# FEL3330 Networked and Multi-agent Control Systems
# Lecture 5: Formation Control 1

September 15, 2016

# Today's lecture

- Position based formations
- Formation infeasibility
- Flocking behavior

# Formation Control

- Convergence to desired relative states with respect to neighbors
- Can be distinguished between position based and distance based
- Other relative states can be considered such as relative orientation

# Position-based formations

- Goal: Convergence to desired relative position *vectors* in $\mathbb{R}^2$.
- Consider undirected and static graph.
- A vector $c_{ij} \in \mathbb{R}^2$ is associated to each edge $(i, j) \in E$, representing the desired relative position of agents $i, j$.
- The formation configuration is called *feasible* if the set $\Phi \triangleq \left\{ q \in \mathbb{R}^{2N} \,|\, q_i - q_j = c_{ij}, \, \forall (i, j) \in E \right\}$ of feasible formation configurations is nonempty.

# Control law

- $u_i = -\frac{\partial \gamma_i}{\partial q_i}, \gamma_i = \frac{1}{2} \sum_{j \in N_i} \|q_i - q_j - c_{ij}\|^2$
- $\dot{q} = \begin{bmatrix} -\frac{\partial \gamma_1}{\partial q_1} & \dots & -\frac{\partial \gamma_N}{\partial q_N} \end{bmatrix}^T = -(Lq + c_I)$
- $c_{ii} = -\sum_{j \in N_i} c_{ij}$ $, c_I = [c_{11}, \dots, c_{NN}]^T$, $L = \mathcal{L} \otimes I_2$
- Use $V = \sum_i \gamma_i$ as a candidate Lyapunov function.

# Results

- Assume that the formation configuration is feasible and that the formation graph is connected. Then, the agents converge to the desired formation configuration.
- If the formation graph is connected, the system reaches a configuration in which all agents have the same velocity vector, i.e., $\dot{q}_i = \dot{q}^*$ for all $i \in \mathcal{N}$ which is given by $\dot{q}^* = -\frac{1}{N}\sum_i c_{ii}$.

# Using the incidence matrix

- Consider a fictitious global coordinate frame and $c_i$ the position of agent $i$ in any formation realization.
- Denote $\tilde{q}_i = q_i - c_i$ and $\tilde{q}$,$\tilde{q}_e$ stack vector and corresponding edge vector.
- Then $\dot{q} = -(Lq + c_l) = -L\tilde{q} = -D\tilde{q}_e$ and thus $\dot{\tilde{q}}_e = -D^T D\tilde{q}_e$
- From previous lecture this implies all $\tilde{q}_i$ go to the same value thus formation is achieved for a connected graph.

# Minimal requirements

- For a connected graph, it suffices to define the formation specification on (one of) its spanning trees.
- In this case $D = D^T$ and the same analysis as before holds.
- In general we can employ linear transformations to move between equivalent representations of the same formation specification.

# Distance-based formations: a first take

- Goal: Convergence to desired relative *distance* in $\mathbb{R}$.
- A scalar weight $d_{ij} > 0$ is associated to each edge $(i, j) \in E$ representing the desired relative distance of agents $i, j$.
- The formation configuration is called *feasible* if the set $\Phi \triangleq \{ q \in \mathbb{R}^{2N} \mid ||q_i - q_j|| = d_{ij}, \ \forall (i, j) \in E \}$ of feasible formation configurations is nonempty.

# Control law

- Formation potential: $\gamma(\beta_{ij}) \in C^1 : \mathbb{R}^+ \to \mathbb{R}^+ \cup \{0\}$, with $\gamma(d_{ij}^2) = 0$ and $\gamma(\beta_{ij}) > 0$ for all $\beta_{ij} \neq d_{ij}^2$, where $\beta_{ij}(q) = \|q_i - q_j\|^2$.
- $\rho_{ij} \stackrel{\Delta}{=} \frac{\partial \gamma(\beta_{ij})}{\partial \beta_{ij}}$. Note that $\rho_{ij} = \rho_{ji}$, for all $i, j \in V, i \neq j$.
- $u_i = -\sum_{j \in \mathcal{N}_i} \frac{\partial \gamma(\beta_{ij}(q))}{\partial q_i} = -\sum_{j \in \mathcal{N}_i} 2\rho_{ij} (q_i - q_j)$

# Analysis

- $u = -2\left(R \otimes I_2\right) q$ where $R$ is given by $R_{ij} = -\rho_{ij}$, for $j \in \mathcal{N}_i$, $R_{ij} = 0$, for $j \notin \mathcal{N}_i$, and $R_{ii} = \sum_{j \in \mathcal{N}_i} \rho_{ij}$, for all $i \in V$.
- Use $V_f(q) = \sum_{i} \sum_{j \in \mathcal{N}_i} \gamma(\beta_{ij}(q))$ as a candidate Lyapunov function.

# Results

- Assume $\Phi \neq \emptyset$. If the communication graph is a tree, then there exists a $\gamma$ such that the agents are driven to the desired formation, i.e., $\lim_{t\to\infty} q(t) = q^* \in \Phi$.
- The tree condition is *necessary* for formation stabilization from all initial conditions.

# Flocking Motion

- Represents Reynolds' model for cohesion, collision avoidance and velocity alignment
- A celebrated application of bioinspired models in multi-agent systems
- Model based on double integrator dynamics
- We focus on the velocity alignment element here

# Flocking Motion

- Model: $\dot{q}_i = u_i, \dot{u}_i = v_i$
- Control law: sum of two terms, one representing position specs and one representing velocity specs
- Control of the form: $v_i = -\sum_{j \in N_i} \frac{\partial \gamma(\beta_{ij}(q))}{\partial q_i} - \sum_{j \in N_i} (u_i - u_j)$
- $\gamma$ can represent any kind of position based spec, such as distance based formation

# Analysis

- Use $V = \frac{1}{2}\sum_{i}\sum_{j\in N_i}\gamma(\beta_{ij}) + \frac{1}{2}\sum_{i}\|u_i\|^2$ as a candidate Lyapunov function
- Main result: For a connected graph, agents converge to a configuration where they share a common velocity

# Next Lecture

**Communication constraints 1**

- Connectivity
- Connectivity maintenance
- Quantization