Reciprocal Safety Velocity Cones for Decentralized Collision Avoidance in Multi-Agent Systems

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Abstract: In this paper, we solve the inter-agent collision avoidance problem in an arbitrary $n-$dimensional Euclidean space using *reciprocal safety velocity cones* (RSVCs). We propose a decentralized feedback control strategy that guarantees simultaneously asymptotic stabilization to a reference and collision avoidance. Our algorithm is purely decentralized in the sense that each agent uses only local information about its neighbouring agents. Moreover, the proposed solution can be implemented using only inter-agent bearing measurements. Therefore, the algorithm is a sensor-based control strategy which is practically implementable using a wide range of sensors such as vision systems and range scanners. Simulation results in a two dimensional environment cluttered with agents shows that the number of possible deadlocks is marginal and decrease with the decrease in the clutteredness of the workspace.

Keywords: Collision avoidance, multi-agent systems, sensor-based methods, motion planning, feedback control.

1. INTRODUCTION

In the last few decades, multi-agent systems have received great attention in the research community due to their plethora of useful applications. In fact, multi-agent systems are able to efficiently carry out tasks that are either impossible or inefficient to carry out using single-agent systems. In this paper, we focus on one of the central problems in the control of multi-agent systems: inter-agent collision avoidance. The approach proposed in this paper aims at addressing both the stabilization (convergence to targets) and the collision-free motion planning problems. Different but closely related problems have been addressed as well in the literature such as collision-free formation control (Tanner and Kumar, 2005; Tanner and Boddu, 2012) and the collision-free velocity synchronization (Chopra, 2008).

One of the important approaches in the area of feedback-based collision avoidance are based on Koditschek–Rimon navigation functions (Koditschek and Rimon, 1990) and control barrier functions (Ames et al., 2017) which were extended to multi-robot systems in (Dimarogonas et al., 2006a,b; Roussos and Kyriakopoulos, 2013; Wang et al., 2017; Verginis and Dimarogonas, 2019, 2021); to name a few. Some approaches such as (Dimarogonas et al., 2006b) consider a “partially” decentralized method where each agent has global knowledge of the position of the others but has knowledge only of its own desired destination. Purely decentralized motion planners such as (Van Berg et al., 2008; Snape et al., 2011) need local knowledge of both the position and the velocity of the neighbouring agents. A totally distributed motion control of multi-agent systems using decentralized navigation functions has been proposed in (Dimarogonas et al., 2006a) but global convergence cannot be guaranteed in all scenarios. Cooperative control laws based on the idea of avoidance control (Leitmann and Skowronski, 1977) has been applied to multi-agent systems in (Stipanovic et al., 2007). The proposed approach may be appended to already designed optimal control laws of independent agents. A differential game approach has been recently proposed in (Mylvaganam et al., 2017) but under a centralized framework, in which the positions of each agent are available to the remaining members of the group at all times. Safety barrier certificates are designed both in a centralized and a decentralized manner in (Wang et al., 2017) to ensure collision-free behaviors in multirobot systems by modifying the nominal controllers to formally satisfy safety constraints. The resulting computation of the safety controllers is done by solving in real-time a quadratic program (QP). Finally, a different type of decentralized barrier functions has been used in (Verginis and Dimarogonas, 2019, 2021) for collision-avoidance in multi-agent systems with uncertain dynamics.

In this paper, we propose a purely decentralized collision-free feedback stabilization approach for multi-agent systems. The proposed strategy relies on projection onto reciprocal safety velocity cones (RSVCs), which are an extension to multi-agent systems of the safety velocity cones (SVCs) (Berkane, 2020)–introduced for safe navigation in unknown environments filled with static obstacles. The proposed algorithm has two main advantages compared to the existing approaches. First, the decentralized control laws can be computed using only inter-agent bearing measurements of those neighbouring agents instead of their full position information. This is useful in sensor-based applications where sensors are limited (*e.g.,* monocular cameras) and do not provide exact agents' positions. Note however that we still have to detect neighboring agents and that proximity sensors can be used for this purpose. Second, the algorithm is very simple both in terms of design (closed-form solution) and efficient real-time computation. Furthermore, one of the most attractive features of the proposed multi-agent navigation algorithm is the fact that collision avoidance is activated only when other agents enter the local sensing region of an individual agent, and, therefore, it does not interfere with the agents' individual nominal controllers outside of these regions.

The key drawback of the proposed algorithm, as in other decentralized schemes (Dimarogonas et al., 2006a; Wang et al., 2017), is that global convergence cannot be guaranteed from all initial conditions. This is the price to pay for the lack of a central coordination signal which leads to possible deadlocks among multiple agents with conflicting tasks. However, Monte Carlo simulations has shown that the number of deadlocks decreases when the environment has fewer agents. In other words, the number of deadlocks is commensurate with the clutteredeness of the environment.

2. NOTATION AND PRELIMINARIES

Let $\mathbb{N}$ denote the set of natural numbers. $\mathbb{R}$ and $\mathbb{R}_+$ denote, respectively, the set of reals and non-negative reals. $\mathbb{R}^n$ is the n-dimensional Euclidean space. For vectors in $\mathbb{R}^n$, the relation operators $<, \leq, >, \geq$ stand for the element-wise relations. For a given matrix $A \in \mathbb{R}^{m\times n}$ and a subset $\mathcal{I} \subseteq \mathbb{N}$, we denote by $A[\mathcal{I}]$ the $|\mathcal{I}| \times n$ matrix obtained by considering only the kth rows of A, with $k \in \mathcal{I}$. The interior (resp. boundary) of a subset $\mathcal{A} \subset \mathbb{R}^n$ is denoted by $\mathbf{int}(\mathcal{A})$ (resp. $\partial\mathcal{A}$). The complement of $\mathcal{A}$ in $\mathbb{R}^n$ is denoted by $\complement\mathcal{A}$ and its closure by $\overline{\mathcal{A}}$. The Euclidean norm of $x \in \mathbb{R}^n$ is defined as $\|x\| := \sqrt{x^\top x}$. We denote by $\mathcal{B}(x, r) := \{y \in \mathbb{R}^n : \|x - y\| < r\}$ the open ball of radius r that is centered at x. Given a non-empty subset $\mathcal{A} \subset \mathbb{R}^n$, the distance function from a point $x \in \mathbb{R}^n$ to $\mathcal{A}$ is defined as $\mathbf{d}_\mathcal{A}(x) := \inf_{y\in\mathcal{A}} \|y - x\|$ and the projection of x onto $\overline{\mathcal{A}}$ is given by the set-valued map $\mathbf{P}_\mathcal{A}(x) := \{y \in \overline{\mathcal{A}} : \|y - x\| = \mathbf{d}_\mathcal{A}(x)\}$.

Definition 1. (Cone). A subset $\mathcal{C}$ of $\mathbb{R}^n$ is a cone if for every $x \in \mathcal{C}$ and $\lambda \in \mathbb{R}_+$, we have $\lambda x \in \mathcal{C}$.

Definition 2. (Convex Cone). A subset $\mathcal{C}$ of $\mathbb{R}^n$ is a convex cone if for every $x_1, x_2 \in \mathcal{C}$ and $\lambda_1, \lambda_2 \in \mathbb{R}_+$, we have $\lambda_1 x_1 + \lambda_2 x_2 \in \mathcal{C}$.

Definition 3. (Polar Cone). The polar cone of a cone $\mathcal{C}$ is the set $\mathcal{C}^o = \{x' \in \mathbb{R}^n : x^\top x' \leq 0,\ \forall x \in \mathcal{C}\}$.

Definition 4. (Finitely Generated Cone). A cone $\mathcal{C} \subset \mathbb{R}^n$ is finitely generated if there is a finite set $\mathcal{S} \subset \mathbb{R}^n$ such that $\mathcal{C} = \mathbf{cone}(\mathcal{S})$ where

$$\mathbf{cone}(\mathcal{S}) := \left\{ y \in \mathbb{R}_n : y = \sum \lambda_i x_i : x_i \in \mathcal{S}, \lambda_i \in \mathbb{R}_+ \right\}.$$

Definition 5. (Hyperplane). An $(n-1)-$dimensional hyperplane in $\mathbb{R}^n$ is defined by

$$\mathcal{P}(\nu, c) := \{x \in \mathbb{R}^n : \nu^\top (x - c) = 0\}, \tag{1}$$

for some $\nu \in \mathbb{R}^n \setminus \{0\}$ and $c \in \mathbb{R}^n$.

Definition 6. (Halfspace). A (closed) halfspace is defined as

$$\mathcal{H}(\nu, c) := \{x \in \mathbb{R}^n : \nu^\top (x - c) \leq 0\}, \tag{2}$$

for some $\nu \in \mathbb{R}^n \setminus \{0\}$ and $c \in \mathbb{R}^n$.

Every hyperplane $\mathcal{P}(n, c)$ divides $\mathbb{R}^n$ into two halfspaces. In particular, $\mathcal{H}_0(n) := \mathcal{H}(n, 0)$ denotes the halfspace that passes through the origin. Also, for convenience, we allow n to be zero in (2) where in this case $\mathcal{H}(0, c) = \mathcal{H}_0(0) \equiv \mathbb{R}^n$.

Definition 7. (Polyhedron). A subset of $\mathbb{R}^n$ is a polyhedron if it can be written as the intersection of finitely many halfspaces, i.e. every polyhedron can be written, for some $m \in \mathbb{N}$, as $\mathcal{P}(A, b) = \{x \in \mathbb{R}^n : Ax \leq b\}$ where $A \in \mathbb{R}^{m\times n}$ and $b \in \mathbb{R}^m$.

In particular, a polyhedral cone is a polyhedron formed by the intersection of halfspaces that pass through the origin.

Definition 8. (Polyhedral Cone). For any matrix A, the set $\mathcal{P}(A, 0)$ defines a polyhedral cone and is denoted as $\mathcal{C}(A) = \{x \in \mathbb{R}^n : Ax \leq 0\}$.

A polyhedral cone is in other words the solution set of a system of homogeneous linear inequalities. One of the most important results in the theory of polyhedra is that every polyhedral cone is a finitely generated cone and vice versa. We end the preliminaries with the following result.

Proposition 1. ((Ujvári, 2016)). The polar cone of a polyhedral cone $\mathcal{C}(A)$ is given by the finitely generate cone $\mathcal{C}^o(A) = \{A^\top \lambda : \lambda \in \mathbb{R}^n_+\}$.

3. PROBLEM FORMULATION

Consider a set of $N \geq 1$ spherical agents with radius $r_i > 0$ and center of mass $x_i \in \mathbb{R}^n$. Each agent i occupies a ball region defined by

$$\mathcal{O}_i := \{x \in \mathbb{R}^n : \|x - x_i\| \leq r_i\}. \tag{3}$$

We consider that each agent's state (position) $x_i \in \mathbb{R}^n$ is subject to the following kinematics equation

$$\dot{x}_i = u_i, \quad i \in \mathbb{I} := \{1, \cdots, N\}. \tag{4}$$

Here $u_i \in \mathbb{R}^n$ denotes the velocity commands (control input) applied to agent i. Our objective is to asymptotically stabilize the position of each agent i to a desired position $x_i^d \in \mathbb{R}^n$ while avoiding their collision, *i.e.* the constraint

$$\|x_i - x_j\| > r_i + r_j, \quad \forall i, j \in \mathbb{I}, i \neq j, \tag{5}$$

must hold for all times. The following is a feasibility assumption.

Assumption 1. For all, $i, j \in \mathbb{I}$, $\|x_i^d - x_j^d\| > r_i + r_j$ and $\|x_i(0) - x_j(0)\| > r_i + r_j$.

In this work, we *only* require that each agent i knows the *inter-agent bearings* of those agents that are in conflict with the agent, *i.e.,* those agents which are close enough to the agent i. This is in contrast to other approaches that assume knowledge of the velocity of the other agents (Van Berg et al., 2008) and/or global position information about all the agents (Dimarogonas et al., 2006b). We also assume that each agent is unaware of the other agents desired destinations. Therefore our problem formulation is completely decentralized. To model this latter fact, we let

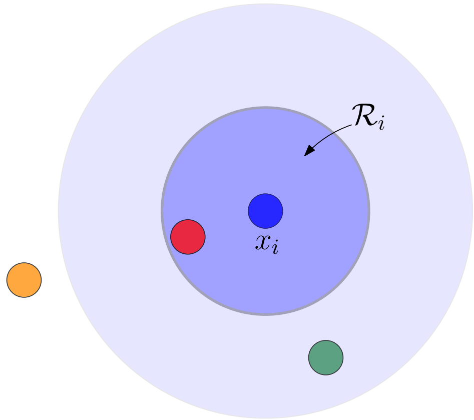

Fig. 1. Agent i (solid blue) is able to sense the position of two agents (green and red) since these are inside its sensing region (light blue). However, agent i will only use the position of the red agent in the collision avoidance controller since it is inside the avoidance set $\mathcal{R}_i$ (dark blue). The avoidance set $\mathcal{R}_i$ can be tuned arbitrary small.

$R_i > r_i > 0$ to be the radius of the *avoidance set* associated to agent i which is defined as

$$\mathcal{R}_i := \{x \in \mathbb{R}^n : \|x - x_i\| \leq R_i\}. \tag{6}$$

We then define the set of *neighbouring agents* as follows:

$$\mathcal{N}_i := \{j \in \mathbb{I} : \mathcal{O}_j \cap \mathcal{R}_i \neq \emptyset\}. \tag{7}$$

Note that for our convenience, the set $\mathcal{N}_i$ contains the index $\{i\}$ of the agent i itself. Our assumption about the available information to each agent is formally written as follows.

Assumption 2. For all $i \in \mathbb{I}$, agent i has access to the inter-agent bearings

$$b_{ij} := \frac{x_i - x_j}{\|x_i - x_j\|}$$

of all agents $j \in \mathcal{N}_i$ and its own desired position x_i^d.

The inter-agent bearings of neighbouring agents can be obtained practically using on-board sensors such as monocular/depth cameras or LiDARs. The neighbouring agents can be detected using proximity sensors or range sensors. LiDARs, for example, can provide both bearing and range measurements. Note that the avoidance set can be tuned as small as desired but no greater than the real sensing region of agent i. In this sense, agents which are inside the sensing region but outside of the avoidance set are not taken into account in the collision avoidance controller of agent i although the latter has access to their positions, see Fig. 1.

4. MAIN RESULTS

Our proposed collision avoidance approach consists in defining, for each agent $i \in \mathbb{I}$, a *velocity cone* that contains all the permissible velocities that guarantee safety with respect to neighbouring agents inside the avoidance set $\mathcal{R}_i$. More concretely, during the avoidance maneuver, each agent will try to keep the relative distances to the other agents (inside $\mathcal{R}_i$) non-decreasing, assuming these agents are static. In fact, for each agent i, and if we assume that the velocity of a neighbouring agent $j \in \mathcal{N}_i$ is zero, we have

$$\frac{1}{2}\frac{d}{dt}\|x_i - x_j\|^2 = u_i^\top (x_i - x_j). \tag{8}$$

Therefore, in order to guarantee the non-increase of the relative distance $\|x_i - x_j\|$, agent i must select its velocity u_i such that it lies in the half-space $\mathcal{H}_0(b_{ji}) = \mathcal{H}_0(x_j - x_i) \subset \mathbb{R}^n$. Now, assuming that the other agent j executes the same collision avoidance strategy, both agents will stay safe from each other: this is the concept of *reciprocity*. In other words, reciprocity allows for an agent to take only half of the responsibility for collision avoidance by implicitly assuming that the other agent takes the other half.

Extending this fact to all the neighbouring agents in $\mathcal{N}_i$, we must restrict the velocity of agent i to lie in the intersection set of all the half-spaces generated by the presence of these agents, *i.e.,* we impose the condition

$$u_i \in \bigcap_{j \in \mathcal{N}_i} \mathcal{H}_0(b_{ji}). \tag{9}$$

Note that, if $\mathcal{N}_i = \{i\}$, the above inclusion yields $u_i \in \mathbb{R}^n$ which reflects the fact that the control input should not be restricted if there are no neighbouring agents for agent i. Since the above set is the intersection of finitely many half-spaces passing through the origin, it defines a convex polyhedral cone (see Definition 8) which has motivated us to name this cone the *reciprocal safety velocity cone* (RSVC). More precisely, if we let $\mathcal{N}_i = \{j_1, \cdots, j_{N_i}\}$, condition (9) is written as

$$u_i \in \mathcal{C}(\mathbf{A}(x_{\mathcal{N}_i})) \tag{10}$$

where $x_{\mathcal{N}_i} = [x_{j_1}^\top, \cdots, x_{j_{N_i}}^\top]^\top \in \mathbb{R}^{nN_i}$ is the concatenated vector of all the neighbouring agents states (as well as x_i) and the matrix-valued function $\mathbf{A}(x_{\mathcal{N}_i})$ is given by

$$\mathbf{A}(x_{\mathcal{N}_i}) := \begin{bmatrix} \frac{(x_{j_1} - x_i)^\top}{\|x_{j_1} - x_i\|} \\ \vdots \\ \frac{(x_{j_{N_i}} - x_i)^\top}{\|x_{j_{N_i}} - x_i\|} \end{bmatrix} = \begin{bmatrix} -b_{i(j_1)} \\ \vdots \\ -b_{i(j_{N_i})} \end{bmatrix} \in \mathbb{R}^{N_i \times n}. \tag{11}$$

The computation of this matrix requires only the inter-agents bearings of those agents in $\mathcal{N}_i$ so this does not violate Assumption 2. To sum up, provided that all other agents restrict their control inputs to satisfy (9), the inter-agent distance between each agent i and its neighbouring agents $j \in \mathcal{N}_i$ cannot decrease; thus safety is ensured.

To further ensure that the desired stabilization tasks are achieved, we should choose a control law that decreases the distance to the desired target x_i^d. When the agent is not in proximity of any other agent, the classical nominal feedback law

$$u_i^0(x_i, x_i^d) := -k_i(x_i - x_i^d), \quad k_i > 0, \tag{12}$$

will achieve the required stabilization task. However, since the input must be constrained to $u_i \in \mathcal{C}(\mathbf{A}(x_{\mathcal{N}_i}))$, the solution we propose is to project the nominal controller u_i^0 onto the closed convex cone $\mathcal{C}(\mathbf{A}(x_{\mathcal{N}_i}))$. This projection is equivalent to solving the following convex optimization problem

$$\min_{u_i} \frac{1}{2}\|u_i - u_i^0\|^2 \quad \text{subject to } u_i \in \mathcal{C}(\mathbf{A}(x_{\mathcal{N}_i})), \tag{13}$$

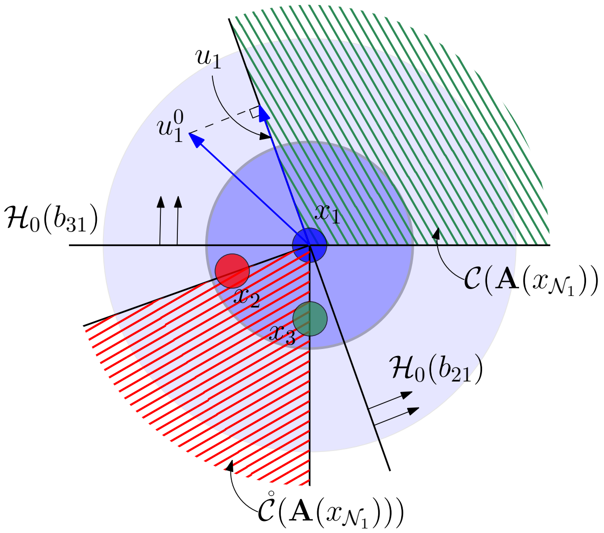

Fig. 2. The velocity cone $\mathcal{C}(\mathbf{A}(x_{\mathcal{N}_1}))$ (hashed green area) induced by the presence of two agents 2 and 3 inside the avoidance set of agent 1. Agent 1 will restrict its velocity to lie within the velocity cone $\mathcal{C}(\mathbf{A}(x_{\mathcal{N}_1}))$ and, assuming that the other agents will reciprocally cooperate in the same avoidance strategy, collision is avoided. To further ensure that the agent converges to its final destination, the nominal controller u_i^0 is projected onto the velocity cone $\mathcal{C}(\mathbf{A}(x_{\mathcal{N}_1}))$. If the nominal controller was in the polar velocity cone $\mathring{\mathcal{C}}(\mathbf{A}(x_{\mathcal{N}_i}))$ (hashed red area), the resulting projection is zero.

which is a quadratic programming problem with a set of linear inequality constraints on $\mathbb{R}^n$. Since $\mathcal{C}(\mathbf{A}(x_{\mathcal{N}_i}))$ is a nonempty closed convex set, and by the Hilbert projection theorem, the projection onto $\mathcal{C}(\mathbf{A}(x_{\mathcal{N}_i}))$ is unique and we will denote it by $\Pi(\mathcal{C}(\mathbf{A}(x_{\mathcal{N}_i})), z)$ for any $z \in \mathbb{R}^n$. Fortunately, a closed-form expression of this projection function has been derived recently in (Rutkowski, 2017) for general polyhedral sets. This result is refined and fine-tuned for a polyhedral cone $\mathcal{C}(A)$ in the following lemma.

Lemma 1. (Derived from (Rutkowski, 2017)). For a given matrix $A \in \mathbb{R}^{m\times n}$ and for any $x \in \mathbb{R}^n \setminus \mathcal{C}(A)$, there exists a non-empty subset $\mathcal{I} \subseteq \{1, \cdots, m\}$, with $|\mathcal{I}| \leq \text{rank}(A)$, such that the projection onto $\mathcal{C}(A)$ is given by the linear map

$$\Pi(\mathcal{C}(A), x) = (I_n - \bar{A}^\top(\bar{A}\bar{A}^\top)^{-1}\bar{A})x, \tag{14}$$

where $\bar{A} = A[\mathcal{I}]$ and the following conditions hold:

- $\bar{A}$ is full row rank
- $(\bar{A}\bar{A}^\top)^{-1}\bar{A}x > 0$
- $A\Pi(\mathcal{C}(A), x) \leq 0$

Basically, the above lemma shows that the projection onto the polyhedral cone is equal to the projection onto one of its *faces*, i.e., there exists a face that lies in one of the subspaces spanned by the rows of A such that the projection onto the polyhedral cone is equal to the projection onto this given face; this fact has been mentioned also in (Ujvári, 2016). The conditions in the above lemma allow to check which face is valid for the projection and are the results of the feasibility and the complementary slackness conditions of the Karush–Kuhn–Tucker conditions (Rutkowski, 2017). This lemma leads to Algorithm 1 which summarizes the procedure to compute the projection map.

Algorithm 1 Projection onto a polyhedral cone

Require: a matrix $A \in \mathbb{R}^{m\times n}$ and a vector $x \in \mathbb{R}^n$
Ensure: $y = \Pi(\mathcal{C}(A), x)$
  if $Ax \leq 0$ **then** $y = x$
  else
    Let $\Delta = \{\mathcal{I}_1, \mathcal{I}_2, \cdots\}$ ▷ Collection of all non-empty subsets $\mathcal{I}_k$ of $\{1, \cdots, m\}$ with cardinality $\leq \text{rank}(A)$.
    for $k \in \{1, 2, \cdots\}$ **do**
      $\bar{A} = A[\mathcal{I}_k]$
      if $\det(\bar{A}) \neq 0$ **then**
        Solve $\bar{A}\bar{A}^\top\nu = \bar{A}x$ for ν
        $y = x - \bar{A}^\top\nu$
        if $\nu > 0$ and $y \in \mathcal{C}(A)$ **then** return y
        end if
      end if
    end for
  end if

To sum up, the proposed control strategy for each agent is given by (see Algorithm 2)

$$u_i(x_{\mathcal{N}_i}, x_i^d) = \Pi(\mathcal{C}(\mathbf{A}(x_{\mathcal{N}_i})), u_i^0(x_i, x_i^d)), \quad i \in \mathbb{I}, \tag{15}$$

where u_i^0 is the nominal controller in (12) and $\Pi(\mathcal{C}(\mathbf{A}(x_{\mathcal{N}_i})), \cdot)$ is the projection function onto the reciprocal velocity cone $\mathcal{C}(\mathbf{A}(x_{\mathcal{N}_i}))$. It is clear that the proposed multi-agent control strategy is purely decentralized. Let us define the vector $\mathbf{x} = [x_1^\top, \cdots, x_N^\top]^\top \in \mathbb{R}^{nN}$ which corresponds to the concatenated vector of all agents' states. The multi-agent closed-loop system can be written as

$$\dot{\mathbf{x}} = \begin{bmatrix} \Pi(\mathcal{C}(\mathbf{A}(x_{\mathcal{N}_1})), u_1^0(x_1, x_1^d)) \\ \vdots \\ \Pi(\mathcal{C}(\mathbf{A}(x_{\mathcal{N}_N})), u_N^0(x_N, x_N^d)) \end{bmatrix} =: f(\mathbf{x}). \tag{16}$$

Note that the vector field $f(\mathbf{x})$ is discontinuous whenever agents enter or leave the sensing region of other agents. The collision avoidance task is then equivalent to the forward invariance of the set under the dynamics (16)

$$\mathcal{W} := \{\mathbf{x} \in \mathbb{R}^{nN} : \|x_i - x_j\| > r_i + r_j, \forall i, j \in \mathbb{I}, i \neq j\}. \tag{17}$$

The global target can be defined as

$$\mathbf{x}_d := [(x_1^d)^\top, \cdots, (x_N^d)^\top]^\top \in \mathbb{R}^{nN}.$$

Algorithm 2 Collision Avoidance with Stabilization

Require: Current position x_i and reference position x_i^d
Ensure: Results of Theorem 1.
  Detect neighbouring agents set $\mathcal{N}_i$
  Compute inter-agents bearings b_{ij} with $j \in \mathcal{N}_i$
  Construct bearing matrix $\mathbf{A}(x_{\mathcal{N}_i})$ in (11).
  Calculate nominal controller $u_i^0(x_i, x_i^d)$ in (12).
  Calculate projection $\Pi(\mathcal{C}(\mathbf{A}(x_{\mathcal{N}_i})), u_i^0(x_i, x_i^d))$ using Algorithm 1.
  Assign agent's velocity $u_i = \Pi(\mathcal{C}(\mathbf{A}(x_{\mathcal{N}_i})), u_i^0(x_i, x_i^d))$.

The following theorem summarizes the main results.

Theorem 1. Consider the multi-agent closed loop system (16). Then:

(1) The set $\mathcal{W}$ is forward invariant (collision avoidance).
(2) All distances $\|x_i - x_i^d\|$ are non-increasing.

(3) The equilibrium $\mathbf{x}_d$ is locally exponentially stable.
(4) All solutions must converge to the largest invariant set contained in $\{\mathbf{x}_d\} \cup \mathcal{E}$ where

$$\mathcal{E} := \{(x_1, \cdots, x_N) : (x_i^d - x_i) \in \mathring{\mathcal{C}}(\mathbf{A}(x_{\mathcal{N}_i}))\}.$$

In other words, each agent will either converge to the desired goal or it will be stuck in a deadlock with a group of other agents characterized by condition (4) in Theorem 1. The deadlock is present if, for each agent i in that group, its nominal velocity is inside the polar velocity cone $\mathring{\mathcal{C}}(\mathbf{A}(x_{\mathcal{N}_i}))$; hence its projection is zero causing the agent to stop. Current investigation is carried out to study the invariant properties of these deadlocks and possible remedies.

5. SIMULATION RESULTS

In this section we conduct some simulations to confirm the effectiveness of the proposed collision avoidance approach. We consider a set of N agents moving inside a two dimensional 1×1 squared region. The radius of each agent is $r_i = 0.05$ and the avoidance radius is $R_i = 0.07$. The *clutteredness* of the environment is defined as:

$$\rho_N := \frac{\text{total area covered by the agents}}{\text{total area of the workspace}} = \frac{\sum_{i=1}^N \pi r_i^2}{1 \times 1}.$$

We assume that all agents are moving at the same speed imposed by the gain $k_i = 0.5$ in (12). The closed-loop dynamics (16) are integrated on the time interval $[0, 30]$ using Euler method with a step size $h = 0.001$. The agents are divided into four colored groups: yellow, green, red, and blue. All the members of a given colored group will be required to stabilize their position to a desired location on the same edge of the square.

We consider 1000 random initial agents' positions taken from N position slots on the edges of the square (total possibilities for the initial condition of the whole system is $N!$). The desired locations coincide with these position slots. For each random initial condition of the multi-agent system, we calculate the *success rate* $\sigma \in [0, 1]$ which is defined as the complement of the ratio of deadlocked agents:

$$\sigma = 1 - \frac{\text{number of deadlocked agents}}{\text{total number of agents}}.$$

We assume that the cumulative distribution function (CDF) of the random variable σ is given by the exponential function

$$F_\beta(\sigma) = \frac{e^{\beta\sigma} - 1}{e^\beta - 1} \in [0, 1],$$

where β is a positive parameter that depends on the given problem's configuration (*e.g.,* number of agents). It is not difficult to show that $F_\beta(\sigma)$ is strictly decreasing with respect to β when $\sigma \in]0, 1[$. Therefore, the higher β, the lower the probability of having deadlocks. We compare two scenarios with $N = 36$ and $N = 20$ corresponding to clutteredness $\rho_{36} \approx 28.3\%$ and $\rho_{20} \approx 15.7\%$, respectively. The histograms of the two obtained cumulative distribution functions are plotted in Fig 5. For each scenario, the parameter β is obtained using maximum likelihood estimation (MLE), where we have obtained $\beta =: \beta_{36} = 6.2$ for $N = 36$ and $\beta =: \beta_{20} = 7.1$ for $N = 20$. Since $\beta_{20} > \beta_{36}$, we have $F_{\beta_{20}}(\sigma) < F_{\beta_{30}}(\sigma)$ for $\sigma \in]0, 1[$, which proves that the likelihood of deadlocks is minimized as the clutteredness of the environment decreases. As an example, we consider two simulation scenarios with 36 agents and different initial conditions. The first scenario, depicted in Fig. 3, results in a successful mission (with $\sigma = 1$) while the second scenario, depicted in Fig. 4, results in a failed mission (with $\sigma = 0.58$). Recall that for all scenarios, safety is 100% guaranteed (hard constraint).

6. CONCLUSION

This work presents a multi-agent cooperative control strategy with collision avoidance. The proposed framework is sensor-based and fully reactive in the sense that each agent acts solely based on the local sensor information to avoid collision with other agents while converging to its desired destination. In particular, we consider agents that are equipped with sensors providing bearing measurements to their neighbouring agents. We have shown that, under this distributed control strategy, collision avoidance is guaranteed while progress towards the desired targets is ensured if the agents do not arrive to a deadlock. Although the deadlocks are explicitly identified, their region of attraction is yet to be characterized. Future work will consider an additional mechanism to resolve deadlocks.

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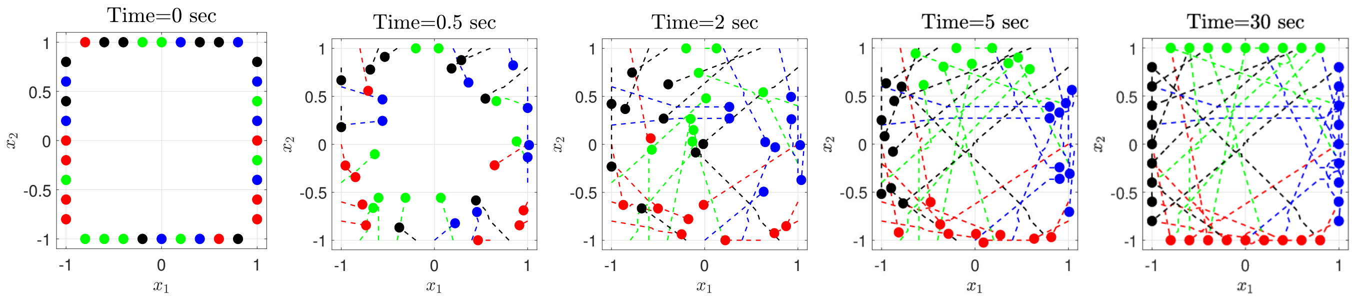

Fig. 3. Simulation results with 36 agents corresponding to a mission with 100% success rate. Complete simulation video can be found at `https://youtu.be/94Ws77ia6I8`.

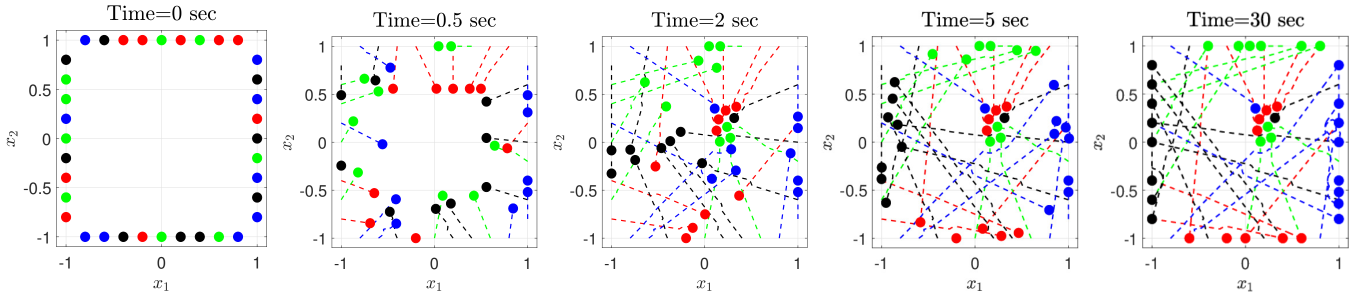

Fig. 4. Simulation results with 36 agents corresponding to a mission with 58% success rate. Complete simulation video can be found at `https://youtu.be/u3qY-vAoTUY`.

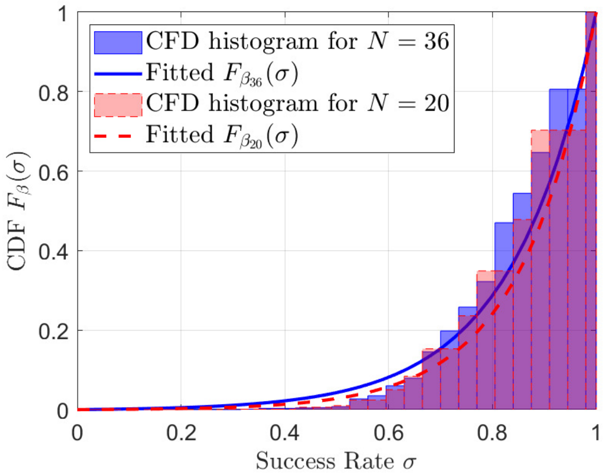

Fig. 5. Comparison between fitted cumulative distribution function $F_\beta(\sigma)$ in two scenarios: $N = 36$ (blue) and $N = 20$ (red). The histogram bars represent the real CDFs when running simulations with 1000 random initial conditions.

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