Distributed Control of Coupled Leader-follower Multi-agent Systems under Spatiotemporal Logic Tasks ⋆

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Abstract: This paper addresses the problem of cooperative control of leader-follower multi-agent systems under local signal temporal logic (STL) tasks in a distributed fashion, where the overall system is composed of several leader-follower subsystems with coupled dynamics. In this work, only the leaders know the related STL specifications and are designed to drive the followers in a way such that the STL specifications are globally satisfied. Under the local feasibility assumption, we propose a funnel-based control approach for each leader-follower subsystem such that the local STL specifications are achieved, which further implies the global satisfaction of all STL specifications. In order to enforce the satisfaction of the STL formulas, the funnel parameters are appropriately designed to prescribe certain transient behavior that constrains the closed-loop trajectories. The proposed approach is illustrated by a simulation example.

Keywords: Leader-follower control, multi-agent systems, signal temporal logic, prescribed performance control.

1. INTRODUCTION

Temporal logics show the ability to express more complex and high-level task specifications that expand beyond the standard control objectives. Compared with the formal methods (Belta et al., 2017) based approaches for single agent systems, e.g., (Kress-Gazit et al., 2009), the consideration in a multi-agent setup can include more real world applications such as multi-robot coordination. The multi-agent case needs further analysis on the couplings with respect to dynamics or task specifications. Signal Temporal Logic (STL) (Maler and Nickovic, 2004) which is based on continuous-time signals can be used to deal with quantitative spatiotemporal constraints for multi-agent systems since it allows both time and space constraints.

In this paper, we consider a leader-follower setup where a group of agents with advanced capabilities is selected as *leaders* in order to drive the remaining *follower* agents in a way such that the specified tasks are satisfied. Such complex and high-level tasks are represented by STL specifications which characterize both time and space constraints. In order to tackle the couplings with respect to the dynamics and STL specifications, a leader-follower multi-agent system is treated as a composition of several leader-follower subsystems through their coupled dynamics. For each subsystem, only the leaders know the STL specifications and their controllers are designed to fulfill the tasks, while the followers are indirectly guided through the dynamic couplings with the controlled leaders.

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Prescribed performance control (PPC) (Bechlioulis and Rovithakis, 2008) or funnel control (Berger et al., 2018; Ilchmann et al., 2005) is utilized in this work in order to enforce the satisfaction of the STL formulas by prescribing certain transient behavior of the funnels that constrain the closed-loop trajectories. Related research within a similar leader-follower framework mainly focuses on controllability (Ji et al., 2006) or leader selection problems (Fitch and Leonard, 2016). When it comes to the control design aspects, (Chen and Dimarogonas, 2020) proposes a distributed prescribed performance control strategy for leader-follower multi-agent systems to achieve formation tasks within predefined transient constraints. On STL specifications for multi-agent systems, a robust funnel-based control strategy is proposed in (Lindemann and Dimarogonas, 2019) for coupled multi-agent systems. In our previous work (Chen and Dimarogonas, 2022), we have considered control of leader-follower multi-agent systems under a single global STL task in a centralized manner. In this work, we instead consider leader-follower multi-agent systems which are composed of several leader-follower subsystems through dynamic couplings. Each subsystem may have multiple leaders and each leader is equipped with a local STL specification. We aim to design only leader controllers and in a distributed fashion such that all local STL specifications are satisfied, which further guarantees the global satisfaction of the STL tasks under the presence of dynamic couplings and task dependency.

The rest of the paper is organized as follows. In Section 2, we introduce preliminaries and formulate the problem. Section 3 presents the main result by proposing a funnel-based approach for coupling leader-follower multi-agent systems under certain fragments of local STL formulas.

The results are verified by a simulation example in Section 4. Section 5 includes conclusions and future work.

2. PRELIMINARIES AND PROBLEM STATEMENT

2.1 Leader-follower Multi-agent Systems

We consider M leader-follower multi-agent subsystems $\mathcal{S}_i$ under undirected communication graphs (Mesbahi and Egerstedt, 2010) $\mathcal{G}_i = (\mathcal{V}_i, \mathcal{E}_i), i \in \mathcal{I} = \{1, \dots, M\}$ with the cardinality of $\mathcal{V}_i$ as $|\mathcal{V}_i| = n_i$ and $\sum_{i\in\mathcal{I}} n_i = N$. Then, the respective vertices sets are indexed as $\mathcal{V}_i = \left\{(\sum_{j=1}^{i} n_j) - n_i + 1, \quad \dots, \sum_{j=1}^{i} n_j\right\}$. The edge sets are $\mathcal{E}_i = \{(a, b) \in \mathcal{V}_i \times \mathcal{V}_i \mid b \in \mathcal{N}_a \subset \mathcal{V}_i\}$ where $\mathcal{N}_a \subset \mathcal{V}_i$ denotes the neighbor of agent a in set $\mathcal{V}_i$. Suppose that for each subsystem we have n_i^L leaders and n_i^F followers with the respective vertices set as $\mathcal{V}_i^L = \left\{(\sum_{j=1}^{i} n_j) - n_i^L + 1, \dots, \sum_{j=1}^{i} n_j\right\}$ and $\mathcal{V}_i^F = \mathcal{V}_i \setminus \mathcal{V}_i^L$ such that $n_i = n_i^L + n_i^F$. Suppose that the overall system $\mathcal{S}$ is composed of the above leader-follower subsystems under the connected and undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with $\mathcal{V} = \cup_{i\in\mathcal{I}}\mathcal{V}_i$ and $\mathcal{E} = \cup_{i\in\mathcal{I}}\mathcal{E}_i \cup \mathcal{E}_c$, where $\mathcal{E}_c = \{(a, b) \in \mathcal{V}_i \times \mathcal{V}_j \mid b \in \mathcal{N}_a \subset \mathcal{V}, i \neq j\}$ is the edge set that connects different subsystems $\mathcal{S}_i, \mathcal{S}_j, i \neq j$. $\mathcal{G}$ is connected in the sense that for any subsystems $\mathcal{S}_i, \mathcal{S}_j, i \neq j$, there exists a path from $\mathcal{G}_i$ to $\mathcal{G}_j$. We can further define the respective leader and follower agents sets of $\mathcal{G}$ as $\mathcal{V}^L = \cup_{i\in\mathcal{I}}\mathcal{V}_i^L$, $\mathcal{V}^F = \cup_{i\in\mathcal{I}}\mathcal{V}_i^F$ and $\mathcal{V} = \mathcal{V}^L \cup \mathcal{V}^F$. Now the neighbor of agent a in set $\mathcal{V}$ is defined as $\mathcal{N}_a = \{b \in \mathcal{V} \mid (a, b) \in \mathcal{E}\}$.

Let $x_k \in \mathbb{R}^n$ be the state of agent $k \in \mathcal{V}$, and the state evolution of agent k is governed by the following dynamics:

$$\dot{x}_k = \sum_{l\in\mathcal{N}_k} (x_l - x_k) + b_k u_k, \tag{1}$$

with $b_k = 1$ if $k \in \mathcal{V}^L$, and $b_k = 0$ if $k \in \mathcal{V}^F$. For each leader-follower subsystem $\mathcal{S}_i, i \in \mathcal{I}$, we derive the dynamics of $\mathcal{S}_i$ by stacking (1) for $k \in \mathcal{V}_i$:

$$\mathcal{S}_i : \dot{\boldsymbol{x}}_{\boldsymbol{i}} = -(L_i \otimes I_n)\boldsymbol{x}_{\boldsymbol{i}} + (C_i \otimes I_n)\boldsymbol{x} + (B_i \otimes I_n)\boldsymbol{u}_{\boldsymbol{i}}, \tag{2}$$

where $\boldsymbol{x}_{\boldsymbol{i}} \in \mathbb{R}^{nn_i}$ is the stacked state of all $x_k, k \in \mathcal{V}_i$, $\boldsymbol{u}_{\boldsymbol{i}} \in \mathbb{R}^{nn_i^L}$ is the input for $\mathcal{S}_i$ by stacking $u_k, k \in \mathcal{V}_i^L$, $\boldsymbol{x} = [\boldsymbol{x}_{\boldsymbol{1}}, \dots, \boldsymbol{x}_{\boldsymbol{M}}]^T \in \mathbb{R}^{nN}$, $L_i \in \mathbb{R}^{n_i\times n_i}$ is the graph Laplacian (Mesbahi and Egerstedt, 2010) for $\mathcal{G}_i$, $C_i \in \mathbb{R}^{n_i\times N}$ represents the dynamic couplings between $\mathcal{S}_i$ and $\mathcal{S}_j$ indicated through $\mathcal{E}_c$, $j \in \mathcal{I} \setminus \{i\}$, and $B_i = \begin{bmatrix} 0_{n_i^F\times n_i^L} \\ I_{n_i^L} \end{bmatrix}$. According to (2), the overall dynamics of the leader-follower multi-agent system $\mathcal{S}$ are derived as follows:

$$\mathcal{S} : \dot{\boldsymbol{x}} = -(L \otimes I_n)\boldsymbol{x} + (B \otimes I_n)\boldsymbol{u}, \tag{3}$$

with $\boldsymbol{u} = [\boldsymbol{u}_{\boldsymbol{1}}, \dots, \boldsymbol{u}_{\boldsymbol{M}}]^T \in \mathbb{R}^{nn^L}$ and $n^L = \sum_{i=1}^{M} n_i^L$, $L = diag(L_1, \dots, L_M) + [C_1^T, \dots, C_M^T]^T$ is the graph Laplacian of $\mathcal{G}$, and $B = diag(B_1, \dots, B_M)$.

2.2 Signal Temporal Logic (STL)

Let $\mathbb{B} := \{\top, \bot\}$ with $\top$ and $\bot$ as the boolean true and false values, respectively. Signal temporal logic (STL) (Maler and Nickovic, 2004) consists of predicates $\mu : \mathbb{R}^n \to \mathbb{B}$ which are obtained by evaluating a continuously differentiable predicate function $h : \mathbb{R}^n \to \mathbb{R}$ and assigning the respective true or false boolean value as: $\mu = \top$ if $h(\mathrm{x}) \geq 0$; $\mu = \bot$ if $h(\mathrm{x}) < 0$, where $\mathrm{x} \in \mathbb{R}^n$. The STL syntax is defined as

$$\phi ::= \top \mid \mu \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid \mathrm{F}_{[a,b]}\phi \mid \mathrm{G}_{[a,b]}\phi, \tag{4}$$

where ϕ_1, ϕ_2 are STL formulas and $\neg, \wedge, \mathrm{F}_{[a,b]}, \mathrm{G}_{[a,b]}$ are the respective negation, conjunction, eventually, always operators with $0 \leq a \leq b < \infty$. The satisfaction relation $(\mathrm{x}, t) \models \phi$ represents that the continuous-time signal $\mathrm{x} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ satisfies ϕ at time t. Robust semantics have been introduced in (Fainekos and Pappas, 2009) in order to quantify how robustly the signal x satisfies the STL formula ϕ at time t. Space robustness semantics (Donzé and Maler, 2010) for STL are defined as: $\rho^{\mu}(\mathrm{x}, t) := h(\mathrm{x}(t))$; $\rho^{\neg\phi}(\mathrm{x}, t) := -\rho^{\phi}(\mathrm{x}, t)$; $\rho^{\phi_1\wedge\phi_2}(\mathrm{x}, t) := \min(\rho^{\phi_1}(\mathrm{x}, t), \rho^{\phi_2}(\mathrm{x}, t))$; $\rho^{\mathrm{F}_{[a,b]}\phi}(\mathrm{x}, t) := \max_{t_1\in[t+a,t+b]} \rho^{\phi}(\mathrm{x}, t_1)$; $\rho^{\mathrm{G}_{[a,b]}\phi}(\mathrm{x}, t) := \min_{t_1\in[t+a,t+b]} \rho^{\phi}(\mathrm{x}, t_1)$. Note that it holds that $(\mathrm{x}, t) \models \phi$ if $\rho^{\phi}(\mathrm{x}, t) > 0$. In this work, we consider a fragment of the STL introduced above, which is defined as follows:

$$\psi := \top \mid \mu \mid \neg\mu \mid \psi_1 \wedge \psi_2 \tag{5a}$$

$$\phi := \mathrm{F}_{[a,b]}\psi \mid \mathrm{G}_{[a,b]}\psi \mid \mathrm{F}_{[a,b]}\mathrm{G}_{[c,d]}\psi \tag{5b}$$

where ψ in (5b) and ψ_1, ψ_2 in (5a) are non-temporal formulas of class ψ as in (5a), and where ϕ as in (5b) are temporal formulas with $[a, b], [c, d]$ as the time intervals. In this work, we focus on the fragment of STL in the form of (5a), (5b), which are expressive enough to tackle leader-follower multi-agent planning tasks, e.g. formation control, collision avoidance and connectivity maintenance.

2.3 Problem Statement

In this work, each subsystem $\mathcal{S}_i$ is assigned with one local STL formula $\phi_i, i \in \mathcal{I}$ as in (5b), which can be further decomposed to n_i^L local STL formulas $\phi_i^k, i \in \mathcal{I}, k \in \mathcal{V}_i^L$ as in (5b) for each leader of $\mathcal{S}_i$ (if $\mathcal{S}_i$ has multiple leaders). The local STL formula ϕ_i^k is only known to the leader $k \in \mathcal{V}_i^L$ in $\mathcal{S}_i$. The local satisfaction of ϕ_i^k depends on only one leader, i.e., k and the neighboring agents of leader k in $\mathcal{S}_i$, which is a subset of agents $\mathcal{V}_{\phi_i^k}$ in $\mathcal{S}_i$, i.e., $\mathcal{V}_{\phi_i^k} \cap \mathcal{V}_i^L = \{k\}$, $\mathcal{V}_{\phi_i^k} \subseteq (\mathcal{N}_k \cap \mathcal{V}_i \cup \{k\}) \subseteq \mathcal{V}_i$. For each leader $k \in \mathcal{V}_i^L$ of the subsystem $\mathcal{S}_i$ which has multiple leaders, we assume that there exists another leader $j \in \mathcal{V}_i^L$ of the same subsystem such that $\mathcal{V}_{\phi_i^k} \cap \mathcal{V}_{\phi_i^j} \neq \emptyset$. This means that for each subsystem $\mathcal{S}_i$, the local STL specifications are coupled within $\mathcal{S}_i$. Otherwise, if $\forall j \in \mathcal{V}_i^L \setminus \{k\}$ for the same subsystem such that $\mathcal{V}_{\phi_i^k} \cap \mathcal{V}_{\phi_i^j} = \emptyset$, we can further decompose the subsystem $\mathcal{S}_i$ into a subsystem with agent set $\mathcal{V}_{\phi_i^k}$ and a subsystem with agent set $\mathcal{V}_i \setminus \mathcal{V}_{\phi_i^k}$. Therefore, the subsystems are defined based on the task dependency, i.e., $\forall k \in \mathcal{V}_i^L, \exists j \in \mathcal{V}_i^L \setminus \{k\}$ such that $\mathcal{V}_{\phi_i^k} \cap \mathcal{V}_{\phi_i^j} \neq \emptyset$. Furthermore, the local satisfaction of ϕ_i depends on a subset of agents $\mathcal{V}_{\phi_i}$ in $\mathcal{S}_i$, i.e., $\mathcal{V}_{\phi_i} \subseteq \mathcal{V}_i$, and also depends on the dynamic couplings with $\mathcal{S}_j, j \in \mathcal{I} \setminus \{i\}$. Next, we define the notion of local feasibility as follows.

Definition 1. (Local satisfaction and local feasibility) The closed-loop signal $\boldsymbol{x}_{\boldsymbol{i}} : [0, \infty) \to \mathbb{R}^{nn_i}$ of $\mathcal{S}_i$ as in (2) locally satisfies ϕ_i if and only if $(\boldsymbol{x}_{\boldsymbol{i}}, 0) \models \phi_i$. The formula ϕ_i for $\mathcal{S}_i$ is locally feasible if and only if $\exists \boldsymbol{u}_{\boldsymbol{i}} : [0, \infty) \to \mathbb{R}^{nn_i^L}$ for (2) such that the closed-loop signal $\boldsymbol{x}_{\boldsymbol{i}} : [0, \infty) \to \mathbb{R}^{nn_i}$ locally satisfies ϕ_i.

If $\forall i \in \mathcal{I}$, $\boldsymbol{x_i}$ locally satisfies ϕ_i for all subsystems $\mathcal{S}_i, i \in \mathcal{I}$, we say that the signal $\boldsymbol{x} : [0, \infty) \to \mathbb{R}^{nN}$ globally satisfies $\{\phi_1, \ldots, \phi_M\}$. The set of STL formulas $\{\phi_1, \ldots, \phi_M\}$ is globally feasible if $\exists \boldsymbol{u} : [0, \infty) \to \mathbb{R}^{nn^L}$ for (3) such that the closed-loop signal $\boldsymbol{x} : [0, \infty) \to \mathbb{R}^{nN}$ globally satisfies $\{\phi_1, \ldots, \phi_M\}$. The objective of this work is defined in Problem 1 under the local feasibility assumption as indicated in Assumption 1.

Assumption 1. The local STL formula ϕ_i for each subsystem $\mathcal{S}_i, i \in \mathcal{I}$ is locally feasible as per Definition 1.

Problem 1. Given a local STL formula ϕ_i as in (5b) for each leader-follower subsystem $\mathcal{S}_i, i \in \mathcal{I}$ as in (2), which can be further decomposed to n_i^L local STL formula $\phi_i^k, i \in \mathcal{I}, k \in \mathcal{V}_i^L$ as in (5b) for each leader. Suppose that Assumption 1 holds, design control strategies $\boldsymbol{u_i}$ for $\mathcal{S}_i$ such that the closed-loop trajectory $\boldsymbol{x} : [0, \infty) \to \mathbb{R}^{nN}$ of (3) globally satisfies $\{\phi_1, \ldots, \phi_M\}$.

3. ENCODING STL WITH PRESCRIBED PERFORMANCE CONTROL

In this section, we propose an approach to synthesising a control strategy $\boldsymbol{u_i}$ for the subsystem $\mathcal{S}_i, i \in \mathcal{I}$ as in (2) such that the local STL formula ϕ_i as in (5b) is satisfied. If $\mathcal{S}_i$ contains more than one leader, we first further decompose ϕ_i to n_i^L local STL formulas $\phi_i^k, k \in \mathcal{V}_i^L$ for each leader according to the task dependency. Suppose that ϕ_i is the temporal formula as in (5b) with the non-temporal formula $\psi_i = \psi_{i,1} \wedge \cdots \wedge \psi_{i,q}$, $\phi_i^k, k \in \mathcal{V}_i^L$ only including the formulas $\psi_{i,j}, j \in \{1, \ldots, q\}$ such that $\mathcal{V}_{\psi_{i,j}} \subseteq (\mathcal{V}_{\phi_i} \cap \mathcal{N}_k \cup \{k\})$, where $\mathcal{V}_{\psi_{i,j}}$ denotes the agents participating in $\psi_{i,j}$. If ϕ_i^k contains conjunctions of the non-temporal formulas as in (5a), e.g., $\phi_i^k := \mathrm{F}_{[a_i,b_i]}\psi_i^k$ and $\psi_i^k := \psi_{i,1}^k \wedge \cdots \wedge \psi_{i,n}^k$, the non-smooth robust semantics $\rho^{\psi_{i,1}^k \wedge \cdots \wedge \psi_{i,n}^k}(\mathrm{x}, t)$ can be replaced by a smooth under-approximation $\rho^{\psi_{i,1}^k \wedge \cdots \wedge \psi_{i,n}^k}(\mathrm{x}, t) \approx -\frac{1}{\eta}\ln(\sum_{j=1}^{n}\exp(-\eta\rho^{\psi_{i,j}^k}(\mathrm{x}, t)))$ with the parameter $\eta > 0$. We then utilize PPC to enforce the satisfaction of temporal STL formulas by prescribing the transient behavior of the non-temporal STL formulas within:

$$-\mathrm{p}_i^k(t) + \rho_i^k < \rho^{\psi_i^k}(\boldsymbol{x_i}, 0) < \rho_i^k, \quad i \in \mathcal{I}, k \in \mathcal{V}_i^L \tag{6}$$

where ψ_i^k is the corresponding non-temporal formula inside the F, G operators as in (5b). In order to guarantee that $\rho^{\phi_i^k}(\boldsymbol{x_i}, 0) > 0$, we first prescribe a temporal behavior to the related $\rho^{\psi_i^k}(\boldsymbol{x_i}, 0)$ by appropriately designing the funnel $\mathrm{p}_i^k(t)$ and the positive scalar ρ_i^k. $\mathrm{p}_i^k(t) : \mathbb{R}_+ \to \mathbb{R}_+ \setminus \{0\}$ is a positive, smooth and strictly decreasing performance function (Bechlioulis and Rovithakis, 2008), and in this work we choose $\mathrm{p}_i^k(t) := (\mathrm{p}_{i,0}^k - \mathrm{p}_{i,\infty}^k)e^{-l_i^k t} + \mathrm{p}_{i,\infty}^k$ with $\mathrm{p}_{i,0}^k, \mathrm{p}_{i,\infty}^k, l_i^k$ as positive funnel parameters and we have that $\mathrm{p}_{i,0}^k > \mathrm{p}_{i,\infty}^k$. In the sequel, these parameters are designed in order to prescribe the behavior of $\rho^{\psi_i^k}(\boldsymbol{x_i}, 0)$. In PPC, we first define an error term for each $\rho^{\psi_i^k}(\boldsymbol{x_i}, 0)$, i.e., $e_i^k(\boldsymbol{x_i}) = \rho^{\psi_i^k}(\boldsymbol{x_i}, 0) - \rho_i^k$, we then obtain the modulated error as

$$\bar{e}_i^k(\boldsymbol{x_i}, t) = \frac{e_i^k(\boldsymbol{x_i})}{\mathrm{p}_i^k(t)} \tag{7}$$

and the related prescribed performance region $\mathcal{D} := (-1, 0)$. The transformed error is then defined as

$$\varepsilon_i^k(\boldsymbol{x_i}, t) = T_i^k(\bar{e}_i^k(\boldsymbol{x_i}, t)) := \ln\left(-\frac{1 + \bar{e}_i^k(\boldsymbol{x_i}, t)}{\bar{e}_i^k(\boldsymbol{x_i}, t)}\right), \tag{8}$$

where T_i^k is the transformation function that defines a smooth and strictly increasing mapping $T_i^k : \mathcal{D} \to \mathbb{R}$. The basic idea of PPC is to derive a control law such that the $\varepsilon_i^k(\boldsymbol{x_i}, t)$ is rendered bounded, which in turn implies (6).

3.1 Control design for temporal formulas using PPC

In this subsection, we propose a funnel-based control strategy for $\mathcal{S}_i, i \in \mathcal{I}$ as in (2) such that the prescribed behavior on $\rho^{\psi_i^k}(\boldsymbol{x_i}, 0)$ described as (6) can be achieved. Then, in combination with the appropriately chosen funnel parameters, we can further guarantee the transient behavior that is characterized by the temporal formulas (5b). The connection between the non-temporal formulas (5a) and the temporal formulas (5b) is established by designing the funnel parameters such that the satisfaction of (6) will guarantee that $0 < \rho^{\phi_i^k}(\boldsymbol{x_i}, 0) < \rho_i^k$ holds. For each temporal STL formula ϕ_i for $\mathcal{S}_i, i \in \mathcal{I}$ in the form of (5b), we first define the so-called crossing time as: $t_{\star,i} = a_i$ if $\phi_i = \mathrm{G}_{[a_i,b_i]}\psi_i$; $t_{\star,i} = a_i'$ if $\phi_i = \mathrm{F}_{[a_i,b_i]}\psi_i$; $t_{\star,i} = a_i''$ if $\phi_i = \mathrm{F}_{[a_i,b_i]}\mathrm{G}_{[c_i,d_i]}\psi_i$, where $a_i' \in [a_i, b_i]$ and $a_i'' \in [a_i + c_i, b_i + c_i]$. Note that for the subsystem $\mathcal{S}_i$ which has multiple leaders, the decomposed STL formulas $\phi_i^k, k \in \mathcal{V}_i^L$ will share the same time interval as ϕ_i. The crossing time $t_{\star,i}$ characterizes the instance that the lower bound of the funnel, i.e., $-\mathrm{p}_i^k(t) + \rho_i^k$ traverses across zero.

Next, the following assumption is assumed in this paper, which also indicates the advanced capability of the leaders.

Assumption 2. Only the leaders $k \in \mathcal{V}_i^L$ of the leader-follower subsystem $\mathcal{S}_i$ know $\phi_i^k, i \in \mathcal{I}$. In addition $\frac{\partial \rho^{\psi_i^k}(\boldsymbol{x_i},0)}{\partial x_k}, k \in \mathcal{V}_i^L$ is a nonzero vector.

Now, recall the performance function $\mathrm{p}_i^k(t) := (\mathrm{p}_{i,0}^k - \mathrm{p}_{i,\infty}^k)e^{-l_i^k t} + \mathrm{p}_{i,\infty}^k$ for each leader $k \in \mathcal{V}_i^L$ of the subsystem $\mathcal{S}_i$. We propose a control strategy such that $\rho^{\psi_i^k}(\boldsymbol{x_i}, 0)$ is always within the funnel (6) for all $i \in \mathcal{I}, k \in \mathcal{V}_i^L$. Furthermore, by choosing the funnel parameters $\mathrm{p}_{i,0}^k, \mathrm{p}_{i,\infty}^k$ and l_i^k appropriately, the satisfaction of (6) for all $k \in \mathcal{V}_i^L$ ensures that $(\boldsymbol{x_i}, t) \models \phi_i$.

Theorem 1. Given a local STL formula ϕ_i as in (5b) for each leader-follower subsystem $\mathcal{S}_i, i \in \mathcal{I}$ as in (2) with the decomposed local STL formula ϕ_i^k for each leader $k \in \mathcal{V}_i^L$. Suppose that Assumptions 1 and 2 hold. If the initial conditions $\rho^{\psi_i^k}(\boldsymbol{x_i}(0), 0)$ are within the funnel (6), and it further holds that

- for $t_{\star,i} = 0$, $\mathrm{p}_{i,0}^k \in (\rho_i^k - \rho^{\psi_i^k}(\boldsymbol{x_i}(0), 0), \rho_i^k]$; $\mathrm{p}_{i,\infty}^k \in (0, \min(\mathrm{p}_{i,0}^k, \rho_i^k))$; $l_i^k > 0$; $\rho_i^k > \rho^{\psi_i^k}(\boldsymbol{x_i}(0), 0)$;
- for $t_{\star,i} > 0$, $\mathrm{p}_{i,0}^k \in (\rho_i^k - \rho^{\psi_i^k}(\boldsymbol{x_i}(0), 0), \infty)$; $\mathrm{p}_{i,\infty}^k \in (0, \min(\mathrm{p}_{i,0}^k, \rho_i^k))$; $l_i^k = -\frac{1}{t_{\star,i}}\ln(\frac{\rho_i^k - \mathrm{p}_{i,\infty}^k}{\mathrm{p}_{i,0}^k - \mathrm{p}_{i,\infty}^k})$; $\rho_i^k > \rho^{\psi_i^k}(\boldsymbol{x_i}(0), 0)$,

then the control strategy

$$u_k(\boldsymbol{x_i},t) = -\varepsilon_i^k(\boldsymbol{x_i},t)\frac{\partial \rho^{\psi_i^k}(\boldsymbol{x_i},0)}{\partial x_k},\ \ i\in\mathcal{I}, k\in\mathcal{V}_i^L \qquad (9)$$

for each subsystem $\mathcal{S}_i$ guarantees that the closed-loop trajectory $\boldsymbol{x} : [0,\infty) \to \mathbb{R}^{nN}$ of (3) globally satisfies $\{\phi_1,\dots,\phi_M\}$, where $\varepsilon_i^k(\boldsymbol{x_i},t)$ is the transformed error defined as in (8).

Proof. The proof is composed of two parts. In the first part, we will prove that by applying (9), (6) is guaranteed for each subsystem $\mathcal{S}_i$. In the second part, we prove that the satisfaction of (6) enforces $0 < \rho^{\phi_i^k}(\boldsymbol{x_i},0) < \rho_i^k$ for all leaders $k \in \mathcal{V}_i^L$ in $\mathcal{S}_i$, thus $\boldsymbol{x_i}$ locally satisfies ϕ_i. Then, since $\forall i \in \mathcal{I}$, $\boldsymbol{x_i}$ locally satisfies ϕ_i for all subsystems $\mathcal{S}_i, i\in\mathcal{I}$, we can further conclude that the closed-loop trajectory $\boldsymbol{x}$ globally satisfies $\{\phi_1,\dots,\phi_M\}$. The advances when compared with our previous work (Chen and Dimarogonas, 2022) involve the consideration of the coupled dynamics between different subsystems and the coupled local STL tasks within the subsystem. Furthermore, the proposed control (9) is distributed.

Part 1: *Step 1.* Since the initial condition $\rho^{\psi_i^k}(\boldsymbol{x_i}(0),0)$ for each $i\in\mathcal{I}, k\in\mathcal{V}_i^L$ is within the funnel (6), this implies that the initial condition $\bar{e}_i^k(\boldsymbol{x_i}(0),t)$ is within the prescribed performance region $\mathcal{D}$ according to (6), (7). Inserting (9) to (2) for each $k\in\mathcal{V}_i^L$, we obtain the closed-loop dynamics for the leader-follower subsystem $\mathcal{S}_i$ as $\dot{\boldsymbol{x}}_{\boldsymbol{i}} = f_i(\boldsymbol{x_i},\bar{e}_i) = -(L_i\otimes I_n)\boldsymbol{x_i} + (C_i\otimes I_n)\boldsymbol{x} - (B_i\otimes I_n)\boldsymbol{u_i}(\boldsymbol{x_i},t)$, where $\bar{e}_i$ is the stacked vector of all $\bar{e}_i^k, k\in\mathcal{V}_i^L$ and $\boldsymbol{u_i}(\boldsymbol{x_i},t)$ is the input vector for $\mathcal{S}_i$ by stacking (9) for all $k\in\mathcal{V}_i^L$. We can denote $\boldsymbol{u_i}(\boldsymbol{x_i},t)$ as $\boldsymbol{u_i}(\boldsymbol{x_i},t) = -\Theta\varepsilon_i$, where $\Theta := diag\left(\frac{\partial\rho^{\psi_i^k}(\boldsymbol{x_i},0)}{\partial x_k}\right) \in \mathbb{R}^{nn_i^L\times n_i^L}$ represents a block diagonal matrix with diagonal entries $\frac{\partial\rho^{\psi_i^k}(\boldsymbol{x_i},0)}{\partial x_k}$ over $k\in\mathcal{V}_i^L$, and ε_i is the stacked vector of all $\varepsilon_i^k, k\in\mathcal{V}_i^L$. By stacking all $\dot{\boldsymbol{x}}_{\boldsymbol{i}}$, we have that $\dot{\boldsymbol{x}} = f(\boldsymbol{x},\bar{e}) := [f_1^T(\boldsymbol{x_1},\bar{e}_1),\dots,f_M^T(\boldsymbol{x_M},\bar{e}_M)]^T$, where $\bar{e} := [\bar{e}_1^T,\dots,\bar{e}_M^T]^T$. By calculating the derivative of $\bar{e}_i^k = (\rho^{\psi_i^k}(\boldsymbol{x_i},0)-\rho_i^k)/\mathrm{p}_i^k(t)$, we obtain $\dot{\bar{e}}_i^k = \left(\frac{\partial\rho^{\psi_i^k}(\boldsymbol{x_i},0)^T}{\partial\boldsymbol{x_i}}\dot{\boldsymbol{x}}_{\boldsymbol{i}} - \dot{\mathrm{p}}_i^k(t)\bar{e}_i^k\right)/\mathrm{p}_i^k(t)$. Replacing $\dot{\boldsymbol{x}}_{\boldsymbol{i}}$, we derive $\dot{\bar{e}}_i^k = \bar{g}_k(\boldsymbol{x_i},\bar{e}_i,t) = \frac{1}{\mathrm{p}_i^k(t)}(\frac{\partial\rho^{\psi_i^k}(\boldsymbol{x_i},0)^T}{\partial\boldsymbol{x_i}} f_i(\boldsymbol{x_i},\bar{e}_i) - \bar{e}_i^k\dot{\mathrm{p}}_i^k(t))$. Denote $g_i(\boldsymbol{x_i},\bar{e}_i,t)$ as the vector of all $\bar{g}_k(\boldsymbol{x_i},\bar{e}_i,t)$, $k\in\mathcal{V}_i^L$, let $g(\boldsymbol{x},\bar{e},t) := [g_1^T(\boldsymbol{x_1},\bar{e}_1,t),\dots,g_M^T(\boldsymbol{x_M},\bar{e}_M,t)]^T$, we then have $\dot{\bar{e}} = g(\boldsymbol{x},\bar{e},t)$. Next, we define $z := [\boldsymbol{x}^T,\bar{e}^T]^T$, and $\dot{z} = h(z,t)$ with $h(z,t) = [f^T(\boldsymbol{x},\bar{e}), g^T(\boldsymbol{x},\bar{e},t)]^T$. The initial condition $\boldsymbol{x_i}(0)$ is such that $\bar{e}_i^k(\boldsymbol{x_i}(0),0)\in\mathcal{D}$, which is an open set. We then define $\mathcal{D}_i := \{\boldsymbol{x_i}\in\mathbb{R}^{nn_i} \mid \bar{e}_i^k(\boldsymbol{x_i}(0),0)\in\mathcal{D}, k\in\mathcal{V}_i^L\}$, which is also an open, non-empty and bounded set. Therefore, $\mathcal{D}_z := \mathcal{D}_x\times\mathcal{D}_{\bar{e}}$ is an open, non-empty and bounded set as well with the initial condition satisfying $z(0) = [\boldsymbol{x}^T(0),\bar{e}^T(\boldsymbol{x}(0),0)]^T\in\mathcal{D}_z$, where $\mathcal{D}_x := \mathcal{D}_1\times\cdots\times\mathcal{D}_M \subset\mathbb{R}^{nN}$ and $\mathcal{D}_{\bar{e}} := \mathcal{D}\times\cdots\times\mathcal{D}\subset\mathbb{R}^{n^L}$. We now consider the initial value problem $\dot{z} = h(z,t)$ with $z(0)\in\mathcal{D}_z$. We can verify that $h(z,t)$ is continuous on t due to continuity of $\mathrm{p}_i^k(t)$ and $\dot{\mathrm{p}}_i^k(t)$. Moreover, since the transformed function $\ln\left(-\frac{1+\bar{e}_i^k}{\bar{e}_i^k}\right)$ is locally Lipschitz continuous and $\frac{\partial\rho^{\psi_i^k}(\boldsymbol{x_i},0)}{\partial\boldsymbol{x_i}}$ is also locally Lipschitz continuous due to the smooth approximation discussed previously, we can conclude that $h(z,t)$ is locally Lipschitz on z. Hence, according to Theorem 54 of (Sontag, 2013), there exists a maximal solution $z(t)$ of the initial value problem $\dot{z} = h(z,t)$ in a time interval $[0,\tau_{\max})$ such that $z(t)\in\mathcal{D}_z, \forall t\in[0,\tau_{\max})$.

Step 2. Based on Step 1, we know that $\rho^{\psi_i^k}(\boldsymbol{x_i}(t),0)$ satisfies (6) for all $t\in[0,\tau_{\max})$. This is due to the fact that $z(t)\in\mathcal{D}_z,\forall t\in[0,\tau_{\max})$, thus $\bar{e}_i^k(\boldsymbol{x_i}(t),t)\in\mathcal{D},\forall t\in[0,\tau_{\max})$, which in turn implies the satisfaction of (6) for all $t\in[0,\tau_{\max})$. We now consider the Lyapunov function candidate $V(\varepsilon_i) = \frac{1}{2}\varepsilon_i^T\varepsilon_i$ with $\dot{V} = \varepsilon_i^T\dot{\varepsilon}_i$. Taking the derivative on (8), we have $\dot{\varepsilon}_i^k = -\frac{\dot{\bar{e}}_i^k}{\bar{e}_i^k(1+\bar{e}_i^k)}$. By replacing $\dot{\bar{e}}_i^k$ which is derived in Step 1, we obtain

$$\dot{\varepsilon}_i^k = -\frac{1}{\mathrm{p}_i^k(t)\bar{e}_i^k(1+\bar{e}_i^k)}\left(\frac{\partial\rho^{\psi_i^k}(\boldsymbol{x_i},0)^T}{\partial\boldsymbol{x_i}}\dot{\boldsymbol{x}}_{\boldsymbol{i}} - \bar{e}_i^k\dot{\mathrm{p}}_i^k(t)\right).$$

Then, stacking all $\dot{\varepsilon}_i^k$ for $k\in\mathcal{V}_i^L$, $\dot{V}$ can be obtained as $\dot{V} = \varepsilon_i^T J(\Gamma\dot{\boldsymbol{x}}_{\boldsymbol{i}} - \mathbf{p})$, where $J\in\mathbb{R}^{n_i^L\times n_i^L}$ is a diagonal matrix with the diagonal entries $-\frac{1}{\mathrm{p}_i^k(t)\bar{e}_i^k(1+\bar{e}_i^k)}, k\in\mathcal{V}_i^L$, $\Gamma\in\mathbb{R}^{n_i^L\times nn_i}$ is a matrix with row vectors $\frac{\partial\rho^{\psi_i^k}(\boldsymbol{x_i},0)^T}{\partial\boldsymbol{x_i}}$, and $\mathbf{p}\in\mathbb{R}^{n_i^L}$ is a vector with entries $\bar{e}_i^k\dot{\mathrm{p}}_i^k(t)$. Replacing $\dot{\boldsymbol{x}}_{\boldsymbol{i}}$ by (2), we further obtain $\dot{V} = \varepsilon_i^T J(\Gamma\big(-(L_i\otimes I_n)\boldsymbol{x_i} + (C_i\otimes I_n)\boldsymbol{x} + (B_i\otimes I_n)\boldsymbol{u_i}\big) - \mathbf{p})$. We next discuss the diagonal entries of J. Since the performance function $\mathrm{p}_i^k(t) := (\mathrm{p}_{i,0}^k - \mathrm{p}_{i,\infty}^k)e^{-l_i^k t} + \mathrm{p}_{i,\infty}^k$ is strictly decreasing, we have $\mathrm{p}_{i,\infty}^k \le \mathrm{p}_i^k(t) \le \mathrm{p}_{i,0}^k$. Moreover, since $\bar{e}_i^k\in(-1,0)$, we achieve that $\frac{4}{\mathrm{p}_{i,0}^k} \le -\frac{1}{\mathrm{p}_{i,0}^k\bar{e}_i^k(1+\bar{e}_i^k)} \le -\frac{1}{\mathrm{p}_i^k\bar{e}_i^k(1+\bar{e}_i^k)} \le -\frac{1}{\mathrm{p}_{i,\infty}^k\bar{e}_i^k(1+\bar{e}_i^k)} < \infty$ holds $\forall\bar{e}_i^k\in\mathcal{D}$. Thus, each diagonal entry of J is always positive and bounded and we know that $\underline{\alpha}_1 I_{n_i^L} \le J \le \bar{\alpha}_1 I_{n_i^L}$ with bounded parameters $\underline{\alpha}_1,\bar{\alpha}_1 > 0$. We can further upper bound $\dot{V}$ by

$$\dot{V} \le \|\varepsilon_i\|\|J\|(\|\Gamma\|\| - (L_i\otimes I_n)\boldsymbol{x_i} + (C_i\otimes I_n)\boldsymbol{x}\| + \|\mathbf{p}\|) + \varepsilon_i^T J\Gamma(B_i\otimes I_n)\boldsymbol{u_i}.$$

Since $\frac{\partial\rho^{\psi_i^k}(\boldsymbol{x_i},0)^T}{\partial\boldsymbol{x_i}}, k\in\mathcal{V}_i^L$ and $(L_i\otimes I_n)\boldsymbol{x_i}$ are both bounded for all $t\in[0,\tau_{\max})$, and $(C_i\otimes I_n)\boldsymbol{x}$ is bounded due to Assumption 1, we can additionally upper bound the term $\|J\|(\|\Gamma\|\| - (L_i\otimes I_n)\boldsymbol{x_i} + (C_i\otimes I_n)\boldsymbol{x}\| + \|\mathbf{p}\|)$ by a positive constant α_2, which results in

$$\dot{V} \le \alpha_2\|\varepsilon_i\| + \varepsilon_i^T J\Gamma(B_i\otimes I_n)\boldsymbol{u_i}. \qquad (10)$$

Next, we replace the control law (9) in a stacked form $\boldsymbol{u_i}(\boldsymbol{x_i},t) = -\Theta\varepsilon_i$ similarly to Step 1 for the leaders in (10), and derive that

$$\dot{V} \le \alpha_2\|\varepsilon_i\| - \varepsilon_i^T J\Gamma(B_i\otimes I_n)\Theta\varepsilon_i. \qquad (11)$$

Recall that $\Gamma\in\mathbb{R}^{n_i^L\times nn_i}$ is a matrix with row vectors $\frac{\partial\rho^{\psi_i^k}(\boldsymbol{x_i},0)^T}{\partial\boldsymbol{x_i}}$ and $\Theta := diag\left(\frac{\partial\rho^{\psi_i^k}(\boldsymbol{x_i},0)}{\partial x_k}\right)\in\mathbb{R}^{nn_i^L\times n_i^L}$. Due to the structure of the B_i matrix and the fact that $\mathcal{V}_{\phi_i^k}\cap\mathcal{V}_i^L = \{k\}$, we have that $\Gamma(B_i\otimes I_n)\Theta = \Theta^T\Theta = diag\left(\|\frac{\partial\rho^{\psi_i^k}(\boldsymbol{x_i},0)}{\partial x_k}\|^2\right)$. According to Assumption 2 that $\frac{\partial\rho^{\psi_i^k}(\boldsymbol{x_i},0)}{\partial x_k}, k\in\mathcal{V}_i^L$ is a nonzero vector, we can obtain that $\Theta^T\Theta \ge \alpha_3 I_{n_i^L}$ for some parameter α_3 such that

$\|\frac{\partial \rho^{\psi_i^k}(\boldsymbol{x_i},0)}{\partial x_k}\|^2 \geq \alpha_3 > 0$ for all $k \in \mathcal{V}_i^L$. Recall J is also a diagonal matrix satisfying $\underline{\alpha}_1 I_{n_i^L} \leq J$, $\dot{V}$ can be further upper bounded by

$$\dot{V} \leq \alpha_2 \|\varepsilon_i\| - \underline{\alpha}_1 \alpha_3 \|\varepsilon_i\|^2. \tag{12}$$

From (12), we know that $\dot{V} \leq 0$ as long as $\|\varepsilon_i\| \geq \alpha_2(\underline{\alpha}_1\alpha_3)^{-1}$. Therefore, we can conclude that the transformed error is upper bounded by

$$\|\varepsilon_i\| \leq \epsilon_{i,\star} = \max\left\{\|\varepsilon_i(0)\|, \alpha_2(\underline{\alpha}_1\alpha_3)^{-1}\right\}, \tag{13}$$

$\forall t \in [0, \tau_{\max})$ (Khalil, 2002). Due to the boundedness of $\|\varepsilon_i\|$ in $t \in [0, \tau_{\max})$, we can conclude the boundedness of each ε_i^k by a constant $\epsilon_{i,\star}^k$. Then, based on properties of T_i^k, we can restrict $\bar{e}_i^k$ in a compact subset of $\mathcal{D}$ as

$$\bar{e}_i^k(\boldsymbol{x_i}, t) \in [\underline{\delta}_i^k, \bar{\delta}_i^k] \triangleq [T_i^{k^{-1}}(-\epsilon_{i,\star}^k), T_i^{k^{-1}}(\epsilon_{i,\star}^k)] \subset \mathcal{D}, \tag{14}$$

where $T_i^{k^{-1}}$ is the inverse function of the transformed function T_i^k; such inverse function always exists since T_i^k is a smooth and strictly increasing function.

Step 3. Finally, we prove that $\tau_{\max}$ can be extended to ∞. According to (14), we know that $\bar{e}_i^k(\boldsymbol{x_i}, t) \in \mathcal{D}_k, \forall t \in [0, \tau_{\max})$, where $\mathcal{D}_k = [\underline{\delta}_i^k, \bar{\delta}_i^k]$. Hence, $\mathcal{D}_k \subset \mathcal{D}$ is a nonempty and compact subset of $\mathcal{D}$ and it can be concluded that $\bar{e}_i^k(\boldsymbol{x_i}, t) \in \mathcal{D}_k, \forall t \in [0, \tau_{\max})$. Now assume that $\tau_{\max} < \infty$, and according to Proposition C.3.6 of (Sontag, 2013), there then exists a $t' \in [0, \tau_{\max})$ such that $\bar{e}_i^k(\boldsymbol{x_i}, t) \notin \mathcal{D}_k$, which leads to a contradiction. Hence, we conclude that $\tau_{\max}$ is extended to ∞, that is $\bar{e}_i^k(\boldsymbol{x_i}, t) \in \mathcal{D}_k \subset \mathcal{D}, \forall t \geq 0$. Therefore ε_i^k is bounded for all $t \geq 0$ and the boundedness of the transformed error ε_i^k implies that $\rho^{\psi_i^k}(\boldsymbol{x_i}, 0)$ satisfies (6) for all $t \geq 0$. We can conclude that the satisfaction of (6) is guaranteed when applying the control strategy (9).

Part 2: In this part, we will prove that the satisfaction of (6) enforces that $0 < \rho^{\phi_i^k}(\boldsymbol{x_i}, 0) < \rho_i^k$ holds for all $k \in \mathcal{V}_i^L$ under the designed funnel parameters, thus $\boldsymbol{x_i}$ locally satisfies ϕ_i. In general, the choices of the parameters should guarantee the initial condition, i.e., $\rho^{\psi_i^k}(\boldsymbol{x_i}(0), 0)$ is within the funnel (6). Moreover, $-\mathrm{p}_i^k(t_{\star,i}) + \rho_i^k \geq 0$ should hold in order to enforce the satisfaction of the STL formula (5b) by prescribing the transient behavior of the funnel. For $t_{\star,i} = 0$, $\mathrm{p}_{i,0}^k \in (\rho_i^k - \rho^{\psi_i^k}(\boldsymbol{x_i}(0), 0), \rho_i^k]$ will ensure that $\mathrm{p}_{i,0}^k > \rho_i^k - \rho^{\psi_i^k}(\boldsymbol{x_i}(0), 0)$, which is equivalent to $-\mathrm{p}_{i,0}^k + \rho_i^k < \rho^{\psi_i^k}(\boldsymbol{x_i}(0), 0)$, and by further choosing ρ_i^k such that $\rho^{\psi_i^k}(\boldsymbol{x_i}(0), 0) < \rho_i^k$, the initial condition is satisfied. Moreover, $\mathrm{p}_{i,0}^k \leq \rho_i^k$ means that $-\mathrm{p}_i^k(t_{\star,i}) + \rho_i^k = -\mathrm{p}_{i,0}^k + \rho_i^k \geq 0$. Next, we use the fact that the function $-\mathrm{p}_i^k(t) + \rho_i^k$ is strictly increasing in order to conclude on the satisfaction of the temporal STL formulas as in (5b). For $t_{\star,i} > 0$, we can check similarly that the initial condition holds, i.e., the initial condition $\rho^{\psi_i^k}(\boldsymbol{x_i}(0), 0)$ is within the funnel (6). In addition $l_i^k = -\frac{1}{t_{\star,i}} \ln(\frac{\rho_i^k - \mathrm{p}_{i,\infty}^k}{\mathrm{p}_{i,0}^k - \mathrm{p}_{i,\infty}^k})$ results in $-\mathrm{p}_i^k(t_{\star,i}) + \rho_i^k = -(\mathrm{p}_{i,0}^k - \mathrm{p}_{i,\infty}^k)e^{-l_i^k t_{\star,i}} - \mathrm{p}_{i,\infty}^k + \rho_i^k = 0$, and thus we can also achieve the satisfaction of the temporal STL formulas as in (5b) according to the fact that $-\mathrm{p}_i^k(t) + \rho_i^k$ is strictly increasing. Thus, we can conclude that the control strategy (9) guarantees that $0 < \rho^{\phi_i^k}(\boldsymbol{x_i}, 0) < \rho_i^k$ holds for each ϕ_i^k as in (5b) by appropriately choosing the parameters $\mathrm{p}_{i,0}^k, \mathrm{p}_{i,\infty}^k$ and l_i^k as above. Therefore, $\boldsymbol{x_i}$ locally satisfies ϕ_i. Then, since $\forall i \in \mathcal{I}$, $\boldsymbol{x_i}$ locally satisfies ϕ_i for all subsystems $\mathcal{S}_i, i \in \mathcal{I}$, we can further conclude that the closed-loop trajectory $\boldsymbol{x}$ globally satisfies $\{\phi_1, \ldots, \phi_M\}$. □

Remark 1. Assumption 2 mentions that $\frac{\partial \rho^{\psi_i^k}(\boldsymbol{x_i},0)}{\partial x_k}$ is a nonzero vector, which is used to avoid local optima that may cause infeasibility issues. This assumption includes some specific classes of robustness functions, e.g., concave functions, and we can design the funnel parameters such that local optima are avoided to guarantee Assumption 2. It also requires that the leader $k \in \mathcal{V}_i^L$ is involved in the task ϕ_i^k, which is indicated by the fact that $\mathcal{V}_{\phi_i^k} \cap \mathcal{V}_i^L = \{k\}$.

4. SIMULATION

In this section, we consider a simulation example which is composed of three subsystems $\mathcal{S}_1$, $\mathcal{S}_2$ and $\mathcal{S}_3$ as shown in Fig. 1, where grey and white nodes represent leaders and followers, respectively. The dynamic couplings between different subsystems come from the edges e_1, e_2, e_3 that connect different subsystems. The agents are initialized as $x_1 = [0,2]^T, x_2 = [4,2]^T, x_3 = [2,0]^T, x_4 = [6,0]^T, x_5 = [10,0]^T, x_6 = [14,0]^T, x_7 = [8,2]^T, x_8 = [12,2]^T, x_9 = [16,0]^T, x_{10} = [20,0]^T, x_{10} = [18,2]^T$. Each subsystem is assigned with a local STL formula (5b). The task for $\mathcal{S}_1$ is $\phi_1 = \mathrm{F}_{[0,2]}\mathrm{G}_{[1,3]}\psi_1$, where $\psi_1 = \psi_{1,1} \wedge \psi_{1,2}$ with $\psi_{1,1} = (||x_3 - x_1||_2 < 2)$, $\psi_{1,2} = (||x_3 - x_2||_2 < 2)$. The task for $\mathcal{S}_2$ is $\phi_2 = \mathrm{G}_{[4,7]}\psi_2$, where $\psi_2 = \psi_{2,1} \wedge \psi_{2,2} \wedge \psi_{2,3} \wedge \psi_{2,4}$ with $\psi_{2,1} = (||x_7 - x_4||_2 < 2)$, $\psi_{2,2} = (||x_7 - x_5||_2 < 2)$, $\psi_{2,3} = (||x_8 - x_5||_2 < 2)$, $\psi_{2,4} = (||x_8 - x_6||_2 < 2)$. ϕ_2 can be decomposed to each leader as $\phi_2^7 = \mathrm{G}_{[4,7]}\psi_2^7 = \mathrm{G}_{[4,7]}(\psi_{2,1} \wedge \psi_{2,2})$ for leader 7, and $\phi_2^8 = \mathrm{G}_{[4,7]}\psi_2^8 = \mathrm{G}_{[4,7]}(\psi_{2,3} \wedge \psi_{2,4})$ for leader 8 according to task dependency. The task for $\mathcal{S}_3$ is $\phi_3 = \mathrm{F}_{[0,6]}\psi_3$, where $\psi_3 = \psi_{3,1} \wedge \psi_{3,2}$ with $\psi_{3,1} = (||x_{11} - [16,2]^T||_2 < 1)$, $\psi_{3,2} = (||x_{11} - x_{10}||_2 < 2)$. We can observe that ϕ_i only depends on the agents in $\mathcal{S}_i, i \in \{1,2,3\}$. $\mathcal{S}_2$ has two leaders such that their tasks ϕ_2^7 and ϕ_2^8 are coupled.

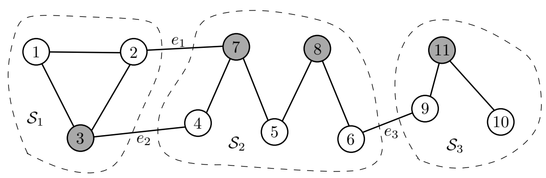

Fig. 1. Leader-follower multi-agent system with 3 subsystems $\mathcal{S}_1$, $\mathcal{S}_2$ and $\mathcal{S}_3$.

Next, we design the funnels for each task according to Theorem 1. The performance function for ϕ_1 is designed as $\mathrm{p}_1^3(t) = 7e^{-l_1^3 t} + 1$ with $t_{\star,1} = 1, \rho_1^3 = 2, \mathrm{p}_{0,1}^3 = 8, \mathrm{p}_{\infty,1}^3 = 1$ and $l_1^3 = \ln(7)$. Regarding subsystem $\mathcal{S}_2$ which has two leaders, the performance function for ϕ_2^7 is designed as $\mathrm{p}_2^7(t) = 14e^{-l_2^7 t} + 2$ with $t_{\star,2} = 4, \rho_2^7 = 2, \mathrm{p}_{0,2}^7 = 16, \mathrm{p}_{\infty,2}^7 = 2$ and $l_2^7 = 0.25\ln(7)$, and these parameters are the same for ϕ_2^8. The performance function for ϕ_3 is designed as $\mathrm{p}_3^{11}(t) = 7e^{-l_3^{11} t} + 1$ with $t_{\star,3} = 5, \rho_3^{11} = 2, \mathrm{p}_{0,3}^{11} = 8, \mathrm{p}_{\infty,3}^{11} = 1$ and $l_3^{11} = 0.2\ln(7)$. The simulation results when applying the control law (9) are shown in Fig. 2 and 3. Fig. 2 shows the evolution of the agents, where the leaders and followers are represented by solid and hollow circles, respectively. The initial formation is in black, while the final formation is in blue. In Fig. 3, we plot the evolution of the robustness functions (red curve) against the corresponding funnels (black curve). We can see that the performance functions enforce the satisfaction of the corresponding tasks by prescribing the temporal behavior of the lower bound of the funnels. Since all the robustness functions evolve within the corresponding funnels, we can conclude that the set of the STL tasks $\phi_i, i \in \{1, 2, 3\}$ is satisfied by applying the control law (9).

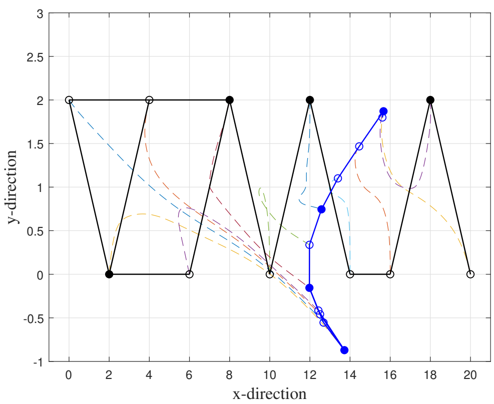

Fig. 2. Formation control of the agents under STL tasks ϕ_1, ϕ_2 and ϕ_3.

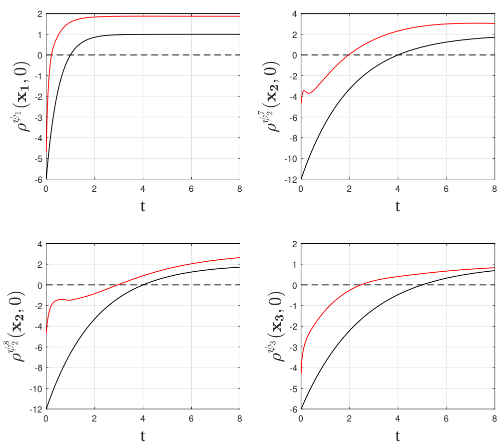

Fig. 3. Evolution of the robustness functions against the funnels (black curves).

5. CONCLUSIONS

In this paper, we have investigated the problem of cooperative control of leader-follower multi-agent systems under local signal temporal logic specifications. The overall leader-follower multi-agent system is composed of several leader-follower subsystems with coupled dynamics and each leader is assigned with a local STL specification. Under a local feasibility assumption, funnel-based distributed control strategies have been proposed for the leaders to enforce the satisfaction of STL formulas by appropriately designing the funnel parameters such that the local STL specifications are achieved, which further implies the global satisfaction. Future work includes considering more general class of STL formulas and deriving conditions on the STL tasks and leader-follower graph topology such that local feasibility is guaranteed.

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