# FEL3330 Networked and Multi-agent Control Systems
# Lecture 10:
# Communication constraints: quantization, event-triggered sampling and control.

October 3, 2016

# Today's lecture

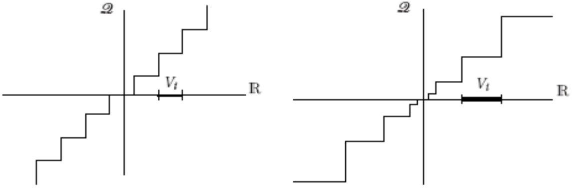
- Quantized Consensus
- Event-triggered multi-agent control

# Quantized consensus

- Each agent has quantized measurements of the form $q(x_i - x_j)$, $q(\cdot)$ quantization function
- Can be extended to multiple dimensions
- Uniform and logarithmic quantizers

# Quantizer types

- Uniform quantizer: $|q_u(a) - a| \leq \delta_u, \forall a \in \mathbb{R}$
- Logarithmic quantizer: $|q_l(a) - a| \leq \delta_l |a|, \forall a \in \mathbb{R}$




(a) Uniform quantizer. The scalar case.

(b) Logarithmic quantizer. The scalar case.

# Closed-loop system

- $\dot{x}_i = u_i = -\sum_{j \in \mathcal{N}_i} a_{ij} q\left(x_i - x_j\right)$
- Stack vector form: $\dot{\bar{x}} = -B^T B \Gamma q\left(\bar{x}\right)$
- We use the positive definiteness of $B^T B$ when $G$ is a tree graph

# Closed-loop system

Theorem: Assume $G$ is a tree.

- In the case of a uniform quantizer, the system converges to the set $\{x \in \mathbb{R}^N : |x_i - x_j| \leq \frac{\delta_u}{2}, \{i,j\} \in \mathcal{E}\}$.
- In the case of a logarithmic quantizer, the system asymptotically converges to $\bar{x} = 0$, for all $\delta_l > 0$.

Use $V = \bar{x}^T M^{-1} \bar{x}$, $M = B^T B$ as a candidate Lyapunov function

# Quantized consensus

- Results extended to time varying communication topolgies (Guo and DVD, Automatica 2013)
- Additional properties of the (common) Lyapunov function $V = x^T H x$, $H = B(B^T B)^{-1} B^T$ are exploited

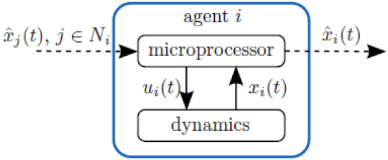
# Event-triggered sampling

- Time-triggered sampling at pre-specified instants: does not take into account optimal resource usage
- A strategy considering better resource usage: event-triggered control
- Actuation updates in asynchronous manner
- Application to multi-agent systems

# Event-triggered sampling

- Each agent $i$ broadcast their state at discrete time instants $t_0^i, t_1^i, \ldots$
- $\hat{x}_i(t) = x_i(t_k^i)$: latest update for agent $i$
- Event-triggered control law: $u_i(t) = -\sum_{j \in N_i} (\hat{x}_i(t) - \hat{x}_j(t))$
- Measurement error: $e_i(t) = \hat{x}_i(t) - x_i(t) = x_i(t_k^i) - x_i(t), t \in [t_k^i, t_{k+1}^i)$
- Next slides ack to Georg Seyboth

# Event based scheduling of measurement broadcasts




**Event-based broadcasting**

$$\hat{x}_i(t) = x_i(t_k^i),\ t \in [t_k^i, t_{k+1}^i[$$

$$0 \le t_0^i \le t_1^i \le t_2^i \le \cdots$$

- Consensus protocol $u_i(t) = -\sum_{i \in N_i} (\hat{x}_i(t) - \hat{x}_j(t))$
- Measurement errors $e_i(t) = \hat{x}_i(t) - x_i(t)$
- Closed-loop $\dot{x}(t) = -L\hat{x}(t) = -L(x(t) + e(t))$
- Disagreement $\delta(t) = x(t) - a\mathbf{1}; \quad \mathbf{1}^T \delta(t) \equiv 0$

# Trigger mechanism-Problem statement

- Define triger functions $f_i(\cdot)$ and triger when

$$f_i(t, x_i(t), \hat{x}_i(t), \hat{x}_{j_1}(t), \ldots, \hat{x}_{j_{N_i}}(t)) > 0$$

- Defines sequence of events $t^i_{k+1} = \inf\{t > t^i_k : f_i(t) > 0\}$
- Problem statement: Find suitable $f_i(\cdot)$ such that
  - no Zeno bevior occurs
  - desired convergence properties are maintained
  - inter-agent communication is reduced as much as possible
- Intuition

$$f_i(e_i(t)) = |e_i(t)| - c_0 \Longrightarrow |e_i(t)| \leq c_0, \forall t \geq 0$$

# Constant thresholds

$$\dot{x}(t) = u(t) \qquad u(t) = -L\hat{x}(t) \tag{1}$$

### Theorem

*Consider system* (1) *with undirected and connced graph* $G$*. Suppose that*

$$f_i(e_i(t)) = |e_i(t)| - c_0$$

*with* $c_0 > 0$*. Then, for all* $x_0 \in \mathbb{R}^N$*, the system does not exhibit Zeno behavior and for* $t \to \infty$*,*

$$||\delta(t)|| \leq \frac{\lambda_N(G)}{\lambda_2(G)} \sqrt{N} c_0$$

Proof ideas:

- Analytical solution of disagreement dynamics yields

$$||\delta(t)|| \leq e^{-\lambda_2(G)t} ||\delta(0)|| + \lambda_N(G) \int_0^t e^{-\lambda_2(G)(t-s)} ||e(s)|| ds$$

- Compute lower bound $\tau$ on the inter-event intervals

# Exponentially decreasing thresholds

Can we achieve asymptotic convergence?

## Theorem

*Consider system* (1) *with undirected and connected graph* $G$*. Suppose that*

$$f_i(e_i(t)) = |e_i(t)| - c_1 e^{-at}$$

*with* $c_1 > 0$ *and* $0 < a < \lambda_2(G)$*. Then, for all* $x_0 \in \mathbb{R}^N$*, the system does not exhibit Zeno behavior and for* $t \to \infty$*,*

$$||\delta(t)|| \to 0$$

Intuition:

- $\lambda_2(G)$ specifies the degree of convrgence for $\delta(t)$
- $a < \lambda_2(G)$ implies that the threshold $c_1 e^{-at}$ decreases slower

Remark: Vanishing thresholds cause problems (measurement noise, numerics)

# Exponentially decreasing thresholds with offset

Combine the advantages

### Theorem

*Consider system* (1) *with undirected and connected graph $G$. Suppose that*

$$f_i(e_i(t)) = |e_i(t)| - (c_0 + c_1 e^{-at})$$

*with $c_1, c_2 \geq 0$, at least one positive, and $0 < a < \lambda_2(G)$. Then, for all $x_0 \in \mathbb{R}^N$, the system does not exhibit Zeno behavior and for $t \to \infty$,*

$$||\delta(t)|| \leq \frac{\lambda_N(G)}{\lambda_2(G)} \sqrt{N} c_0$$

Remarks:

- Size of final region is tunable
- Larger thresholds for small $t$ increase inter-event times
- Problems due to noise or numerics are avoided

# Next lecture

Guest lecture: Prof. Petter Ögren