents $i \in \mathcal{V}$, as

$$\dot{x}^s = \mathfrak{f}^s(x^s) + \mathfrak{g}^s(x^s)u, \tag{4}$$

for the 1^{st} order dynamics and

$$\dot{x}^d = \mathfrak{f}^d(x^d) + \mathfrak{g}^d(x^d)u, \tag{5}$$

for the 2^{nd} order dynamics, where $x^s := [x_i^s]_{i\in\mathcal{V}} = [p_i]_{i\in\mathcal{V}} \in \mathcal{S}^s \subseteq \mathbb{R}^n$, $\mathfrak{f}^s(\cdot) = [\mathfrak{f}_i^s(\cdot)]_{i\in\mathcal{V}} \in \mathbb{R}^n$, $x^d := \left[x_i^d\right]_{i\in\mathcal{V}} = [p_i; v_i]_{i\in\mathcal{V}} \in \mathcal{S}^d \subseteq \mathbb{R}^{2n}$, $\mathfrak{f}^d(\cdot) = \left[\mathfrak{f}_i^d(\cdot)\right]_{i\in\mathcal{V}} \in \mathbb{R}^{2n}$. Without loss of generality, we consider functions $\mathfrak{f}_i^s(x^s)$ and $\mathfrak{f}_i^d(x^d)$ as $\mathfrak{f}_i^s(x^s) = \mathfrak{f}_{i,i}^s(x_i^s) + \sum_{j\in\mathcal{V}, j\neq i} \mathfrak{f}_{i,j}^s(x_i^s, x_j^s)$ and $\mathfrak{f}_i^d(x^d) = \mathfrak{f}_{i,i}^d(x_i^d) + \sum_{j\in\mathcal{V}, j\neq i} \mathfrak{f}_{i,j}^d(x_i^d, x_j^d)$, respectively. The local dynamic function $\mathfrak{f}_{i,i}^s(x_i^s)$ corresponds to the terms of $\mathfrak{f}_i^s(x^s)$ which are only dependent on x_i^s (p_i), and $\mathfrak{f}_{i,j}^s(x_i^s, x_j^s)$ contains the terms of $\mathfrak{f}_i^s(x^s)$ which are dependent on agent $j \in \mathcal{V}, j \neq i$ as well. The same holds for $\mathfrak{f}_i^d(x^d)$. For the case of one leader, with follower and leader sets $\mathcal{V}_f := \{1, \cdots, n-1\}$ and $\mathcal{V}_l := \{n\}$, respectively, the input matrices and control input signal are defined as $\mathfrak{g}^s(\cdot) := \left[\begin{array}{c} 0_{n-1\times 1}^T, \mathfrak{g}_n^s(\cdot) \end{array}\right]^T$, $\mathfrak{g}^d(\cdot) := \left[\begin{array}{c} 0_{2n-1\times 1}^T, \mathfrak{g}_n^d(\cdot) \end{array}\right]^T$, and $u := u_n \in \mathbb{R}$. In addition, for networks containing more than one leader, with $\mathcal{V}_f := \{1, \cdots, n_f\}$ and $\mathcal{V}_l := \{n_f+1, \cdots, n\}$, $n = n_f + n_l$, the associated matrices are denoted as $\mathfrak{g}^s(\cdot) := \left[\begin{array}{c} 0_{n_f\times n_l}^T, \mathfrak{g}_{n_l}^s(\cdot) I_{n_l} \end{array}\right]^T$, $\mathfrak{g}_{n_l}^s(\cdot) = [\mathfrak{g}_i^s(\cdot)]_{i\in\{n_f+1,\cdots,n\}}$, $\mathfrak{g}^d(\cdot) := \left[\begin{array}{c} 0_{n_e+n_f\times n_l}^T, \mathfrak{g}_{n_l}^d(\cdot) I_{n_l} \end{array}\right]^T$, $\mathfrak{g}_{n_l}^d(\cdot) = \left[\mathfrak{g}_i^d(\cdot)\right]_{i\in\{n_f+1,\cdots,n\}}$, and $u := [u_i]_{i\in\{n_f+1,\cdots,n\}} \in \mathbb{R}^{n_l}$. Note also that the input matrices $\mathfrak{g}^s(\cdot)$ and $\mathfrak{g}^d(\cdot)$ are not full row rank. We also denote by m_i as the minimum of the length of paths between agent i and the leaders of the network.

We assume that the dynamics of the agents are input-to-state stable (ISS). In other words, consider the neighbor agents' states x_j^s and x_j^d, $j \in \mathcal{N}_i$, as inputs to the functions $f_i^s(x^s)$ and $f_i^d(x^d)$, respectively, where $i \in \mathcal{V}$. Then, ISS implies that $f_i^s(x^s)$ ($f_i^d(x^d)$), $i \in \mathcal{V}$, is asymptotically stable whenever $x_j^s = 0$ ($x_j^d = 0$), $j \in \mathcal{N}_i$. Moreover, we assume that the small-gain condition for the network consisting of n ISS agents is satisfied according to [21]. In particular, for the case of first order dynamics, consider the local dynamics $\dot{x}_i = \mathfrak{f}_i^s(x^s) + b_i\mathfrak{g}_i^s(x_i^s)u_i = \mathfrak{f}_{i,i}^s(x_i^s) + \sum_{j\in\mathcal{V}, j\neq i} \mathfrak{f}_{i,j}^s(x_i^s, x_j^s) + b_i\mathfrak{g}_i^s(x_i^s)u_i$, with $b_i = 0, i \in \mathcal{V}_f$ and $b_i = 1, i \in \mathcal{V}_l$. Moreover, the stacked dynamics is given as (4).
Then, the local ISS property for the dynamics $\dot{x}_i = \mathfrak{f}_i^s(x^s) + b_i\mathfrak{g}_i^s(x_i^s)u_i$ with internal inputs $x_j^s, j \in \mathcal{N}_i$ and external input $u_i \in \mathbb{R}$, states that there exist continuously differentiable functions $V_i : \mathbb{R} \to \mathbb{R}_+$ and functions $\alpha_i, \gamma_{iu} \in \mathcal{K}_\infty$ and $\gamma_{ij} \in \mathcal{K}_\infty \cup \{0\}$, $i, j = 1, \cdots, n$ such that

$$\dot{V}_i(x_i^s) \leq -\alpha_i(V_i(x_i^s)) + \sum_{i\neq j} \gamma_{ij}(V_j(x_j^s)) + \gamma_{iu}(\|u\|) \tag{6}$$

for all $x_i^s \in \mathbb{R}$, $i = 1, \cdots, n$. Now define the matrices $\Gamma(s) := (\gamma_{ij}(s))_{i,j=1,\cdots,n} \in (\mathcal{K}_\infty \cup \{0\})^{n\times n}$ and $A(s) := \operatorname{diag}(\alpha_1(s), \cdots, \alpha_n(s))$. Then, the small-gain property [21] is formulated as

$$\Gamma \circ A^{-1}(s) < s, \forall s \in \mathbb{R}_+^n \backslash \{0\}. \tag{7}$$

In addition, the inequality (6) can be written in the vector form below.

$$\dot{V}_{vec}(x^s) \leq (-A + \Gamma)(V_{vec}(x^s)) + \gamma_u(\|u\|), \tag{8}$$

where $V_{vec}(x^s) := (V_1(x_1^s), \cdots, V_n(x_n^s))^T$ and $\gamma_u(\cdot) := [\gamma_{iu}(\cdot)]_{i=1,\cdots,n}$. Consider α_i and γ_{ij} as linear gains. Then, according to [21, Lemma 3.1], there exists a vector $\mu \in \mathbb{R}_+^n$, $\mu > 0$ such that $\mu^T(-A + \Gamma) < 0$, if and only if the small-gain condition (7) holds. By considering the candidate ISS-Lyapunov function $V(x^s) := \mu^T V_{vec}(x^s)$ and applying the result of [21, Lemma 3.1] to the inequality (8) corresponding to the stacked dynamics of the network, the ISS property of the network (i.e., boundedness of the state vector x^s) is concluded. For the case of nonlinear gains α_i and γ_{ij}, the small-gain condition (7) is not sufficient to obtain the desired robustness with respect to the external input. Thus, a *robust small gain condition* is imposed in [21] which requires that for some $D = \operatorname{diag}(\mathrm{id} + \beta_1, \cdots, \mathrm{id} + \beta_n)$, $\beta_i \in \mathcal{K}_\infty$ we have

$$D \circ \Gamma \circ A^{-1}(s) < s, \forall s \in \mathbb{R}_+^n \backslash \{0\}, \tag{9}$$

where id denotes the identity function. Under the *robust small gain condition* (9), the ISS property of the whole network is guaranteed according to [21, Theorem 4.1]. Similar arguments are valid for the network (5) consisting of the second order dynamics agents.

Definition 2. *[14] A continuous function $\lambda : (-b, a) \Rightarrow \mathbb{R}$ for some $a, b > 0$ is called an extended class $\mathcal{K}$ function if it is strictly increasing and $\lambda(0) = 0$.*

C. Time-varying barrier functions

Let $\mathfrak{h}^s(x^s, t) : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ (resp. $\mathfrak{h}^d(x^d, t) : \mathbb{R}^{2n} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$) be a piece-wise differentiable function. The time-varying barrier function $\mathfrak{h}^s(x^s, t)$ (resp. $\mathfrak{h}^d(x^d, t)$) is built corresponding to the STL task ϕ^s (resp. ϕ^d) related to the multi-agent system (4) (resp. (5)). Consider the 1^{st} order dynamic network (4). Following the procedure in [20], we construct the barrier function, piece-wise continuous in the second argument, for the conjunctions of a number of q^s single temporal operators in ϕ^s, by using a smooth under-approximation of the min-operator. Accordingly, consider the continuously differentiable barrier functions $\mathfrak{h}_j^s(x^s, t)$, $j \in \{1, \cdots, q^s\}$, corresponding to each temporal operator in ϕ^s. Then, we have

$\min_{j\in\{1,\cdots,q^s\}} \mathfrak{h}_j^s(x^s,t) \approx -\frac{1}{\eta^s}\ln(\sum_{j=1}^{q^s}\exp(-\eta^s\mathfrak{h}_j^s(x^s,\mathrm{t})))$, with parameter $\eta^s>0$ that is proportionally related to the accuracy of this approximation. In view of [20, Steps A, B, and C], the corresponding barrier function to ϕ^s could be constructed as

$$\mathfrak{h}^s(x^s,t) := -\frac{1}{\eta^s}\ln(\sum_{j=1}^{q^s}\exp(-\eta^s\mathfrak{h}_j^s(x^s,\mathrm{t}))), \qquad (10)$$

where each $\mathfrak{h}_j^s(x^s,t)$ is related to an always or eventually operator specified for the time interval $[a_j,b_j]$. Whenever the jth temporal operator is satisfied, its corresponding barrier function $\mathfrak{h}_j^s(x^s,t)$ is deactivated and hence a switching occurs in $\mathfrak{h}^s(x^s,t)$. This time-varying strategy helps reducing the conservatism in the presence of large numbers of conjunctions [20]. Due to the knowledge of $[a_j,b_j]$, the switching instants can be known in advance. Denote the switching sequence as $\{\tau_0:=t_0,\tau_1,\cdots,\tau_{p^s}\}$. At time $t\geq\tau_l$, the next switch occurs at $\tau_{l+1} := \mathrm{argmin}_{b_j\in\{b_1,\ldots,b_{q^s}\}}\zeta(b_j,t)$, $l\in\{0,\cdots,p^s-1\}$, where $\zeta(b_j,t) := \begin{cases} b_j-t, & b_j-t>0 \\ \infty, & \text{otherwise} \end{cases}$.

Definition 3. *[22] (Forward Invariance) Consider the set*

$$\mathfrak{C}^s(t) := \{x^s\in\mathbb{R}^n|\mathfrak{h}^s(x^s,t)\geq 0\}. \qquad (11)$$

The set $\mathfrak{C}^s(t)$ is forward invariant with a given control law u for (4)*, if for each initial condition $x_0^s\in\mathfrak{C}^s(t_0)$, there exists a unique solution $x^s:[t_0,t_1]\to\mathbb{R}^n$ with $x(t_0)=x_0^s$, such that $x^s(t)\in\mathfrak{C}^s(t)$ for all $t\in[t_0,t_1]$.*

If $\mathfrak{C}^s(t)$ is forward invariant, then it holds that $x^s \models \phi^s$. Note that since at each switching instant, one control barrier function $\mathfrak{h}_j^s(x^s,t))$ is removed from $\mathfrak{h}^s(x^s,t) := -\frac{1}{\eta^s}\ln(\sum_{j=1}^{q^s}\exp(-\eta^s\mathfrak{h}_j^s(x^s,\mathrm{t})))$, the set $\mathfrak{C}^s(t)$ is non-decreasing at these switching instants. Hence, for each switching instant τ_l, it holds that $\lim_{t\to\tau_l^-}\mathfrak{C}^s(t)\subseteq\mathfrak{C}^s(\tau_l)$, where $\lim_{t\to\tau_l^-}\mathfrak{C}^s(t)$ is the left-sided limit of $\mathfrak{C}^s(t)$ at $t=\tau_l$.

We also assume that the set $\mathfrak{C}^s$ is compact and non-empty.

Definition 4. *We denote the set $\mathfrak{C}^s(t)$ to be **fixed-time convergent** for* (4)*, if there exists a user-defined, independent of the initial condition, and finite time $T^s>t_0$, such that $\lim_{t\to T^s}x^s(t)\in\mathfrak{C}^s(t)$. Moreover, the set $\mathfrak{C}^s(t)$ is **robust fixed-time convergent** if $\lim_{t\to T^s}x^s(t)\in\mathfrak{C}_{rf}^s(t)$, where $\mathfrak{C}_{rf}^s(t)\supset\mathfrak{C}^s(t)$, and **robust convergent** for* (4)*, if $\lim_{t\to\infty}x^s(t)\in\mathfrak{C}_{rf}^s(t)$. The set $\mathfrak{C}_{rf}^s(t)$ is characterized as $\mathfrak{C}_{rf}^s(t) := \{x^s\in\mathbb{R}^n|\mathfrak{h}^s(x^s,t)\geq-\epsilon_{\max}^s\}$, where $\epsilon_{\max}^s$ is a bounded and positive value.*

The same properties hold for the barrier functions $\mathfrak{h}^d(x^d,t)$ and the set $\mathfrak{C}^d(t)$ corresponding to the 2^{nd} order dynamic network (5) under the task ϕ^d.

III. First order leader-follower multi-agent systems

In this section, we provide conditions to guarantee the *fixed-time convergence* property of the set $\mathfrak{C}^s(t)$ corresponding to the STL specification of the form (3b), using control barrier certificates for a network of 1^{st} order leader-follower agents, based on the leader information of the involved followers. Consider the leader-follower network (4) under the task ϕ^s. Let $\mathfrak{h}^s(x^s,t)$ define a time-varying barrier function for this system. Next, we provide a Lemma to guarantee the *fixed-time convergence and forward invariance* of the set $\mathfrak{C}^s(t)$ given in (11) for system (4), under the following assumption.

Assumption 2. *The leader agent corresponding to the graph $\mathcal{G}:=(\mathcal{V},\mathcal{E})$ subject to the task ϕ^s has knowledge of the functions $\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_i^s}$ and dynamics $\mathfrak{f}_i^s(x^s)$, $i\in\{1,\cdots,n\}$.*

A special case satisfying Assumption 2, is the star topology network with a leader in the middle.

Lemma 1. *Consider a leader-follower multi-agent system subject to the dynamics* (4) *containing one leader, under STL task ϕ^s of the form* (3b) *satisfying Assumption 1. Suppose that the leader satisfies Assumption 2. Let $\mathfrak{h}^s(x^s,t)$ be a time-varying barrier function associated with the task ϕ^s, specified in Section II-C. If for some open set $\mathcal{S}^s$ with $\mathcal{S}^s\supset\mathfrak{C}^s(t)$, $\forall t\geq t_0$, and for all $(x^s,t)\in\mathcal{S}^s\times[\tau_l,\tau_{l+1})$, $l\in\{0,\cdots,p^s-1\}$, for some constants $0<\gamma_1^s<1$, $\gamma_2^s>1$, $\alpha^s>0$, $\beta^s>0$ such that $\frac{1}{\alpha^s(1-\gamma_1^s)}+\frac{1}{\beta^s(\gamma_2^s-1)}\leq\min_{l\in\{0,\cdots,p^s-1\}}\{\tau_{l+1}-\tau_l\}$, there exists a control law u_n satisfying*

$$\begin{aligned}&\textstyle\sum_{i\in\mathcal{V}}\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_i^s}\mathfrak{f}_i^s(x^s)+\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_n^s}\mathfrak{g}_n^s(x_n^s)u_n\\ &+\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial t}\geq-\alpha^s\,sgn(\mathfrak{h}^s(x^s,t))|\mathfrak{h}^s(x^s,t)|^{\gamma_1^s}\\ &\qquad-\beta^s\,sgn(\mathfrak{h}^s(x^s,t))|\mathfrak{h}^s(x^s,t)|^{\gamma_2^s},\end{aligned} \qquad (12)$$

then the set $\mathfrak{C}^s(t)$ is fixed-time convergent and forward invariant. Hence, $(x^s,t)\models\phi^s$.

Proof. Consider the inequality (12) and dynamics (4). Since the leader control signal u_n is the only external input responsible for controlling the network, and under Assumption 2, the inequality (12) can be written as follows:

$$\begin{aligned}&\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x^s}(\mathfrak{f}^s(x^s)+\mathfrak{g}^s(x^s)u)+\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial t}\\ &\quad+\alpha^s sgn(\mathfrak{h}^s(x^s,t))|\mathfrak{h}^s(x^s,t)|^{\gamma_1^s}\\ &\quad+\beta^s sgn(\mathfrak{h}^s(x^s,t))|\mathfrak{h}^s(x^s,t)|^{\gamma_2^s}\geq 0.\end{aligned} \qquad (13)$$

Now, consider the satisfaction of (13) for all $(x^s,t)\in\mathcal{S}^s\times[\tau_l,\tau_{l+1})$ under a control input $u:=u_n\in\mathbb{R}$ with positive constants $\gamma_1^s<1$, $\gamma_2^s>1$, α^s, β^s. Note that by $\lim_{t\to\tau_l^-}\mathfrak{C}^s(t)\subseteq\mathfrak{C}^s(\tau_l)$, it is sufficient to ensure convergence and forward invariance of $\mathfrak{C}^s(t)$ for each $[\tau_l,\tau_{l+1})$. This is due to the fact that if $\mathfrak{h}^s(x^s,t)\in\mathfrak{C}^s(t)$ for all $t\in[\tau_l,\tau_{l+1})$, then $\mathfrak{h}^s(x^s,\tau_{l+1})\in\mathfrak{C}^s(\tau_{l+1})$. Consider the function $V^s(x^s,t)=\max\{0,-\mathfrak{h}^s(x^s,t)\}$. Then, for $x^s(t_0)\in\mathfrak{C}^s(t_0)$ ($\mathfrak{h}^s(x^s,t)\geq 0$) we have $V^s(x^s,t)=0$ for all $t\geq t_0$ by the Comparison Lemma [23]. Hence, the set $\mathfrak{C}^s(t)$ is forward-invariant. Moreover, for $x^s(t_0)\in\mathcal{S}^s\backslash\mathfrak{C}^s(t_0)$ ($\mathfrak{h}^s(x^s,t)<0$), we get $V^s(x^s,t)=-\mathfrak{h}^s(x^s,t)$. Thus, (13) can be written as

$$\dot{V}^s(x^s,t)\leq-\alpha^s V^s(x^s,t)^{\gamma_1^s}-\beta^s V^s(x^s,t)^{\gamma_2^s},$$

which guarantees the fixed-time convergence of x^s to the set $\mathfrak{C}^s(t)$ within $T^s\leq\frac{1}{\alpha^s(1-\gamma_1^s)}+\frac{1}{\beta^s(\gamma_2^s-1)}$ and staying there onwards, according to [24]. The proof is complete. $\square$

Accordingly, the following definition is provided.

Definition 5. *The time-varying barrier function $\mathfrak{h}^s(x^s,t)$ is called a time-varying fixed-time convergent control barrier function (TFCBF) for system* (4)*, if there exist positive constants $\gamma_1^s<1$, $\gamma_2^s>1$, α^s, β^s such that*

$$\sup_{u\in\mathbb{R}}\{\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x^s}\mathfrak{f}^s(x^s)+\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x^s}\mathfrak{g}^s(x^s)u \\ +\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial t}+\alpha^s\text{sgn}(\mathfrak{h}^s(x^s,t))|\mathfrak{h}^s(x^s,t)|^{\gamma_1^s} \\ +\beta^s\text{sgn}(\mathfrak{h}^s(x^s,t))|\mathfrak{h}^s(x^s,t)|^{\gamma_2^s}\geq 0\},$$

for all $(x^s,t)\in\mathfrak{C}^s(t)\times[t_0,t_1]$.

Inspired by [25, Theorem 2], we extend the results of Lemma 1 to the case of leader partial information from the subgraph, i.e., there exist followers that aren't neighbors of the leader, denoted by $i\notin\mathcal{N}_n$. In this case, the *robust fixed-time convergence* property of the set $\mathfrak{C}^s(t)$ is guaranteed.

In the following, we impose some relaxations on Assumption 2 and provide further results on task satisfaction under new conditions.

Assumption 3. *Consider the 1^{st} order leader-follower network* (4) *with a single leader $i=n$. We assume that there exists a positive constant δ^s satisfying $\|\sum_{i\in\mathcal{N}_n,j\notin\mathcal{N}_n}\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_j^s}\mathfrak{f}_j^s(x^s)+\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_i^s}\mathfrak{f}_{i,j}^s(x_i^s,x_j^s)\|\leq\delta^s$, $\forall(x^s,t)\in\mathcal{S}^s\times[\tau_l,\tau_{l+1})$, $l\in\{0,\cdots,p^s-1\}$.*

Remark 1. *Note that the function $\mathfrak{h}^s(x^s,t)$ is differentiable $\forall(x^s,t)\in\mathcal{S}^s\times[\tau_l,\tau_{l+1})$. Moreover, according to the detailed explanations provided in Section II-B, by the ISS property of the agents' dynamics and satisfaction of the small-gain condition* (9) *for the network* (4)*, the ISS property of the multi-agent system is concluded [21, Theorem 4.1]. Therefore, the boundedness of the stack vector x^s is fulfilled and hence, the inequality in Assumption 3 is feasible. Moreover, there is no necessity for the leader to know δ^s. This term is used in determining the ultimate convergent set, as will be demonstrated in the following theorem.*

Theorem 1. *Consider a leader-follower multi-agent network subject to the dynamics* (4) *containing one leader, under STL task ϕ^s of the form* (3b) *satisfying Assumption 1. Let $\mathfrak{h}^s(x^s,t)$ be a time-varying barrier function associated with the task ϕ^s, specified in Section II-C. Suppose that Assumption 3 is satisfied for the network* (4)*. If for some constants $\mu^s>1$, $k^s>1$, $\gamma_1^s=1-\frac{1}{\mu^s}$, $\gamma_2=1+\frac{1}{\mu^s}$, $\alpha^s>0$, $\beta^s>0$, for some open set $\mathcal{S}^s$ with $\mathcal{S}^s\supset\mathfrak{C}^s(t)$, $\forall t\geq 0$, and for all $(x^s,t)\in\mathcal{S}^s\times[\tau_l,\tau_{l+1})$, $l\in\{0,\cdots,p^s-1\}$, there exists a control law u_n such that*

$$\sum_{i\in\mathcal{N}_n}\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_i^s}\mathfrak{f}_{i,i}^s(x_i^s)+(\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_n^s}+\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_i^s})\mathfrak{f}_{n,i}^s(x_n^s,x_i^s) \\ +\frac{\partial\mathfrak{h}_e^s(x^s,t)}{\partial x_n^s}\mathfrak{f}_{n,n}^s(x_n^s)+\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial t}+\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_n^s}\mathfrak{g}_n^s(x_n^s)u_n \\ \geq-\alpha^s\,sgn(\mathfrak{h}^s(x^s,t))|\mathfrak{h}^s(x^s,t)|^{\gamma_1} \\ -\beta^s\,sgn(\mathfrak{h}^s(x^s,t))|\mathfrak{h}^s(x^s,t)|^{\gamma_2^s}, \tag{14}$$

with

$$T^s\leq\begin{cases}\frac{\mu^s}{\alpha^s(c^s-b^s)}\log(\frac{|1+c^s|}{|1+b^s|}) & ;\delta^s>2\sqrt{\alpha^s\beta^s}\\ \frac{\mu^s}{\sqrt{\alpha^s\beta^s}}(\frac{1}{k^s-1}) & ;\delta^s=2\sqrt{\alpha^s\beta^s}\\ \frac{\mu^s}{\alpha^s k_1^s}(\frac{\pi}{2}-\tan^{-1}k_2^s) & ;0\leq\delta^s<2\sqrt{\alpha^s\beta^s}\end{cases} \\ \leq\min_{l\in\{0,\cdots,p^s-1\}}\{\tau_{l+1}-\tau_l\}, \tag{15}$$

where b^s,c^s are the solutions of $\gamma^s(s)=\alpha^s s^2-\delta^s s+\beta^s=0$, $k_1^s=\sqrt{\frac{4\alpha^s\beta^s-\delta^{s2}}{4\alpha^{s2}}}$, $k_2^s=-\frac{\delta^s}{\sqrt{4\alpha^s\beta^s-\delta^{s2}}}$, and δ^s is introduced in Assumption 3, then, the set $\mathfrak{C}_{rf}^s(t)\supset\mathfrak{C}^s(t)$ defined by

$$\mathfrak{C}_{rf}^s(t):=\{x^s\in\mathbb{R}^n|\mathfrak{h}^s(x^s,t)\geq-\epsilon_{\max}^s\}$$

with

$$\epsilon_{\max}^s=\begin{cases}(\frac{\delta^s+\sqrt{\delta^{s2}-4\alpha^s\beta^s}}{2\alpha^s})^{\mu^s} & ;\delta^s>2\sqrt{\alpha^s\beta^s}\\ k^{s\mu^s}(\frac{\beta^s}{\alpha^s})^{\frac{\mu^s}{2}} & ;\delta^s=2\sqrt{\alpha^s\beta^s}\\ \frac{\delta^s}{2\sqrt{\alpha^s\beta^s}} & ;0\leq\delta^s<2\sqrt{\alpha^s\beta^s},\end{cases} \tag{16}$$

is forward invariant and fixed-time convergent within T^s time units, defined in (15)*.*

Proof. Inequality (14) can be written as

$$\sum_{i\in\mathcal{N}_n}\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_i^s}\mathfrak{f}_{i,i}^s(x_i^s)+(\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_n^s}+\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_i^s})\mathfrak{f}_{n,i}^s(x_n^s,x_i^s) \\ +\sum_{i\in\mathcal{N}_n,j\notin\mathcal{N}_n}\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_j^s}\mathfrak{f}_j^s(x^s)+\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_i^s}\mathfrak{f}_{i,j}^s(x_i^s,x_j^s) \\ +\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_n^s}\mathfrak{f}_{n,n}^s(x_n^s)+\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_n^s}\mathfrak{g}_n^s(x_n^s)u_n+\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial t} \\ \geq-\alpha^s\,sgn(\mathfrak{h}^s(x^s,t))|\mathfrak{h}^s(x^s,t)|^{\gamma_1} \\ -\beta^s\,sgn(\mathfrak{h}^s(x^s,t))|\mathfrak{h}^s(x^s,t)|^{\gamma_2^s}+\sum_{j\notin\mathcal{N}_n}\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_j^s}\mathfrak{f}_j^s(x^s) \\ +\sum_{i\in\mathcal{N}_n,j\notin\mathcal{N}_n}\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_i^s}\mathfrak{f}_{i,j}^s(x_i^s,x_j^s). \tag{17}$$

Then, we get

$$\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x^s}(\mathfrak{f}^s(x^s)+\mathfrak{g}_n^s(x_n^s)u_n)+\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial t} \\ \geq-\alpha^s\,sgn(\mathfrak{h}^s(x^s,t))|\mathfrak{h}^s(x^s,t)|^{\gamma_1} \\ -\beta^s\,sgn(\mathfrak{h}^s(x^s,t))|\mathfrak{h}^s(x^s,t)|^{\gamma_2^s} \\ +\sum_{i\in\mathcal{N}_n,j\notin\mathcal{N}_n}\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_j^s}\mathfrak{f}_j^s(x^s)+\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_i^s}\mathfrak{f}_{i,j}^s(x_i^s,x_j^s). \tag{18}$$

It is apparent that the left hand side of (18) is equal to the one in (13), since $\mathfrak{g}_n^s(x_n^s)u_n=\mathfrak{g}^s(x^s)u$. Following the proof of Lemma 1, function $V^s(x^s,t)=\max\{0,-\mathfrak{h}^s(x^s,t)\}$ is considered. This function satisfies $V^s(x^s,t)=0$ for $x^s(t_0)\in\mathfrak{C}^s(t_0)$. Therefore, as long as $\mathfrak{h}^s(x^s,t)\geq 0$, V^s remains 0 and then $x^s(t)\in\mathfrak{C}^s(t)$, $t\geq t_0$. This ensures the forward invariance of $\mathfrak{C}^s(t)$. Moreover, $V^s(x^s,t)>0$ for $x^s\in\mathcal{S}^s\backslash\mathfrak{C}^s(t)$ and by Assumption 3, (18) can be written as

$$\dot{V}^s(x^s,t)\leq-\alpha^s V^s(x^s,t)^{\gamma_1^s}-\beta^s V^s(x^s,t)^{\gamma_2^s}+\delta^s.$$

Thus, according to [26, Lemma 1], the convergence of $V^s(x^s,t)$ to the set $\mathfrak{C}_{rf}^s(t)\supset\mathfrak{C}^s(t)$ in a fixed-time interval $t\leq T^s$, as in (15), is achieved. In addition, considering the forward-invariance of $\mathfrak{C}^s(t)$ besides the convergence property of $\mathfrak{C}_{rf}^s(t)$, ensures forward-invariance of $\mathfrak{C}_{rf}^s(t)$ □

Remark 2. *Note that due to the lack of full information of the leader from other agents of the network, a violation in* *the constraints satisfaction for ϕ^s might occur. This violation has been quantified as a function of δ^s, demonstrated in* (16). *Furthermore,* (15) *is feasible provided that the minimum time interval between successive switchings is sufficiently large, such that the user defined constants α^s, β^s, μ^s, k^s fulfill* (14) *and* (15).

Remark 3. *The constraint* (14) *might be satisfied more easily if more than one leader agent for each subgroup exists, provided that each follower is a neighbor to one leader. This setting will be considered in Section IV-E and appropriate solutions will be provided.*

IV. Higher order leader-follower multi-agent systems

In this section, we consider higher order dynamics multi-agent systems and in order to tackle higher relative degree specifications, provide a class of higher order control barrier functions with the property of convergence to the desired sets and robustness with respect to uncertainties.

A. Convergent higher order control barrier functions

Consider the autonomous system

$$\dot{\mathbf{x}} = f(\mathbf{x}), \tag{19}$$

with $\mathbf{x} \in \mathbb{R}^{\mathrm{n}}$ and locally Lipschitz continuous function $f : \mathbb{R}^{\mathrm{n}} \to \mathbb{R}^{\mathrm{n}}$. We introduce class C^m functions $\mathfrak{h}(\mathbf{x}, t) : \mathbb{R}^{\mathrm{n}} \times [t_0, \infty) \to \mathbb{R}$, later called time-varying convergent higher order control barrier functions, to satisfy STL task ϕ of the form (3b). Define a series of functions $\psi_k : \mathbb{R}^{\mathrm{n}} \times [t_0, \infty) \to \mathbb{R}^{\mathrm{n}}$, $0 \le k \le m$, as

$$\begin{aligned}
&\psi_0(\mathbf{x}, t) := \mathfrak{h}(\mathbf{x}, t),\\
&\psi_k(\mathbf{x}, t) := \dot{\psi}_{k-1}(\mathbf{x}, t)\\
&\qquad\qquad + \lambda_k(\psi_{k-1}(\mathbf{x}, t)),\ 1 \le k \le m-1,\\
&\psi_m(\mathbf{x}, t) := \dot{\psi}_{m-1}(\mathbf{x}, t)\\
&\qquad\qquad + \alpha_m sgn(\psi_{m-1}(\mathbf{x}, t))|\psi_{m-1}(\mathbf{x}, t)|^{\gamma_{1m}}\\
&\qquad\qquad + \beta_m sgn(\psi_{m-1}(\mathbf{x}, t))|\psi_{m-1}(\mathbf{x}, t)|^{\gamma_{2m}},
\end{aligned} \tag{20}$$

where $\lambda_k(\cdot)$, $k = 1, \cdots, m-1$, are $(m-k)^{th}$-order differentiable extended class $\mathcal{K}$ functions and $0 < \gamma_{1m} < 1, \gamma_{2m} > 1$, $\alpha_m > 0, \beta_m > 0$, are user specified constants. We define a series of sets $\mathfrak{C}_k(t)$, $k = 1, \cdots, m$, assumed to be compact, as

$$\mathfrak{C}_k(t) := \{\mathbf{x} \in \mathbb{R}^{\mathrm{n}} | \psi_{k-1}(\mathbf{x}, t) \ge 0\}. \tag{21}$$

Definition 6. *A class C^m function $\mathfrak{h}(\mathbf{x}, t) : \mathbb{R}^{\mathrm{n}} \times [t_0, \infty) \to \mathbb{R}$ is a time-varying convergent higher order barrier function (TCHBF) of degree m for the system* (19)*, if there exist extended class $\mathcal{K}$ functions $\lambda_k(\cdot)$, $k = 1, \cdots, m-1$, constants $0 < \gamma_{1m} < 1, \gamma_{2m} > 1, \alpha_m > 0, \beta_m > 0$, and an open set $\mathfrak{D}$ with $\mathfrak{C} := \cap_{k=1}^{m} \mathfrak{C}_k \subset \mathfrak{D} \subset \mathbb{R}^{\mathrm{n}}$ such that*

$$\psi_m(\mathbf{x}, t) \ge 0,\ \forall (\mathbf{x}, t) \in \mathfrak{D} \times \mathbb{R}_{\ge 0},$$

where $\psi_k(\mathbf{x}, t)$, $k = 0, \cdots, m$, are given in (20).

In the following, we aim to show the *convergence and forward invariance* of the set $\mathfrak{C}$.

Proposition 1. *The set $\mathfrak{C} := \cap_{k=1}^{m} \mathfrak{C}_k \subset \mathfrak{D} \subset \mathbb{R}^{\mathrm{n}}$ is convergent and forward invariant for system* (19)*, if $\mathfrak{h}(\mathbf{x}, t)$ is a TCHBF.*

Proof. First, we show forward invariance of the set $\mathfrak{C}$. If $\mathfrak{h}(\mathbf{x}, t)$ is a TCHBF, then $\psi_m(\mathbf{x}, t) \ge 0,\ \forall (\mathbf{x}, t) \in \mathfrak{D} \times [t_0, \infty)$ according to Definition 6. Then,

$$\begin{aligned}
\dot{\psi}_{m-1}(\mathbf{x}, t) &+ \alpha_m sgn(\psi_{m-1}(\mathbf{x}, t))|\psi_{m-1}(\mathbf{x}, t)|^{\gamma_{1m}}\\
&+ \beta_m sgn(\psi_{m-1}(\mathbf{x}, t))|\psi_{m-1}(\mathbf{x}, t)|^{\gamma_{2m}} \ge 0.
\end{aligned}$$

By the proof of Lemma 1, it is concluded that if $\mathbf{x}(t_0) \in \mathfrak{C}_m(t_0)$, then we get $\psi_{m-1}(\mathbf{x}, t) \ge 0,\ \forall t \in [t_0, \infty)$. Then, by [15, Lemma 2] and considering $\psi_{m-1}(\mathbf{x}, t)$ given by (20), since $x(t_0) \in \mathfrak{C}_{m-1}(t_0)$, we also have $\psi_{m-2}(\mathbf{x}, t) \ge 0,\ \forall t \in [t_0, \infty)$. Iteratively, we can show $\psi_{k-1}(\mathbf{x}, t) \ge 0,\ \forall t \in [t_0, \infty)$ for all $k \in \{1, 2, \cdots, m\}$ which certifies $\mathbf{x}(t) \in \mathfrak{C}_k(t)$. Therefore, the set $\mathfrak{C} := \cap_{k=1}^{m} \mathfrak{C}_k \subset \mathfrak{D} \subset \mathbb{R}^{\mathrm{n}}$ is *forward invariant*. The proof of convergence property follows similar arguments as in [27, Proposition 3]. □

Definition 7. *Consider the system*

$$\dot{\mathbf{x}} = f(\mathbf{x}) + g(\mathbf{x})\mathbf{u}(\mathbf{x}), \tag{22}$$

with locally Lipschitz continuous functions f and g. A class C^m function $\mathfrak{h}(\mathbf{x}, t) : \mathbb{R}^{\mathrm{n}} \times [t_0, \infty) \to \mathbb{R}$, associated with the task ϕ of the form (3b)*, is called a time-varying convergent higher order control barrier function (TCHCBF) of degree m for this system under task ϕ of the form* (3b)*, if for some constants $0 < \gamma_{1m} < 1$, $\gamma_{2m} > 1$, $\alpha_m > 0$, $\beta_m > 0$, and an open set $\mathfrak{D}$ with $\mathfrak{C} := \cap_{k=1}^{m} \mathfrak{C}_k \subset \mathfrak{D} \subset \mathbb{R}^{\mathrm{n}}$, $\mathfrak{C}_k, k = 1, \cdots, m$, defined as in* (21)*, there exists a control law $\mathbf{u}(\mathbf{x})$ such that*

$$\begin{aligned}
\frac{\partial \psi_{m-1}(\mathbf{x}, t)}{\partial \mathbf{x}} &(f(\mathbf{x}) + g(\mathbf{x})\mathbf{u}(\mathbf{x})) + \frac{\partial \psi_{m-1}(\mathbf{x}, t)}{\partial t}\\
&\ge -\alpha_m sgn(\psi_{m-1}(\mathbf{x}, t))|\psi_{m-1}(\mathbf{x}, t)|^{\gamma_{1m}}\\
&- \beta_m sgn(\psi_{m-1}(\mathbf{x}, t))|\psi_{m-1}(\mathbf{x}, t)|^{\gamma_{2m}},
\end{aligned} \tag{23}$$

where $\psi_{m-1}(\mathbf{x}, t)$ is given by (20).

Remark 4. *Given a TCHCBF $\mathfrak{h}(\mathbf{x}, t)$ and a control signal $\mathbf{u}(\mathbf{x})$ that provides fixed-time convergence to the set $\mathfrak{C}_m$ and renders the system* (22) *forward complete [28, Theorem III.2], which is a required condition for the set convergence and forward invariance property [17]. Then, it follows directly from Proposition 1 that the set $\mathfrak{C}$ is convergent and forward-invariant.*

Next, we use the introduced TCHCBFs to derive similar results to Section III for 2^{nd} order leader-follower networks.

B. Second order leader-follower multi-agent systems

Consider a group of n number of agents with 2^{nd} order dynamics as in (5), under the task ϕ^d. We will formulate a quadratic program that renders the set $\mathfrak{C}^d := \cap_{k=1}^{2} \mathfrak{C}_k^d \subset \mathcal{S}^d \subset \mathbb{R}^{2n}$ corresponding to functions $\mathfrak{h}^d(x^d, t)$ and $\psi_1(x^d, t)$, defined by (21), *robust convergent*, under the following Assumption.

Assumption 4. *Consider the 2^{nd} order leader-follower network* (5) *with the leader $i = n$. There exists a positive constant δ^d satisfying $\| \sum_{i \in \mathcal{N}_n, j \notin \mathcal{N}_n} \frac{\partial \psi_1(x^d, t)}{\partial x_j^d} \mathfrak{f}_j^d(x^d) +$*

$\frac{\partial \psi_1(x^d,t)}{\partial x_i^d}\mathfrak{f}_{i,j}^d(x_i^d, x_j^d)\| \leq \delta^d$, $\forall (x^d,t) \in \mathcal{S}^d \times [\tau_l, \tau_{l+1})$, $l \in \{0, \cdots, p^d-1\}$).

Remark 5. *Note that Assumption 4 is the equivalent of Assumption 3 for the second order system dynamics. Moreover, $\lambda_1(\cdot)$ in* (20) *is a user defined function. Then, in view of Assumption 3, and the defined sets in* (20)*, Assumption 4 can be rendered feasible, too. In addition, there is no need for the leader to know δ^d.*

In the following, a control input u_n will be found such that for all initial conditions $x^d(t_0)$, and under Assumption 4, the trajectories of (5) converge to a the set $\mathfrak{C}^d_{1,rf}(t) \supset \mathfrak{C}^d(t)$ and $\psi_1(x^d,t) \in \mathfrak{C}^d_{2,rf}$, $\mathfrak{C}^d_{2,rf}(t) \supset \mathfrak{C}^d(t)$ in a fixed-time $t \leq T^d + t_0$, $T^d > 0$. The sets $\mathfrak{C}^d_{1,rf}$ and $\mathfrak{C}^d_{2,rf}$ will be characterized in the sequel.

QP formulation: Define $z^d = \begin{bmatrix} u_n, \varepsilon^d \end{bmatrix}^T \in \mathbb{R}^2$, and consider the following optimization problem.

$$\min_{u_n \in \mathbb{R}, \varepsilon^d \in \mathbb{R}_{\geq 0}} \frac{1}{2} {z^d}^T z^d$$

s.t.

$$\begin{aligned} &\textstyle\sum_{i \in \mathcal{N}_n} \{ \frac{\partial \psi_1(x^d,t)}{\partial x_i^d} \mathfrak{f}_{i,i}^d(x_i^d) \\ &+ (\frac{\partial \psi_1(x^d,t)}{\partial x_n^d} + \frac{\partial \psi_1(x^d,t)}{\partial x_i^d}) \mathfrak{f}_{n,i}^d(x_n^d, x_i^d) \} \\ &+ \frac{\partial \psi_1(x^d,t)}{\partial x_n^d} \mathfrak{g}_n^d(x_n^d) u_n + \frac{\partial \psi_1(x^d,t)}{\partial x_n^d} \mathfrak{f}_{n,n}^d(x_n^d) \\ &+ \frac{\partial \psi_1(x^d,t)}{\partial t} \geq -\alpha_2^d \, sgn(\psi_1(x^d,t)) |\psi_1(x^d,t)|^{\gamma_{12}^d} \\ &\qquad - \beta_2^d \, sgn(\psi_1(x^d,t)) |\psi_1(x^d,t)|^{\gamma_{22}^d} - \varepsilon^d, \end{aligned} \tag{24}$$

where $\alpha_2^d > 0, \beta_2^d > 0$, $0 < \gamma_{12}^d < 1$, $\gamma_{22}^d > 1$.

Theorem 2. *Consider a given TCHCBF $\mathfrak{h}^d(x^d,t)$ from Definition 7 with the associated functions $\psi_k(x^d,t)$, $k \in \{1,2\}$, as defined in* (20)*. Any control signal $u_n : \mathbb{R} \to \mathbb{R}$ which solves the quadratic program* (24) *renders the set $\mathfrak{C}^d(t)$ robust convergent for the leader-follower network* (5)*, under Assumption 4.*

Proof. In view of Theorem 1, constraint (24) corresponds to the fixed-time convergence of the closed-loop trajectories of network (5) to the set $\mathfrak{C}^d_{2,rf}(t) := \{x^d \in \mathbb{R}^{2n} | \psi_1(x^d,t) \geq -\epsilon^d_{\max}\}$, where $\epsilon^d_{\max}$ is defined by the same formulation as in (16), within the fixed-time T^d with similar expression as in (15), built by parameters $\alpha_2^d, \beta_2^d > 0$, $\gamma_{12}^d = 1 - \frac{1}{\mu^d}$, $\gamma_{22}^d = 1 + \frac{1}{\mu^d}$, $\mu^d > 1$, $k^d > 1$ and δ^d. These parameters are substitutions of α^s, β^s, γ_1^s, γ_2^s, μ^s, k^s and δ^s, respectively, in (15) and (16). Then, according to (20), we get $\dot{\mathfrak{h}}^d(x^d,t) + \lambda_1(\mathfrak{h}^d(x^d,t)) \geq -\epsilon^d_{\max}$. Let $\lambda_1(\cdot)$ a linear extended class $\mathcal{K}$ function. Inspired by the notion of *input-to-state safety* [29] and using the Comparison Lemma [30, Lemma 3.4], the set $\mathfrak{C}^d_{1,ref}(t) := \{x^d \in \mathbb{R}^{2n} | \mathfrak{h}^d(x^d,t) \geq \lambda_1^{-1}(-\epsilon^d_{\max})\}$, $t \geq T_e^d + t_0$ is forward-invariant and convergence of $\mathfrak{h}^d(x^d,t)$ to this set is achieved asymptotically. Moreover, $\varepsilon^d > 0$ relaxes (24) in the presence of conflicting specifications and its minimization results in a least violating solution to ensure the feasibility of (24). ☐

Corollary 1. *Consider TCHCBF $\mathfrak{h}^d(x^d,t)$ from Definition 7 with the associated functions $\psi_k(x^d,t)$, $k \in \{1,2\}$, as defined in* (20) *for network* (5)*. Then, any control signal u_n satisfying*

$$\begin{aligned} &\textstyle\sum_{i \in \mathcal{V}} \frac{\partial \psi_1(x^d,t)}{\partial x_i^d} \mathfrak{f}_i^d(x^d) + \frac{\partial \psi_1(x^d,t)}{\partial x_n^d} \mathfrak{g}_n^d(x_n^d) u_n \\ &+ \frac{\partial \psi_1(x^d,t)}{\partial t} \geq -\alpha_2^d \, sgn(\psi_1(x^d,t)) |\psi_1(x^d,t)|^{\gamma_{12}^d} \\ &\qquad - \beta_2^d \, sgn(\psi_1(x^d,t)) |\psi_1(x^d,t)|^{\gamma_{22}^d}, \end{aligned}$$

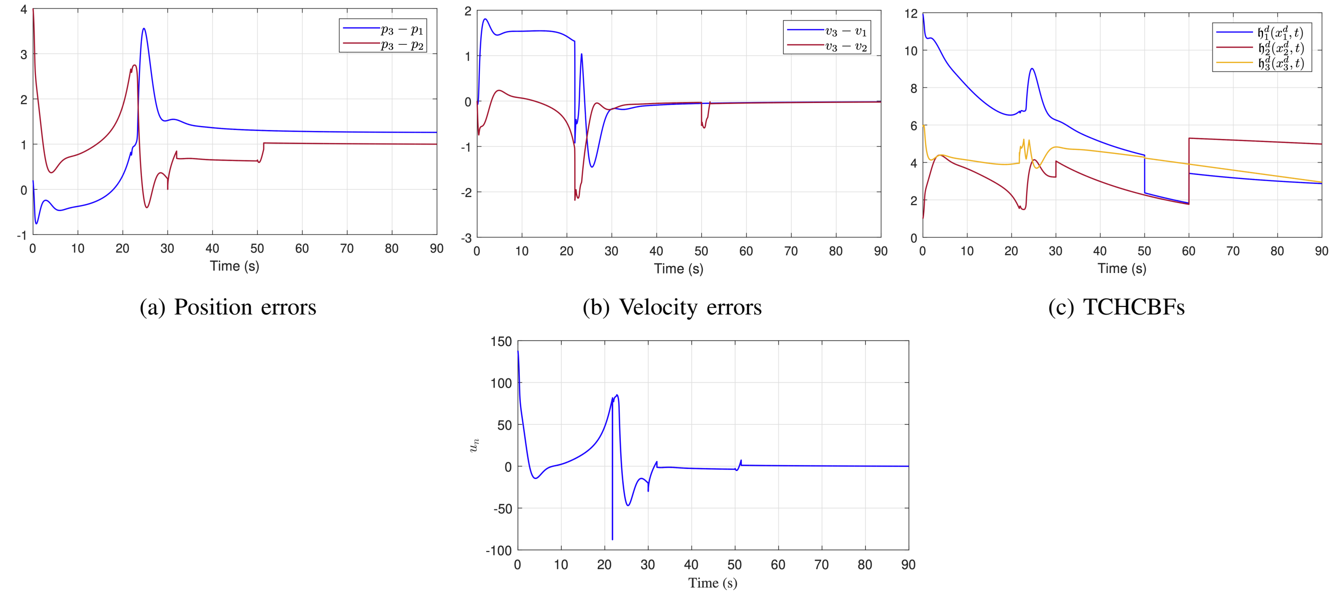
for constants $\alpha_2^d > 0$, $\beta_2^d > 0$, $0 < \gamma_{12}^d < 1$, $\gamma_{22}^d > 1$, renders the set $\mathfrak{C}^d := \cap_{k=1}^2 \mathfrak{C}_k^d \subset \mathcal{S}^d \subset \mathbb{R}^{2n}$ convergent and forward invariant. Moreover, it holds that $x^d \models \phi^d$ within $T^d \leq \frac{1}{\alpha_2^d(1-\gamma_{12}^d)} + \frac{1}{\beta_2^d(\gamma_{22}^d - 1)}$

Proof. Follows by the proof of Lemma 1 with incorporating the arguments in Proposition 1. ☐

Remark 6. *Note that in this work we considered the treatment of singularities that happen due to 1) the higher order constraints in the specifications, e.g., the position constraints in a second order system dynamics, and 2) the under-actuated property of the system caused by the follower agents which are not influenced by direct control actuation. However, there might exist singularities in the solution of the higher order barrier certificate* (24) *in a set of points of measure zero. In particular, whenever $\frac{\partial \psi_1(x^d,t)}{\partial x_n^d} g_n^d(x_n^d) = 0$. Under the assumption that this type of singular points lie inside the safe sets, it can be shown that the required inequalities remain feasible and can be satisfied [27, Proposition 4].*

C. Simulations (I)

Consider a leader-follower multi-agent system consisting of $M := 3$ number of second order dynamics agents. We consider dependent tasks, where the third agent acts as the leader. Consider the formula $\phi^d = \phi_1^d \wedge \phi_2^d \wedge \phi_3^d$ with $\phi_1^d := G_{[10,30]}(|v_3 - v_2| \leq 2) \wedge F_{[10,90]}(|p_1 + 1 - p_3| \leq 1)$, $\phi_2^d := F_{[10,30]}(|v_3 - v_2| \leq 1) \wedge G_{[30,90]}(|v_1 - v_3| \leq 2)$, $\phi_3^d := F_{[10,60]}(|v_3 - v_1| \leq 1) \wedge G_{[60,90]}(|v_2 - v_3| \leq 1) \wedge G_{[50,60]}(|p_2 + 1 - p_3| \leq 1)$. As the position dependent formulas are of relative degree 2, we use TCHCBFs of order $m = 2$. Furthermore, TCHCBFs of order $m = 1$ are considered for velocity dependent specifications. We have considered η^d (the equivalence of η^s for the second order dynamics) as $\eta^d = 1$. We choose the parameters of the QP formulation as $\mu^d = 2$, $\alpha_2^d = \beta_2^d = 1$, and $\lambda_1(r) := r$. We focus on the effect of leader agent information on the group task satisfaction. First, we consider the network (5), where $L := \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ is the laplacian matrix and the input matrix $\mathfrak{g}^d(x^d) := \begin{bmatrix} 0_{1\times 5}, 1 \end{bmatrix}^T$, where leader has knowledge of the functions $\frac{\partial \psi_1(x^d,t)}{\partial x_i^d}$ and dynamics $\mathfrak{f}_i^d(x^d), i \in \{1, \cdots, n\}$ (an equivalent condition to Assumption 2 for 2^{nd} order dynamics), where the convergent and forward invariance property of set $\mathfrak{C}^d(t)$, as well as task satisfaction are concluded as shown in Fig. 1. Next, we consider the agent $i = 1$ as the leader's neighbor and $i = 2$ as a neighbor to $i = 1$ under Assumption 4, where



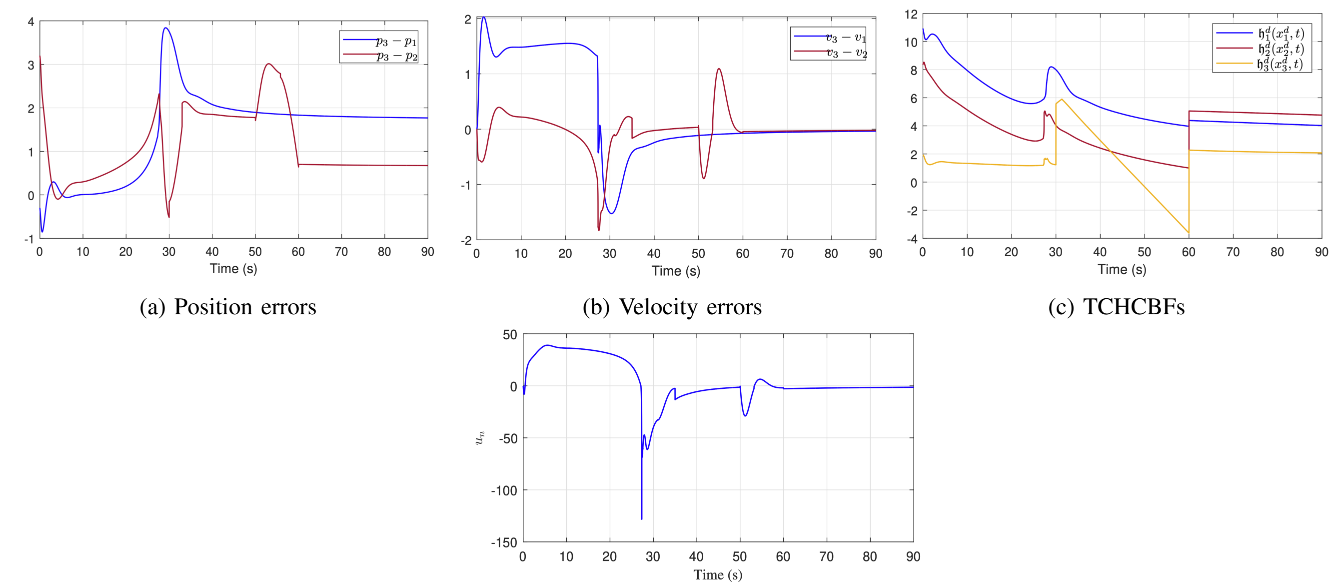
(a) Position errors

(b) Velocity errors

(c) TCHCBFs

(d) Leader control signal

Fig. 1: Leader-follower network (5) under full information of the leader from the network. All considered tasks are satisfied.



(a) Position errors

(b) Velocity errors

(c) TCHCBFs

(d) Leader control signal

Fig. 2: Leader-follower network (5) under Assumption 4. The third task is violated and is satisfied robustly.

$$\mathfrak{f}^d(x^d) := \begin{bmatrix} 0_3 & I_3 \\ -L & -L \end{bmatrix} x^d \text{ with } L := \begin{bmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

By solving (24) where $\delta^d = 2.86$, the fixed-time convergence to the set $\mathfrak{C}^d_{2,ref}(t) \supset \mathfrak{C}^d(t)$ is achieved with $\epsilon^d_{max} = 6.01$ using (16), which gives $\mathfrak{h}^d(x^d, t) \geq \lambda_1^{-1}(-\epsilon^d_{\max}) = -6.01$. Fig. 2 shows a violation in satisfaction of the third task in $t \in [50, 60]$ which certifies this result, although it is less conservative than the estimation. The computation times on an Intel Core i5-8365U with 16 GB of RAM are about 2.1ms.

D. Individual barrier certificates

The barrier certificates proposed in the previous sections provide one constraint, for the networks (4) or (5), which relies on the leader agent control signal as a central coordination unit. This may cause limitations on the multi-agent system's scalability and robustness properties. To address these issues, we next provide individual barrier certificates for each agent according to the tasks that it is involved in, and based on the formation structure of the multi-agent system, in order to guarantee the satisfaction of ϕ^s (resp. ϕ^d). In particular, according to the length of the path between each follower and the leader, higher order barrier certificates for each of the followers are built. Consider first order dynamics in the leader-follower multi-agent system (4), under the task ϕ^s of the form (3b) satisfying Assumption 1, and the corresponding barrier function $\mathfrak{h}^s(x^s,t)$. We denote $\mathfrak{h}_i^s(x^s,t) := \frac{\partial \mathfrak{h}^s(x^s,t)}{\partial x_i^s}\mathfrak{f}_i^s(x^s)$ and a series of functions $\psi_{k,i} : \mathcal{S}^s \times [t_0,\infty) \to \mathbb{R}^n$, $0 \le k \le m_i$ for agent $i \in \{1,\cdots,n-1\}$ as

$$\begin{aligned}
&\psi_{0,i}(x^s,t) := \mathfrak{h}_i^s(x^s,t),\\
&\psi_{k,i}(x^s,t) := \dot{\psi}_{k-1,i}(x^s,t)\\
&\qquad\qquad + \lambda_{k,i}(\psi_{k-1,i}(x^s,t)),\ 1 \le k \le m_i - 1,\\
&\psi_{m_i,i}(x^s,t) := \dot{\psi}_{m_i-1,i}(x^s,t)\\
&\qquad\qquad + \alpha_i^s sgn(\psi_{m_i-1,i}(x^s,t))|\psi_{m_i-1,i}(x^s,t)|^{\gamma_{1,i}^s}\\
&\qquad\qquad + \beta_i^s sgn(\psi_{m_i-1,i}(x^s,t))|\psi_{m_i-1,i}(x^s,t)|^{\gamma_{2,i}^s},
\end{aligned} \tag{25}$$

where $\lambda_{k,i}(\cdot)$, $k = 1,\cdots,m_i-1$, are $(m_i-k)^{th}$-order differentiable extended class $\mathcal{K}$ functions, and m_i is the length of path between the follower $i \in \{1,\cdots,n-1\}$ and the leader, as mentioned in Section II-B. In addition, $0 < \gamma_{1,i}^s < 1$, $\gamma_{2,i}^s > 1$, $\alpha_i^s > 0$, $\beta_i^s > 0$ are user-specified constants, such that $\frac{1}{\alpha^s(1-\gamma_1^s)} + \frac{1}{\beta^s(\gamma_2^s-1)} \le \min_{l\in\{0,\cdots,p^s-1\}}\{\tau_{l+1} - \tau_l\}$. In addition, we define the corresponding sets $\mathfrak{C}_{k,i}$, $k \in \{1,\cdots,m_i\}$, $i \in \{1,\cdots,n-1\}$, assumed to be compact, as follows:

$$\mathfrak{C}_{k,i}(t) := \{x^s \in \mathcal{S}^s | \psi_{k-1,i}(x^s,t) \ge 0\}. \tag{26}$$

Then, the individual barrier certificate for each agent $i \in \{1,\cdots,n-1\}$ can be given as

$$\psi_{m_i,i}(x^s,t) \ge 0. \tag{27}$$

Utilizing the analysis of the higher order barrier functions demonstrated in Section IV, the corresponding constraints for each agent could be satisfied by the leader external input signal u_n that appears in (27) through $\dot{\psi}_{m_i-1,i}$ in (25). This strategy has been formulated in the following Lemma.

Lemma 2. *Consider the first order dynamics leader-follower multi-agent system* (4) *containing one leader $i = n$, under the task ϕ^s of the form* (3b) *satisfying Assumption 1. Let $\mathfrak{C}_i^{ind} := \cap_{k=1}^{m_i}\mathfrak{C}_{k,i}$ and $\mathfrak{C}^{ind} := \cap_{i=1}^{n-1}\mathfrak{C}_i^{ind}$ with $\mathfrak{C}_{k,i}$, $i \in \{1,\cdots,n-1\}$, defined as in* (26)*, and the set $\mathfrak{C}^s$ as in* (11)*. A control input u_n that satisfies* (27) *for all $i \in \{1,\cdots,n-1\}$, and*

$$\begin{aligned}
&\frac{\partial \mathfrak{h}^s(x^s,t)}{\partial x_n^s}\mathfrak{f}_{n,n}^s(x_n^s) + \frac{\partial \mathfrak{h}^s(x^s,t)}{\partial x_n^s}\mathfrak{g}_n^s(x_n^s)u_n\\
&+ \frac{\partial \mathfrak{h}^s(x^s,t)}{\partial t} \ge -\alpha^s\ sgn(\mathfrak{h}^s(x^s,t))|\mathfrak{h}^s(x^s,t)|^{\gamma_1^s}\\
&\qquad\qquad - \beta^s\ sgn(\mathfrak{h}^s(x^s,t))|\mathfrak{h}^s(x^s,t)|^{\gamma_2^s},
\end{aligned} \tag{28}$$

renders the set $\mathfrak{C}^s \cap \mathfrak{C}^{ind}$ forward invariant and convergent.

Proof. Consider the first order dynamics agents and decouple the leader-follower network barrier constraint (12) to be defined for the followers and the leader agent separately, as follows:

$$\frac{\partial \mathfrak{h}^s(x^s,t)}{\partial x_i^s}\mathfrak{f}_i^s(x^s) \ge 0,\quad i \in \{1,\cdots,n-1\}, \tag{29a}$$

$$\begin{aligned}
&\frac{\partial \mathfrak{h}^s(x^s,t)}{\partial x_n^s}\mathfrak{f}_{n,n}^s(x_n^s) + \frac{\partial \mathfrak{h}^s(x^s,t)}{\partial x_n^s}\mathfrak{g}_n^s(x_n^s)u_n\\
&+ \frac{\partial \mathfrak{h}^s(x^s,t)}{\partial t} \ge -\alpha^s\ sgn(\mathfrak{h}^s(x^s,t))|\mathfrak{h}^s(x^s,t)|^{\gamma_1^s}\\
&\qquad\qquad - \beta^s\ sgn(\mathfrak{h}^s(x^s,t))|\mathfrak{h}^s(x^s,t)|^{\gamma_2^s}.
\end{aligned} \tag{29b}$$

It is obvious that if (29a) and (29b) are satisfied, then (12) is satisfied, too. Constraint (29b) provides a condition for satisfying the corresponding tasks to the leader agent, that could be satisfied by choosing appropriate control signal u_n. On the other hand, as can be seen from (29a), which determines the barrier certificates of the followers, there is no control input signal involved to satisfy these inequalities. However, the follower dynamics are dependent to the leader state trajectories through the formation structure of the leader-follower multi-agent systems according to the graph topology. Thus, we can use the barrier certificates of the followers, described in (29a), and construct higher order barrier functions of $\mathfrak{h}_i^s(x^s,t)$ according to (25) and their corresponding sets $\mathfrak{C}_{k,i}$, $k \in \{1,\cdots,m_i\}$ which are defined by (26). This procedure leads to the desired result and is as follows. Consider (29a) and let

$$\mathfrak{h}_i^s(x^s,t) = \frac{\partial \mathfrak{h}^s(x^s,t)}{\partial x_i^s}\mathfrak{f}_i^s(x^s),\quad i \in \{1,\cdots,n-1\}, \tag{30}$$

as the new barrier function corresponding to the follower i. It is apparent from (29a) that if $\mathfrak{h}_i^s \ge 0$, $i \in \{1,\cdots,n-1\}$, the follower barrier certificates are satisfied. Provided that agent i is a neighbor to the leader (i.e., $m_i = 1$), (27) could be satisfied provided that for some constants $0 < \gamma_{1,i}^s < 1$, $\gamma_{2,i}^s > 1$, $\alpha_i^s > 0$, $\beta_i^s > 0$, the following inequality is established:

$$\begin{aligned}
\dot{\mathfrak{h}}_i^s(x^s,t) \ge &-\alpha_i^s\ sgn(\mathfrak{h}_i^s(x^s,t))|\mathfrak{h}_i^s(x^s,t)|^{\gamma_{1,i}^s}\\
&- \beta_i^s\ sgn(\mathfrak{h}_i^s(x^s,t))|\mathfrak{h}_i^s(x^s,t)|^{\gamma_{2,i}^s}.
\end{aligned} \tag{31}$$

The point is that the control input u_n appears in (31), provided that the agent i is a neighbor of the leader. The reason is that if the agent i is a neighbor to the leader n, then $f_i(x^s)$ is a function of the leader state x_n. Therefore, the derivative of (30) (i.e., (31)) will be a function of $\dot{x}_n$, and hence, a function of u_n. If agent i would not be a neighbor of the leader, by using higher order barrier functions constructed from $\mathfrak{h}_i^s(x^s,t)$ using (25), and based on distance of agent i from the leader, the control input u_n will be presented in the constraints. In other words, the higher order barrier function $\psi_{m_i,i}(x^s,t)$ that is the first one which is dependent on the control signal u_n, is of order m_i+1 with respect to $\mathfrak{h}^s(x^s,t)$ (i.e., $L_{\mathfrak{g}_n(x_n^s)}L_{\mathfrak{f}_i(x^s)}^{m_i}\mathfrak{h}^s(x^s,t) \neq 0$), where m_i is the length of path from the follower $i \in \{1,\cdots,n-1\}$ to the leader. Thus, the functions $\mathfrak{h}_i^s(x^s,t)$ are TCHCBFs, and hence, the forward invariance and convergence property of the sets $\mathfrak{C}_i^{ind}$, $i \in \{1,\cdots,n-1\}$, are concluded according to Proposition 1. Moreover, the barrier certificate for leader $i = n$ is first order with respect to $\mathfrak{h}^s(x^s,t)$ as $m_n = 0$, and will be written as in (28). Then, the forward invariance and convergence of the set $\mathfrak{C}^s \cap \mathfrak{C}^{ind}$ is concluded, too. □

Remark 7. *Whenever $\frac{\partial \mathfrak{h}^s(x^s,t)}{\partial x_n^s} = 0$, the inequality* (28) *might not be satisfied. In this case, we construct the higher order barrier functions as in* (20) *to guarantee the forward invariance* *and convergence of the set* $\mathfrak{C} := \cap_{k=1}^{m}\mathfrak{C}_k \subset \mathfrak{D} \subset \mathbb{R}^n$*, where the sets* $\mathfrak{C}_k$ *are defined as in* (21) *and* m *is the order of the barrier function* $\mathfrak{h}^s(x^s, t)$*.*

We clarify the latter case using the following example: **Example 1:** Consider a network consisting of $n = 3$ agents with $i = 3$ as the leader and formation structure $\mathfrak{f}^s(x^s) := -Lx^s$ with $L := \begin{bmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and the input matrix $\mathfrak{g}^s(x^s) := \begin{bmatrix} 0, 0, 1 \end{bmatrix}^T$. Assume that $\mathfrak{h}^s(x^s, t) = p_c - p_2$, where p_c is constant. Then, (12) is written as

$$\begin{aligned} &-p_1 + p_2 \geq -\alpha_2^d \; sgn(-p_2 + p_c)|p_2 - p_c|^{\gamma_{1,2}^s} \\ &-\beta_2^s \; sgn(-p_2 + p_c)| - p_2 + p_c|^{\gamma_{2,2}^s}. \end{aligned} \tag{32}$$

As the control input doesn't appear in (32), this task is of relative degree greater than one and $\mathfrak{C}_1(t) := \{x^s \in \mathbb{R}^3 | \mathfrak{h}^s(x^s, t) \geq 0\}$ according to (21). Building the second order barrier function of $\mathfrak{h}^s(x^s, t)$ using (20), we reach to $\psi_1(x^s, t) = \dot{\mathfrak{h}}^s(x^s, t) + \lambda_1(\mathfrak{h}^s(x^s, t)) = -p_1 + p_2 + \lambda_1(-p_2 + p_c)$ with $\mathfrak{C}_2(t) := \{x^s \in \mathbb{R}^3 | \psi_1(x^s, t) \geq 0\}$, where $\lambda_1(\cdot)$ is an extended class $\mathcal{K}$ function, that here is defined as $\lambda_1(r) := r$. As the leader state p_3 doesn't appear in $\psi_1(x_e^s, t)$, we build the third order barrier function as $\psi_2(x^s, t) = \dot{\psi}_1^s(x^s, t) + \lambda_2(\psi_1^s(x^s, t)) = 2p_1 - p_2 - p_3 + \lambda_2(\psi_1^s(x^s, t))$ with $\mathfrak{C}_3(t) := \{x^s \in \mathbb{R}^3 | \psi_2(x^s, t) \geq 0\}$, where $\lambda_2(\cdot)$ is an extended class $\mathcal{K}$ function, defined as $\lambda_2(r) := r$. According to the graph topology, we have $m = 3$. Then, the corresponding barrier certificate $\psi_3 \geq 0$ by Definition 6, can be written as

$$\begin{aligned} &-3p_1 + 2p_2 + p_3 - u_3 \geq \\ &-\alpha_2^s \; sgn(\psi_2(x^s, t))|\psi_2(x^s, t)|^{\gamma_{13}^s} \\ &-\beta_2^s \; sgn(\psi_2(x^s, t))|\psi_2(x^s, t)|^{\gamma_{23}^s}. \end{aligned}$$

Then, according to Proposition 1, the set $\mathfrak{C}^s := \cap_{k=1}^{3}\mathfrak{C}_k$ is convergent and forward invariant.

Note that higher order barrier certificates allow for taking into account more general task specifications and graph topologies as there is no necessity that all formulas be dependent on the leader.

E. Decentralized barrier certificates

In the previous sections, the multi-agent system contained only one leader. Hence, the satisfaction of specifications had to be achieved in a centralized way. It is apparent that a larger number of leaders increases the ability of the multi-agent system to consider more complex and general specifications. Hence, we next formulate the barrier certificates in a decentralized scheme for multi-leader scenarios.

Lemma 3. *Consider the leader-follower network* (4) *with the first* n_f *agents as followers and the last* n_l *ones as leaders.*

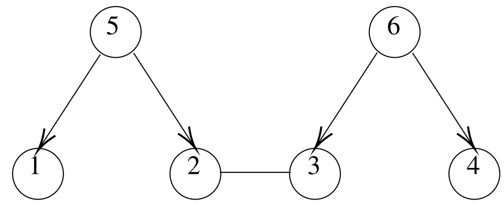


Fig. 3: Leader-follower network with the nodes 5 and 6 as the leaders.

The decentralized barrier certificates can be written by n_l *inequalities with respect to each leader, as below:*

$$\begin{aligned} &\textstyle\sum_{i \in \mathcal{N}_{n_f+1}} \{ \frac{\partial \mathfrak{h}^s(x^s,t)}{\partial x_i^s} \mathfrak{f}_{i,i}^s(x_i^s) \\ &+ (\frac{\partial \mathfrak{h}^s(x^s,t)}{\partial x_{n_f+1}^s} + \frac{\partial \mathfrak{h}^s(x^s,t)}{\partial x_i^s}) \mathfrak{f}_{n_f+1,i}^s(x_{n_f+1}^s, x_i^s) \} \\ &+ \frac{\partial \mathfrak{h}^s(x^s,t)}{\partial x_{n_f+1}^s} \mathfrak{f}_{n_f+1,n_f+1}^s(x_{n_f+1}^s) + \frac{\partial \mathfrak{h}^s(x^s,t)}{\partial x_{n_f+1}^s} \mathfrak{g}_{n_f+1}^s u_{n_f+1} \\ &\geq -\frac{\tau_1}{\sum_{j \in \{1,\cdots,n_l\}} \tau_j} [\frac{\partial \mathfrak{h}^s(x^s,t)}{\partial t} + \alpha^s \; sgn(\mathfrak{h}^s(x^s, t))|\mathfrak{h}^s(x^s, t)|^{\gamma_1^s} \\ &\qquad + \beta^s \; sgn(\mathfrak{h}^s(x^s, t))|\mathfrak{h}^s(x^s, t)|^{\gamma_2^s}], \\ &\vdots \\ &\textstyle\sum_{i \in \mathcal{N}_{n_f+n_l}} \{ \frac{\partial \mathfrak{h}^s(x^s,t)}{\partial x_i^s} \mathfrak{f}_{i,i}^s(x_i^s) \\ &+ (\frac{\partial \mathfrak{h}^s(x^s,t)}{\partial x_{n_f+n_l}^s} + \frac{\partial \mathfrak{h}^s(x^s,t)}{\partial x_i^s}) \mathfrak{f}_{n_f+n_l,i}^s(x_{n_f+n_l}^s, x_i^s) \} \\ &+ \frac{\partial \mathfrak{h}^s(x^s,t)}{\partial x_{n_f+n_l}^s} \mathfrak{f}_{n_f+n_l,n_f+n_l}^s(x_{n_f+n_l}^s) + \frac{\partial \mathfrak{h}^s(x^s,t)}{\partial x_{n_f+n_l}^s} \mathfrak{g}_{n_f+n_l}^s u_{n_f+n_l} \\ &\geq -\frac{\tau_{n_l}}{\sum_{j \in \{1,\cdots,n_l\}} \tau_j} [\frac{\partial \mathfrak{h}^s(x^s,t)}{\partial t} + \alpha^s \; sgn(\mathfrak{h}^s(x^s, t))|\mathfrak{h}^s(x^s, t)|^{\gamma_1^s} \\ &\qquad + \beta^s \; sgn(\mathfrak{h}^s(x^s, t))|\mathfrak{h}^s(x^s, t)|^{\gamma_2^s}], \end{aligned} \tag{33}$$

for positive constants τ_j*,* $j \in \{1, \cdots, n_l\}$*.*

Proof. Consider the general inequality (12) that should be satisfied for the network. According to its equivalent inequality provided in Theorem 1, we can split the expressions in the left hand side of the inequality (14) with respect to the related terms to each leader and its corresponding followers. The right hand side expression could also be divided using the positive weights $\tau_1, \cdots, \tau_{n_l}$. Then, the inequalities in (33) are acquired. $\square$

Remark 8. *Individual barrier certificates for the followers of each leader are given in a similar way to what was proposed in Lemma 2. In this case,* m_i *is considered as the minimum of the path lengths between follower* i *and the leaders of the multi-agent system. Then, the corresponding individual barrier certificate is constructed with respect to the closest leader according to the graph topology.*

We clarify the barrier certificate formulation of multi-leader networks using the following example:

Example 2: Consider Figure 3 consisting of 1^{st} order dynamics agents, with agents $i = \{5, 6\}$ as the leaders. According to the graph topology, we have $\mathfrak{f}^s(x^s) := -Lx^s$, with $L := \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 2 & -1 & 0 & -1 & 0 \\ 0 & -1 & 2 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ and the input matrix $\mathfrak{g}^s := \left[\begin{array}{c} 0_{2\times 4}, I_2 \end{array}\right]^T$. Assume that $\mathfrak{h}^s(x^s,t) = p_2 - p_3$. In order to write the higher order barrier certificate according to (20), we have $\psi_1(x^s,t) = \dot{\mathfrak{h}}^s(x^s,t) + \lambda_1(\mathfrak{h}^s(x^s,t)) = 3(p_2-p_3) - p_5 + p_6 + \lambda_1(p_2-p_3)$. By using $\lambda_1(r) := r$, we get $\psi_1(x^s,t) = -2(p_2-p_3) + p_5 - p_6$. Therefore, according to $f^s(x^s)$, the second order barrier certificate $\psi_2(x^s,t) \geq 0$ can be written as:

$$\begin{aligned} &6(p_2-p_3) - 2(p_5-p_6) + u_5 - u_6 \geq \\ &-\alpha(-2(p_2-p_3) + p_5 - p_6), \end{aligned} \tag{34}$$

for a positive constant α, and extended class $\mathcal{K}$ function $\lambda_2(r) := r$. Note that we used the function $\lambda_2(r)$ instead of $\alpha^s sgn(\psi_1(r))|\psi_1(r)|^{\gamma_1^s} + \beta^s sgn(\psi_1(r))|\psi_1(r)|^{\gamma_2^s}$, which guarantees a fixed-time convergence to the set $\psi_2(x^s,t)$. The reason of this choice is that function $\lambda_2(r)$ allows us to decentralize (34) with respect to leader's control inputs u_5, u_6 and the graph structure. In this way, the inequalities

$$\begin{aligned} &6p_2 - 2p_5 + u_5 \geq -\alpha(-2p_2 + p_5), \\ &-6p_3 + 2p_6 - u_6 \geq -\alpha(2p_3 - p_6), \end{aligned}$$

imply (34). Therefore, we relax the fixed-time convergence property for the set $\psi_2(x^s,t)$ in order to derive the decentralized barrier certificates.

Note that the substitutions of extended class $\mathcal{K}$ functions are used for the barrier certificates in which more than one leader's states are involved. In this example for instance, the barrier certificates corresponding to the specifications dependent on agents $i=1$ or $i=4$, that are directly connected to one leader, can be written similar to (33), which give a fixed-time convergence property to the specified sets.

Remark 9. *The procedures introduced for the individual and decentralized barrier certificates can be extended to higher order dynamics. In that case, higher order constraints are formulated using higher order barrier functions, as specified in Section IV-A.*

In the following simulation example, higher order individual barrier certificates are considered to guarantee the task satisfaction.

F. Simulations (II)

We consider a leader-follower multi-agent system consisting of $M := 6$ nonlinear second order dynamics' agents with the graph topology represented in Fig. 3 with $\mathcal{V}_l := \{5,6\}$. According to (5). The overall dynamics of the network is considered as $\dot{x}^d := \mathfrak{f}^d(x^d) + \mathfrak{f}^d_{NL}(x^d) + \mathfrak{g}^d(x^d)u$, $u \in \mathbb{R}^2$, where $\mathfrak{f}^d(x^d) := \begin{bmatrix} 0_{6\times 6} & I_6 \\ -L & -L \end{bmatrix} x^d$, with L introduced in Example 2, $\mathfrak{f}^d_{NL}(x^d) := [0_{1\times 10}, -v_5^2, -v_6^2]^T$ and the input matrix $\mathfrak{g}^d(x^d) := \left[\begin{array}{c} 0_{2\times 10}, I_2 \end{array}\right]^T$. Consider the formula $\phi^d = \phi_1^d \wedge \phi_2^d \wedge \phi_3^d$ with $\phi_1^d := G_{[60,90]}(p_2 - p_3 - 1.8v_3 \geq 2)$, $\phi_2^d := G_{[0,60]}(v_2 - v_3 \geq 10)$, $\phi_3^d := G_{[30,90]}(p_1 - p_3 \geq 10))$. The first formula, corresponds to the safety constraint for two CAVs (connected and automated vehicles), where $i=2$ physically immediately precedes $i=3$. This formula is a function of velocity (which is of relative degree 1) of the follower $i=3$ as the neighbor of leader $i=6$. Then, according to Lemma 2, the corresponding barrier function to this task would be of order $m=2$. The second formula, is dependent on the velocities of agents that are neighbors to different leaders. Then, the relative degree of this constraint is again $m=2$ and the decentralization of the barrier certificate is established according to (33). The third formula, has degree $m=4$ since it considers the position states (which are of relative degree 2) of two followers as neighbors of leaders (which gives $m_i = 2$, $i=1,3$). This could be deduced using the dynamic matrix $\mathfrak{f}^d(x^d)$ and the input matrix $\mathfrak{g}^d(x^d)$. We choose the parameters of the QP formulation for ϕ_1^d as $\mu^d = 2$, $\alpha^d = \beta^d = 1$, and $\lambda_1(r) := r$. In addition, for ϕ_2^d and ϕ_3^d, we consider extended class $\mathcal{K}$ functions $\lambda(r) := r$ for the decentralization purpose as explained in Example 2. The results are presented in Fig. 4 which show the satisfaction of specified tasks.

V. Conclusion

Based on a class of time-varying convergent higher order control barrier functions, we have presented feedback control strategies to find solutions for the leader-follower multi-agent systems performance under STL tasks, based on the knowledge of leader from the network. Appropriate individual and decentralized barrier certificates are also introduced to maintain more general formulas in a simpler framework. Future work will extend these results to high level specifications including planning fulfilment for leader-follower topologies; e.g., leader selection methods to find the optimal solution with respect to task specifications.

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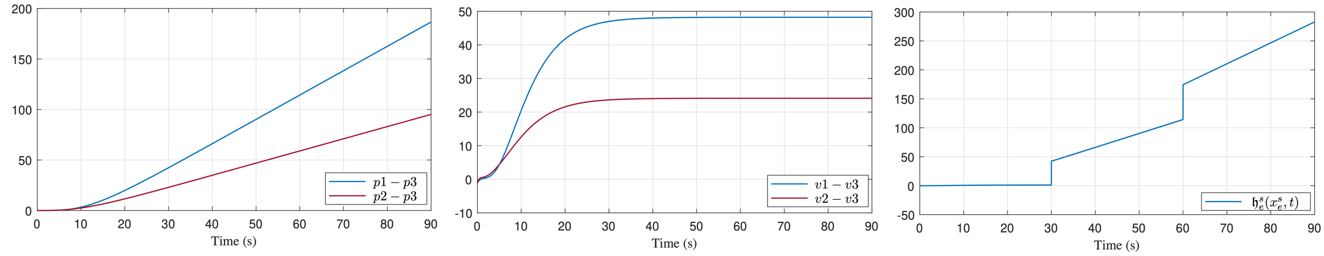

(a) Position errors (b) Velocity errors (c) Barrier function

Fig. 4: Leader-follower network of Fig. 3 under decentralized barrier certificates.

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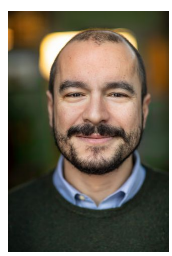
Maryam Sharifi is a Postdoctoral Researcher at the Division of Decision and Control Systems, KTH Royal Institute of Technology. She received her B.Sc. and M.Sc. degress in Electrical Engineering , form Amirkabir University of Technology (Iran) in 2013 and 2015, respectively, and Ph.D. in Control Engineering from Tehran University (Iran) in 2020. Her research interests include nonlinear control theory, robotics, formal synthesis, multi-agent systems, hybrid systems and abstraction methods for infinite networks.



Dimos V. Dimarogonas (Senior Member, IEEE) was born in Athens, Greece, in 1978. He received the Diploma degree in electrical and com- puter engineering and the Ph.D. degree in me- chanical engineering from the National Technical University of Athens, Athens, Greece, in 2001 and 2007, respectively. Between 2007 and 2010, he held Postdoctoral positions with the Department of Automatic Control, KTH Royal Institute of Technology and the Laboratory for Information and Decision Systems, Massachusetts Institute of Technology, Cambridge, MA, USA. He is currently a Professor with the Division of Decision and Control Systems, School of Electrical Engineering and Computer Science, KTH Royal Institute of Technology. His current research interests include multiagent systems, hybrid systems and control, robot navigation and manipulation, humanrobot interaction, and networked control. Prof. Dimarogonas serves in the Editorial Board of the Automatica and IEEE Transactions on Control of Network Systems. He was a recipient of the ERC starting Grant in 2014, the ERC Consolidator Grant in 2019, and the Knut och Alice Wallenberg Academy Fellowship in 2015.