FEL3330 Networked and Multi-agent Control Systems
Lecture 9: Sensing constraints 2

September 29, 2016

Today's lecture

- Sensing constraints
- Aggregation
- Dispersion
- Collision avoidance

Limited sensing constraints

- Usually modelled as a circular region of radius d_i around each agent i
- $N_i = \{j \in \mathcal{N}, j \neq i : \|q_i - q_j\| \leq d_i\}$
- If $d_i = d$ for all i, then $j \in N_i \Leftrightarrow i \in N_j$: undirected graph
- If $d_i \neq d_j$ in general then directed graph

Modelling of sensing limitations

- Introduce a constant term whenever $\beta_{ij} = ||q_i - q_j||^2 > d^2$
- Example potential:

$$\gamma_{ij}(\beta_{ij}) = \begin{cases} \frac{1}{2}\beta_{ij},\ 0 \leq \beta_{ij} \leq c^2 \\ \phi(\beta_{ij}),\ c^2 \leq \beta_{ij} \leq d^2 \\ h,\ d^2 \leq \beta_{ij} \end{cases}$$

- Control law contains terms of the form $\frac{\partial \gamma_{ij}}{\partial q_i} = 2\rho_{ij}(q_i - q_j)$ where $\rho_{ij} \triangleq \frac{\partial \gamma_{ij}}{\partial \beta_{ij}}$ is zero whenever j is outside the sensing zone of i.

Swarms

- Analysis of group behavior when defining only local behaviors (no precise global objective)
- Two examples: dispersion and aggregation in mobile networks
- Another example is opinion dynamics

Dispersion

- Can be seen as the inverse of agreement or as a form of disagreement in 2D
- Local control law: $u_i = -\sum_{j \in N_i} \left(-\frac{1}{\gamma_{ij}^2} \right) \frac{\partial \gamma_{ij}}{\partial q_i} = \sum_{j \in N_i} \frac{2\rho_{ij}}{\gamma_{ij}^2}(q_i - q_j)$
- Since $\rho_{ij} = 0$ for $j \notin N_i$, the control can be rewritten as

$$u_i = \sum_{j \neq i} \frac{2\rho_{ij}}{\gamma_{ij}^2}(q_i - q_j)$$

Results

- The collision avoidance set $\mathcal{I}(q) = \{q | \, \|q_i - q_j\| > 0, \forall i, j \in \mathcal{N}, i \neq j\}$ is invariant for the trajectories of the closed loop system.
- Main result: Agents converge to a configuration of the form

$$\|q_i - q_j\| \geq d, \forall i, j \in \mathcal{N}, i \neq j.$$

- Each agent occupies a disjoint disc of radius $d/2$.

Extensions and issues

- Extension to a bounded workspace.
- Open issues: how can we obtain the "optimal final configuration"?
- Is the collision avoidance property necessary for convergence proof?

Swarm Aggregation

- Objective 1: maintenance of edges with initial neighbors (cf. connectivity maintenance lecture)
- Objective 2: collision avoidance

Aggregation objective

- $N_i = \{j \in \mathcal{N}, j \neq i : \|q_i(0) - q_j(0)\| \leq d\}$
- Objective: keep all initial neighbors within distance d from one another
- We use the connectivity maintenance potential

Connectivity maintenance potential

- Define W_{ij} between i and $j \in N_i$
- $W_{ij} = W_{ij}\left(\|x_i - x_j\|^2\right) = W_{ij}\left(\beta_{ij}\right)$,
- W_{ij} is defined on $\beta_{ij} \in [0, d^2)$,
- $W_{ij} \rightarrow \infty$ whenever $\beta_{ij} \rightarrow d^2$,
- it is C^1 for $\beta_{ij} \in [0, d^2)$ and
- the term $p_{ij} \stackrel{\Delta}{=} \frac{\partial W_{ij}}{\partial \beta_{ij}}$ satisfies $p_{ij} > 0$ for $0 \leq \beta_{ij} < d^2$.

Collision avoidance objective

- Desired property: the collision avoidance set $\mathcal{I}(q) = \{q | \, \|q_i - q_j\| > 0, \forall i, j \in \mathcal{N}, i \neq j\}$ being invariant for the trajectories of the closed loop system.
- Agent i takes into account the agents within $M_i = \{j \in \mathcal{N}, j \neq i : \|q_i - q_j\| \leq d_1\}$ for the collision avoidance objective.

Collision avoidance potential

- $V_{ij} = V_{ij}(\beta_{ij})$, where $\beta_{ij} = \|q_i - q_j\|^2$
- $V_{ij} \to \infty$ whenever $\beta_{ij} \to 0$.
- It is everywhere continuously differentiable.
- $\frac{\partial V_{ij}}{\partial q_i} = 0$ and $V_{ij} = 0$ whenever $\|q_i - q_j\| > d_1$

Collision avoidance potential

- We have $\sum_{j \in M_i} \frac{\partial V_{ij}}{\partial \boldsymbol{q}_i} = \sum_{j \neq i} \frac{\partial V_{ij}}{\partial \boldsymbol{q}_i}$
- $\rho_{ij} \triangleq \frac{\partial V_{ij}}{\partial \beta_{ij}}$
- $\rho_{ij} < 0$ for $0 < \beta_{ij} < d_1^2$,$\rho_{ij} = 0$ for $0 < \beta_{ij} > d_1^2$,

Control law for aggregation

$$u_i = -\sum_{j\in N_i} \frac{\partial W_{ij}}{\partial q_i} - \sum_{j\in M_i} \frac{\partial V_{ij}}{\partial q_i} = -\sum_{j\in N_i} \frac{\partial W_{ij}}{\partial q_i} - \sum_{j\neq i} \frac{\partial V_{ij}}{\partial q_i}$$

The control law can be written in stack vector form as

$$u = -2\left(P \otimes I_2\right) q - 2\left(R \otimes I_2\right) q$$

where $R_{ii} = \sum_{j\neq i} \rho_{ij}$, $R_{ij} = -\rho_{ij}, i \neq j$, $P_{ii} = \sum_{j\in N_i} p_{ij}$, $P_{ij} = -p_{ij}$ for $i \neq j, j \in N_i$ and $P_{ij} = 0$ for $j \notin N_i$.

Results

- The collision avoidance set $\mathcal{I}(q) = \{q | \, \|q_i - q_j\| > 0, \forall i, j \in \mathcal{N}, i \neq j\}$ is invariant for the trajectories of the closed loop system.
- The connectivity maintenance set $\mathcal{J}(q) = \{q | \, \|q_i - q_j\| < d, \forall (i, j) \in E\}$ is invariant for the trajectories of the closed loop system.
- The system reaches a static configuration.

Equilibria

- The swarm center $\bar{q} \triangleq \frac{1}{N} \sum_{i=1}^{N} q_i$ can be shown to be invariant.
- Bounds on the swarm size can be derived based on the properties of the potentials.
- Assumption: $p_{ij} \geq a > 0$, $|\rho_{ij}| \leq \frac{\rho}{\beta_{ij}}$, where $\rho > 0$.

Bounds on the swarm size

- Under the previous assumptions we can show that all initial edges satisfy $\beta_{ij} \leq \frac{\rho}{a} N (N-1)$ at steady state.
- If the graph formed by the initial edges is connected, then

$$\beta_{\max} \leq \frac{\rho}{a} N (N-1)^2$$

at steady state, where $\beta_{\max} = \max_{i,j \in \mathcal{N}} \|q_i - q_j\|^2$.

Next Lecture

Sensing Constraints 2: Swarming-sensor networks

- Swarm aggregation
- Swarm dispersion
- Collision avoidance