FEL3330 Networked and Multi-Agent Control Systems

Lecture 5: Formation control

- Position-Based formations
- Formation infeasibility
- Distance-Based formation elements
- Flocking

Formation Control

- Convergence to desired relative states with respect to neighbors
- Can be distinguished between position based and distance based
- Other relative states can be considered such as relative orientation

Position-based formations

- Goal: Convergence to desired relative position *vectors* in $\mathbb{R}^2$.
- A vector $c_{ij} \in \mathbb{R}^2$ is associated to each edge $(i, j) \in E$, representing the desired relative position of agents i, j.
- The formation configuration is called *feasible* if the set $\Phi \triangleq \left\{ q \in \mathbb{R}^{2N} \,|q_i - q_j = c_{ij},\ \forall\, (i,j) \in E \right\}$ of feasible formation configurations is nonempty.

Control law

- $u_i = -\frac{\partial \gamma_i}{\partial q_i}, \gamma_i = \frac{1}{2} \sum_{j \in N_i} \left\| q_i - q_j - c_{ij} \right\|^2$

- $\dot{q} = \begin{bmatrix} -\frac{\partial \gamma_1}{\partial q_1} & \dots & -\frac{\partial \gamma_N}{\partial q_N} \end{bmatrix}^T = -\left(Lq + c_l \right)$

- $c_{ii} = -\sum_{j \in N_i} c_{ij}$,$c_l = \left[c_{11}, \dots, c_{NN} \right]^T$, $L = \mathcal{L} \otimes I_2$

- Use $V = \sum_i \gamma_i$ as a candidate Lyapunov function.

Results

- Assume that the formation configuration is feasible and that the formation graph is connected. Then, the agents converge to the desired formation configuration.

- If the formation graph is connected, the system reaches a configuration in which all agents have the same velocity vector, i.e., $\dot{q}_i = \dot{q}^*$ for all $i \in \mathcal{N}$ which is given by $\dot{q}^* = -\frac{1}{N}\sum_i c_{ii}$.

Distance-based formations

- Goal: Convergence to desired relative *distance* in $\mathbb{R}$.
- A scalar weight $d_{ij} > 0$ is associated to each edge $(i, j) \in E$ representing the desired relative distance of agents i, j.
- The formation configuration is called *feasible* if the set $\Phi \triangleq \{q \in \mathbb{R}^{2N} \mid ||q_i - q_j|| = d_{ij}, \ \forall (i, j) \in E\}$ of feasible formation configurations is nonempty.

Control law

- Formation potential: $\gamma(\beta_{ij}) \in C^1 : \mathbb{R}^+ \to \mathbb{R}^+ \cup \{0\}$, with $\gamma(d_{ij}^2) = 0$ and $\gamma(\beta_{ij}) > 0$ for all $\beta_{ij} \neq d_{ij}^2$, where $\beta_{ij}(q) = \|q_i - q_j\|^2$.

- $\rho_{ij} \triangleq \frac{\partial \gamma(\beta_{ij})}{\partial \beta_{ij}}$. Note that $\rho_{ij} = \rho_{ji}$, for all $i, j \in V, i \neq j$.

- $u_i = -\sum_{j \in \mathcal{N}_i} \frac{\partial \gamma(\beta_{ij}(q))}{\partial q_i} = -\sum_{j \in \mathcal{N}_i} 2\rho_{ij}\left(q_i - q_j\right)$

Analysis

- $u = -2\,(R \otimes I_2)\,q$ where R is given by $R_{ij} = -\rho_{ij}$, for $j \in \mathcal{N}_i$, $R_{ij} = 0$, for $j \notin \mathcal{N}_i$, and $R_{ii} = \sum_{j \in \mathcal{N}_i} \rho_{ij}$, for all $i \in V$.

- Use $V_f(q) = \sum_i \sum_{j \in \mathcal{N}_i} \gamma(\beta_{ij}(q))$ as a candidate Lyapunov function.

Results

- Assume $\Phi \neq \emptyset$. If the communication graph is a tree, then there exists a γ such that the agents are driven to the desired formation, i.e., $\lim_{t\to\infty} q(t) = q^* \in \Phi$.
- The tree condition is *necessary* for formation stabilization from all initial conditions.

Flocking Motion

- Represents Reynolds' model for cohesion, collision avoidance and velocity alignment
- A celebrated application of bioinspired models in multi-agent systems
- Model based on double integrator dynamics
- We focus on the velocity alignment element here

Flocking Motion

- Model: $\dot{q}_i = u_i, \dot{u}_i = v_i$
- Control law: sum of two terms, one representing position specs and one representing velocity specs
- Control of the form: $v_i = -\sum_{j \in N_i} \frac{\partial \gamma(\beta_{ij}(q))}{\partial q_i} - \sum_{j \in N_i} (u_i - u_j)$
- γ can represent any kind of position based spec, such as distance based formation

Analysis

- Use $V = \frac{1}{2}\sum_i \sum_{j\in N_i} \gamma(\beta_{ij}) + \frac{1}{2}\sum_i \|u_i\|^2$ as a candidate Lyapunov function
- Main result: For a connected graph, agents converge to a configuration where they share a common velocity

Next Lecture

Network Controllability-Leader Follower Networks

- Containement control
- Controllability
- Number of Leaders for Network Connectivity