





Guarding, Searching and Pursuing Evaders using Multiagent Systems

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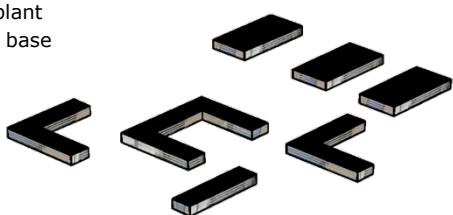

Today's topics

- **Cooperative guarding**
 - Static guards
- **Cooperative search**
 - Static targets
- **Cooperative pursuit evasion**
 - Moving targets and guards

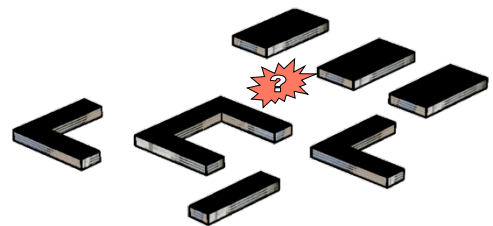





Example Scenario

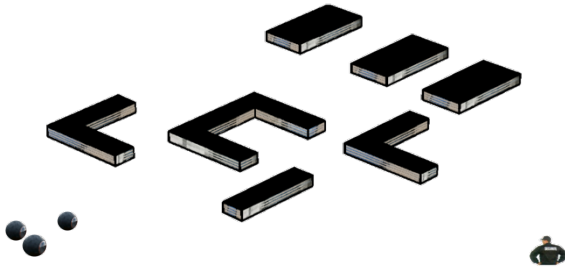

- Airport
- Power plant
- Military base
- Port
- Factory
- ...

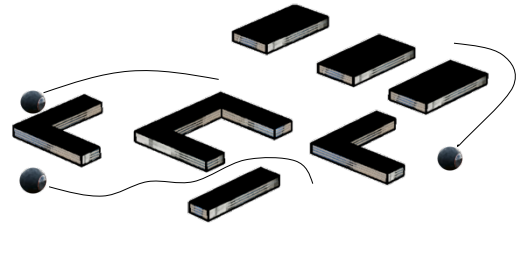
An intruder alarm is set of ...

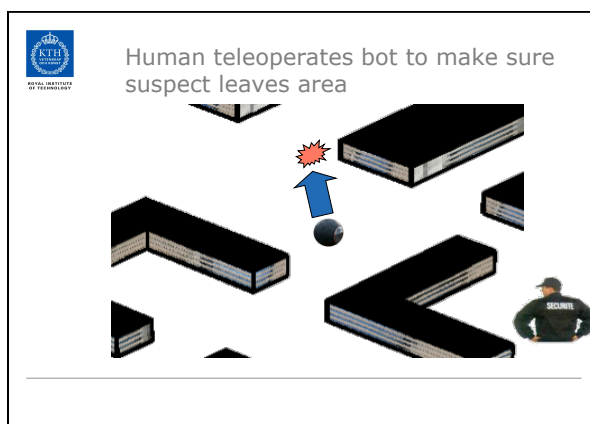
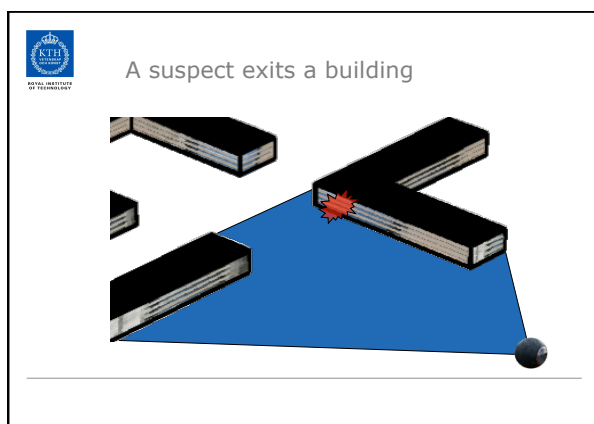
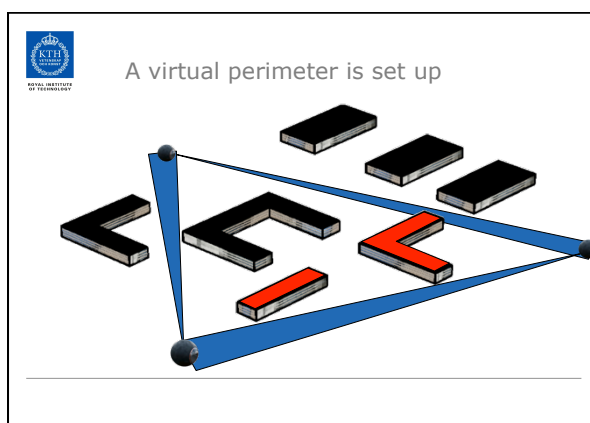
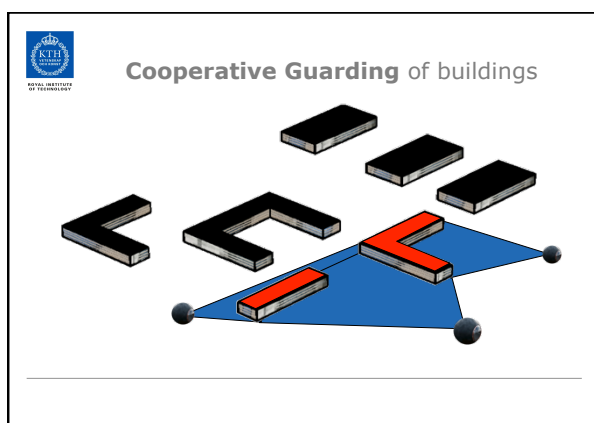
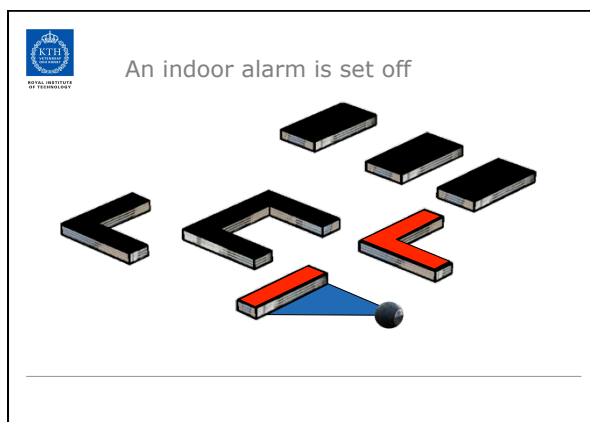
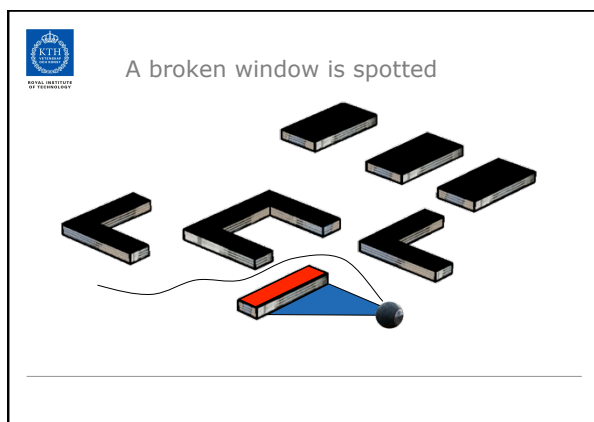




Multiagent surveillance bots are remotely activated

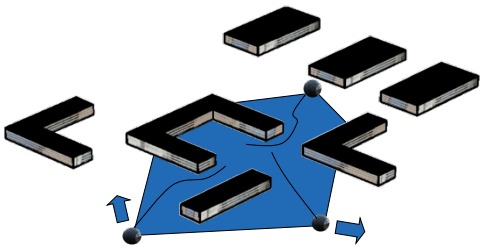

Cooperative search of area



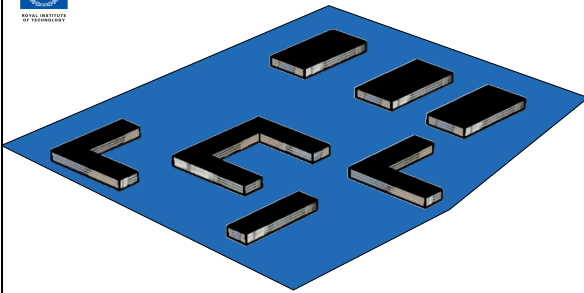





Area is searched for remaining intruders (**pursuit evasion problem**)





All is clear





Today's topics


- **Cooperative guarding**
 - Static guards
- **Cooperative search**
 - Static targets
- **Cooperative pursuit evasion**
 - Moving targets and guards
- This field is very broad
 - Overview of problems, results and tools

But first: Bullo



- Beautiful theory
- Relevant application

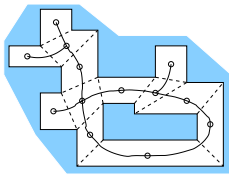


Common theme: Discretizing the search space


- Partitioning search space into convex sets is often useful
- Create a graph
 - Set \leftrightarrow vertex
 - Neighbor \leftrightarrow edge

Some naïve solutions:

- Guarding
 - Put a guard in each vertex
- Search
 - Travelling salesman




Can we improve on these conservative solutions?





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 **Coordinated Guarding/Coverage**


- Applications: Art gallery, Industrial Area, Camera positioning
- Possible objectives:
 - Min no of cameras,
 - Max coverage with N cameras,
 - Weighted coverage
- Environment: 2D/3D




 **Bounds on number of Guards**

- The **General Art Gallery Problem**: What is the smallest number of guards needed to cover **any** polygon with n vertices and h holes.
- For h=0, Chvatal (1975) proved bound: Floor(n/3)
- Hoffmann (1991) proved bound: Floor ((n+h)/3)



 **Minimize number of guards (3D etc)**


- Problem: (Min number of guards)
- Problem (Minimum set cover) Let $E = \{e_1, \dots, e_n\}$ be a finite set of elements, and let $S = \{s_1, \dots, s_m\}$ be a collection of subsets of E, i.e. $s_j \subseteq E$. The problem minimum set cover is the problem of finding a minimum subset $S' \subseteq S$ such that every elements $e_i \in E$ belongs to at least one subset in S' . We say that E is covered by S' .
 - NP-hard
 - Greedy algorithm performs well, Eidenbenz (2002)



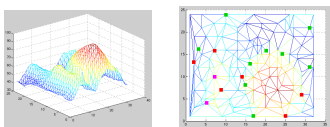
Movie: Guarding with resolution constraints



 **Guarding with resolution constraints**



 **Minimizing number of guards (3D etc)**

- Marangoni (2000)
 - Triangulation of 3D environment
 - Vertex coloring to find subset
 - Visibility computation to get candidates
- Efrat (2002) randomized search instead of the greedy



 **Non-line-of-sight guarding** 

- Distribute agents p_i to
- Minimize Expected squared distance
 - From random event
 - To nearest agent

$$\mathcal{H}(P, \mathcal{W}) = \sum_{i=1}^n \int_{W_i} \|q - p_i\|^2 \phi(q) dq$$

Non-line-of-sight guarding

$$\mathcal{H}(P, \mathcal{W}) = \sum_{i=1}^n \int_{W_i} \|q - p_i\|^2 \phi(q) dq$$

Non-line-of-sight guarding: Discretized

- Distribute agents p_i to
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$$\mathcal{H}(P, \mathcal{W}) = \sum_{i=1}^n \int_{W_i} d(q, p_i) \phi(q) dq$$

Non-line-of-sight guarding: Discretized

Further reading on Guarding

- V. Chvatal. A Combinatorial Theorem in Plane Geometry. *Journal of Combinatorial Theory Series B*, 18:39–41, 1975.
- F. Hoffmann, M. Kaufmann, and K. Kriegel. The Art Gallery Theorem for Polygons With Holes. *Proceedings of the 32nd Annual Symposium on Foundations of Computer Science*, pages 39–48, 1991.
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- A. Efrat and S. Har-Peled. Guarding Galleries and Terrains. *Proceedings of the IFIP 17th World Computer Congress-TC1 Stream*, 2002.
- U. Nilsson, P. Ögren, and I. Thunberg. "Optimal positioning of surveillance UGVs," presented at the 2008 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2008), pp. 2539–2544.
- Y. Wang, M. Colledanchise, A. Marzotto, and P. Ögren. A Distributed Convergent Solution to the Ambulance Positioning Problem on a Streetmap Graph. *World Congress of the International Federation of Automatic Control (IFAC)*, 2014
- W.R. Franklin. Siting Observers on Terrain. *Symposium on Spatial Data Handling*, Ottawa, pages 109–120, 2002.

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Cooperative Search (Background: TSP)

- Travelling Salesperson Problem (TSP)
- Variations
 - Multi-TSP
 - Metric TSP
 - Vehicle routing problem
 - Max capacity
 - Time windows
 - ...

13000 cities. Applegate, Bixby, Cook and Chvatal

"It involves ideas from polyhedral combinatorics and combinatorial optimization, integer and linear programming, computer science data structures and algorithms, parallel computing, software engineering, numerical analysis, graph theory, and more."

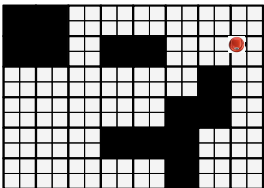
Cooperative Search

- Sensor range gives two cases
- Range \ll environment size
 - Lawn mowing
 - Vacuum cleaning
 - Seeding, harvesting
 - Mine clearing
 - UAV search
- Range similar to environment size
 - UGV search



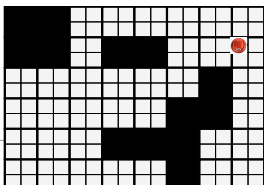
Cooperative search (short range)

- Search the area in minimum time



Spanning Tree Coverage (Search)

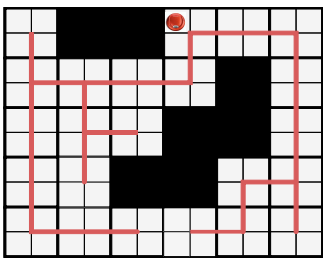
1. Cover the search area with squares 4x the sensor footprint
2. Create a graph $G = (V, E)$
3. V - Centers of each large square
4. E - Adjacent squares
5. Find a spanning tree T
6. Move clockwise around T



Running the algorithm

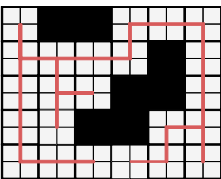
The graph G

The tree T



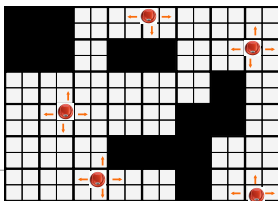
Algorithm Properties

1. The algorithm completes the search
2. The algorithm completes the search in minimum time



Multi-robot search

- Given N robots
- Can we complete the task in $1/N$ of the time?



Multi agent search... (extension)

- Same as single agent algorithms
- Agents stop when reaching already covered square

Tree 1: 31 steps Tree 2: 15 steps

Multi agent search... (extension)

- What is the optimal spanning tree?
- Optimal Multi-agent search is NP-hard
- P-time Heuristics have been proposed - (Agmon and Kaminka 2006)

Seabed mapping (search)

- Given **polygon** of GPS coordinates
- No map
- Plan and execute a path to **cover the whole seabed**
 - **mapping depth**
 - **mines**
- Avoid
 - getting grounded
 - other moving vessels
- Use a bare minimum of sensors
 - GPS
 - downward looking sonars
 - (transponder)

Seabed mapping (search)

- Classical Search Patterns
 - Scanning
 - Spiral
- Disadvantages
 - Land...
- Solution
 - Reactive Greedy approach

Reactive Greedy approach

Cooperative Search

- Sensor range gives two cases
 - Range \ll environment size
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Cooperative Search (long sensor range)

Possible approaches:

- Use Guard positions and solve m-TSP
- Discretize to a graph and solve m-TSP
- Use convex cover and solve m-TSP ...

Example of Cooperative search (Anisi 2010)

How?

Cooperative Search

- How can we make search less conservative?
- Replace partition with overlapping convex cover

```

    graph TD
      A[Create Convex Cover Sets] --> B[Perform Tabu Search on Order of Set Visitation]
      B --> C[Solve Shortest Path Problem to find Paths Visiting Sets]
      C --> B
  
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Cooperative Search

- How can we make search less conservative?
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Further reading on cooperative search

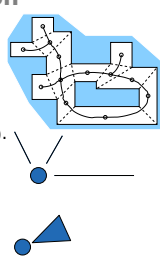
- E. Frazzoli and F. Bullo. Decentralized algorithms for vehicle routing in a stochastic time-varying environment. In Proc. of the 43rd IEEE Conference on Decision and Control, CDC, 2004.
- Maria John, David Panton, and Kevin White. Mission planning for regional surveillance. *Annals of Operations Research*, 108:157–173, Nov. 2001.
- Shuzhi Sam Ge and Cheng-heng Fua. Complete Multi-Robot Coverage of Unknown Environments with Minimum Repeated Coverage. In IEEE International Conference on Robotics and Automation, Barcelona, Spain, pages 727–732, April 2005.
- N Agmon, N Hazon, GA Kaminka, Constructing spanning trees for efficient multi-robot coverage, IEEE International Conference on Robotics and Automation (ICRA), 2006
- I. I. Hussein and Stipanovic, "Effective Coverage Control using Dynamic Sensor Networks," presented at the Decision and Control, 2006 45th IEEE Conference on, 2006.
- D. A. Anisi, P. Ogren, and X. Hu, "Cooperative Minimum Time Surveillance With Multiple Ground Vehicles," *Automatic Control, IEEE Transactions on*, vol. 55, no. 12, pp. 2679–2691, 2010.

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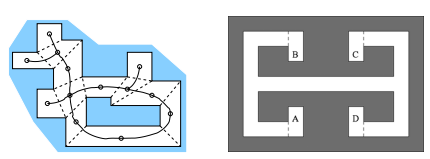
Cooperative Pursuit Evasion

- First introduced by Parsons (1976)
 - Problem on a graph
 - Multiple searchers
- A continuous version: Suzuki et al. (1992).
 - simple polygon
 - single searcher (k-searcher)
- Limited field of view: Gerkey et al. (2006)
 - capability of a robot with a camera
 - (phi-searcher)



Randomized Pursuit Evasion


- Randomized strategy: Isler et al. (2005).




- By repeating a randomized strategy, capture probability can be made arbitrarily high (if simply connected)

Cooperative Pursuit Evasion

- Efrat et al. (2000) consider chains of searchers in simple polygons

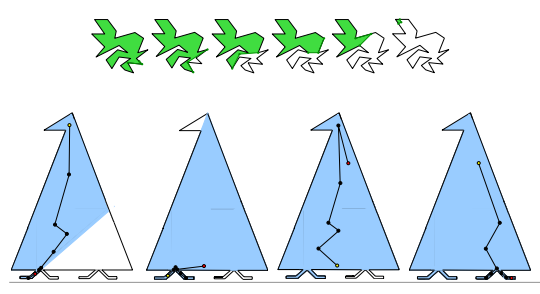


- Hollinger et al. (2007) a probabilistic approach inspired by *Markov Decision processes (MDP)* and *partially observable MDP (POMDP)*



- Thunberg (2011) MILP/MPC formulation ...

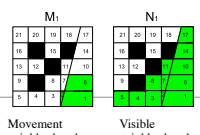
Recontamination may be needed...



Greedy fails, planning horizon needed ...

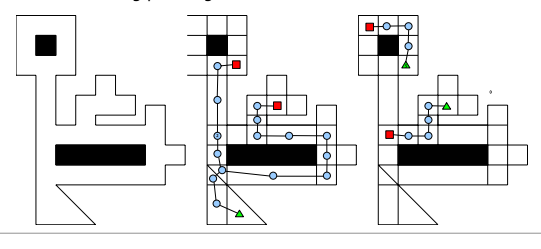
MILP/MPC approach to Pursuit Evasion


- Classical model: p_i pursuer location
- MILP model:
 - $p_{i,t}$ a pursuer at location i and time t
 - Label nodes as (cleared/seen/contaminated)
 - MILP constraints Capture
 - Agent motion
 - Contaminated area dynamics
 - Minimize Contaminated area



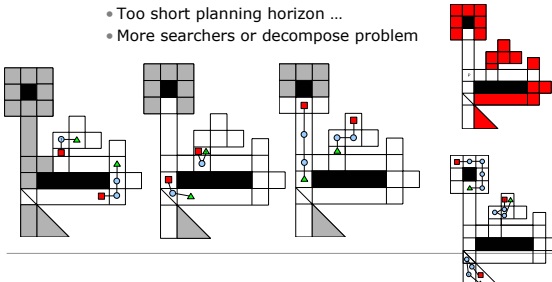
MILP/MPC example execution


- Long planning horizon ...



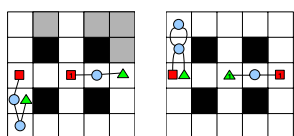
 MILP approach to Pursuit Evasion

- Too short planning horizon ...
- More searchers or decompose problem




 MILP: Additional constraint

- Additional constraint can be added:
 - **Line of sight** every 4th timestep



 MILP/MPC Pros and Cons


- Handles recontamination (planning horizon)
- Handles connectivity constraints
- Scales poorly with planning horizon length
- Scales poorly with environment size
 - Add static "guard" to partition environment
- Scales well with number of agents

 Further reading on pursuit evasion

- T. D. Parsons. Theory and applications of graphs, Lecture Notes in Mathematics, chapter Pursuit-evasion in a graph, pages 426–441. Springer, 1976.
- I. Suzuki and M. Yamashita. Searching for a mobile intruder in a polygonal region. *SIAM Journal on Computing*, 21(5), 1992.
- V. Isler, S. Kannan, and S. Khanna. Randomized pursuit-evasion in a polygonal environment. *IEEE Transactions on Robotics*, 21(5), 2005.
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 Thank you ...