FEL3330 Networked and Multi-Agent Control Systems

Lecture 5: Network Controllability-Leader Follower Networks

- Containment control
- Controllability
- Number of Leaders for Network Connectivity

Networks with leaders

Leaders can be important in various properties

- Network Controllability
- Containment control
- Leader-follower ratio can be also a factor in various problems

Network Controllability

- Goal: how to control a multi-agent system to any configuration with some agents as free inputs.
- Simplest case:
  $\dot{x}_i = -\frac{1}{|N_i|}\sum_{j\in\mathcal{N}_i}(x_i - x_j), i = 1, \ldots, N-1$
- One leader (free input): $\dot{x}_N = u_N$
- Denote $y_i = x_i, i = 1, \ldots, N-1$,$z = x_N$.
- Then $\dot{y} = -Fy - rz$ where F is the matrix obtained from $\Delta^{-1}L$ after deleting the last row and column, and r is the vector of the first $N-1$ elements of the deleted column.

Network Controllability

- Controllability of the followers through the motion of the leader z is examined through the controllability matrix of the system:

$$C = \begin{pmatrix} -r & Fr & -F^2 r & \dots & (-1)^n (UDU^T)^{n-1} r \end{pmatrix}$$

- Main result: The system is controllable iff (i) F has distinct eigenvalues and (ii) the eigenvectors of F are not orthogonal to r.

Remarks

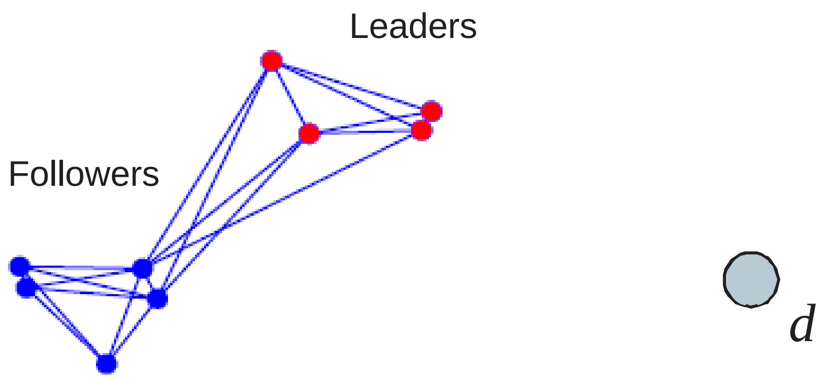
- It can be shown that the complete graph is uncontrollable, whereas a tree graph is controllable (connectivity is not always good).
- Extensions to multiple leaders in the work by Rahmani et al.

Number of Leaders for Network Connectivity

- Previous approach: direct approaches for connectivity maintenance through infinite potentials
- Idea: connectivity maintenance through indirect metrics such as the leader to follower number ratio
- Leaders are aware of the global objective: convergence to a desired destination d, whereas followers are unaware of the global
- The control law of either type of agents does not enforce connectivity maintenance

Control law

- Followers: $\dot{x}_i = -\sum_{j \in \mathcal{N}_i} (x_i - x_j), i \in \mathcal{N}^f$
- Leaders: $\dot{x}_i = -\sum_{j \in \mathcal{N}_i} (x_i - x_j) - a(x_i - d), i \in \mathcal{N}^l$
- Neighboring relation: $\mathcal{N}_i = \{j \in \mathcal{N} : |x_i - x_j| \leq \Delta\}$



Results

- The convex hull of the agents initial positions and the target is invariant for the trajectories of the closed loop system.
- Assume that $G(0)$ is complete and that $N_l \geq \frac{\alpha d}{\Delta} - N_f$ holds. Then $G(t)$ is complete for all $t \geq 0$ and $\lim_{t \to \infty} x_i(t) = d$ for all i.

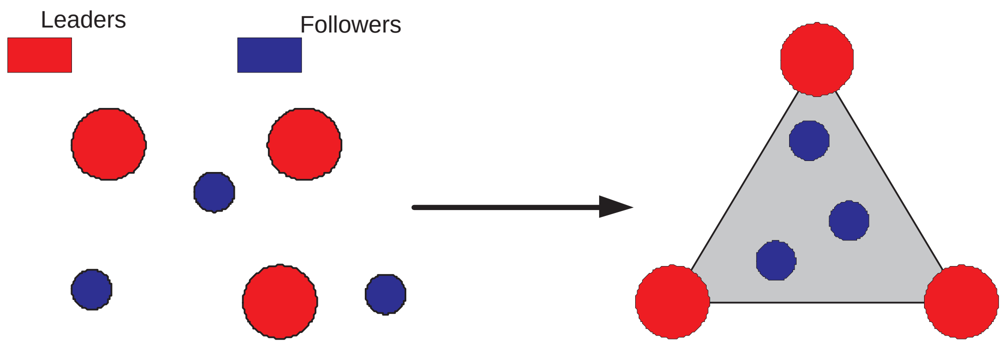
Results

- The results are extended to incomplete graphs.
- Assume that $G(0)$ is connected, and the two subgraphs of leaders and followers are complete. Then if for all initial edges between leader follower pairs satisfy $N_{lj} + N_{fi} \geq \frac{N}{2} + \frac{ad}{2\Delta}$ and $a \geq N_f - N_l \geq 0$, where N_{lj} are the leaders seen by follower j and N_{fi} are the followers seen by leader i, then then all edges are maintained (and hence the graph remains connected), and $\lim_{t\to\infty} x_i(t) = d$ for all i.

Containment control

- Control goal: the leaders force the followers to be contained in the convex hull of the leaders' (final) positions

- For example, the leaders could converge to a desired formation through an appropriate control law



Containment control

- $q_i = (x_i, y_i)^T$, i
- Leaders are considered static: $q_i(t) = q_i(0), i \in N_l$
- Followers are controlled through standard agreement protocol: $\dot{q}_i = -\sum_{j \in N_i} (q_i - q_j)$

Analysis

- Use $V = \frac{1}{2}q^T(L \otimes I_2)q$ as a candidate Lyapunov function
- Main result: For a connected graph, agents converge to the configuration

$$(Lx)_i = (Ly)_i = 0, \forall i \in N_f$$
$$q^l = q^l\,(0)$$

where q^l is the stack vector of the leaders' configuration.

- From Ferrari-Trecate et al., we know that for a connected graph and a nonempty set of leaders, the position of each follower lies in the convex hull of the leaders' positions.

Next Lecture

Formation control 2: distance based formations, rigidity.

- Lecture to be given by Dr. Iman Shames