FEL3330 Networked and Multi-Agent Control Systems

Lecture 4: Communication constraints

- Maintaining connectivity
- Quantized Consensus
- Event-triggered multi-agent control

Communication and sensing limitations

- Constraints in the neighboring relations: sensing radius, lossy links
- Constraints in the communication exchange between neighbors: quantization, time-delays, sampled data, packet losses

Background: Metzler matrices

- A real matrix with zero row sums and non-positive off-diagonal elements.
- A symmetric Metzler matrix is a weighted Laplacian.
- If the graph corresponding to a symmetric Metzler matrix is connected, then zero is a simple eigenvalue of the matrix with corresponding eigenvector having its elements equal.

Connectivity maintenance

- A general assumption for the validity of most results: the graph stays connected (a path exists between any two nodes)
- How to render connectivity from an assumption to an invariant property?
- Direct strategies: the control law guarantees that if the initial communication graph is connected, then it remains connected for all time
- How to achieve that? First approach: once an edge, always an edge!

Connectivity maintenance

- From $u_i = -\sum_j a_{ij}(x_i - x_j)$ to $u_i = -\sum_j a_{ij}(||x_i - x_j||)(x_i - x_j)$
- Apply attraction force that is strong enough whenever an edge between the agents tends to be lost
- Edge definition: $||x_i - x_j|| \leq d \Leftrightarrow (i,j) \in E$

Connectivity maintenance potential

- Define W_{ij} between i and $j \in N_i$
- $W_{ij} = W_{ij}\left(\|x_i - x_j\|^2\right) = W_{ij}\left(\beta_{ij}\right)$,
- W_{ij} is defined on $\beta_{ij} \in [0, d^2)$,
- $W_{ij} \to \infty$ whenever $\beta_{ij} \to d^2$,
- it is C^1 for $\beta_{ij} \in [0, d^2)$ and
- the term $p_{ij} \triangleq \frac{\partial W_{ij}}{\partial \beta_{ij}}$ satisfies $p_{ij} > 0$ for $0 \leq \beta_{ij} < d^2$.

Connectivity maintenance control law

- $\dot{x}_i = u_i = -\sum_{j \in N_i} \frac{\partial W_{ij}}{\partial x_i} = 2 \sum_{j \in N_i} p_{ij} (x_i - x_j)$
- $\dot{x} = -2Px$, P Metzler matrix
- Use $V = \sum_i \sum_{j \in N_i} W_{ij}$ as a candidate Lyapunov function
- It can be shown that $\nabla V = 4Px$
- What are the dynamics in the $\bar{x}$ space?

Results

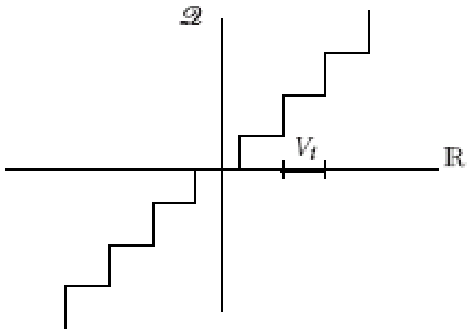
- All agents converge to the initial (invariant) average
- All initial edges are invariant
- Extension: can be extended to consider dynamic addition of edges

Quantized consensus

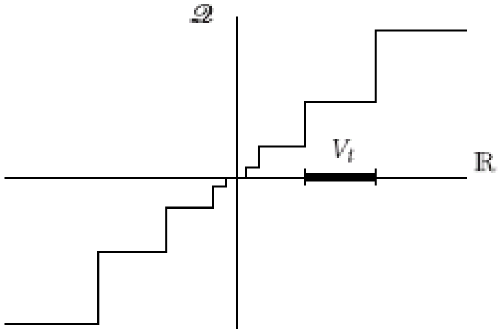
- Each agent has quantized measurements of the form $q(x_i - x_j)$, $q(.)$ quantization function
- Can be extended to multiple dimensions as in the previous cases
- Uniform and logarithmic quantizers

Quantizer types

- Uniform quantizer: $|q_u(a) - a| \leq \delta_u, \forall a \in \mathbb{R}$
- Logarithmic quantizer: $|q_l(a) - a| \leq \delta_l |a|, \forall a \in \mathbb{R}$



(a) Uniform quantizer. The scalar case.



(b) Logarithmic quantizer. The scalar case.

Closed-loop system

- $\dot{x}_i = u_i = -\sum_{j\in\mathcal{N}_i} q\left(x_i - x_j\right)$
- Stack vector form: $\dot{\bar{x}} = -B^T B q\left(\bar{x}\right)$
- We use the positive definiteness of $B^T B$ when G is a tree graph

Results

Theorem: Assume G is a tree.

- In the case of a uniform quantizer, the system converges to a ball of radius $\frac{\left\| B^T B \right\| \delta_u \sqrt{m}}{\lambda_{\min}\left(B^T B\right)}$ around $\bar{x} = 0$ in finite time.

- In the case of a logarithmic quantizer, the system is exponentially stabilized to $\bar{x} = 0$, provided that satisfies $\delta_l < \frac{\lambda_{\min}\left(B^T B\right)}{\|B^T B\|}$ Use $V = \frac{1}{2}\bar{x}^T \bar{x}$ as a candidate Lyapunov function

Use $V = \frac{1}{2}\bar{x}^T \bar{x}$ as a candidate Lyapunov function

Quantized consensus

- Conditions only sufficient
- Use the $V = \frac{1}{2}\delta^T\delta$ as a candidate Lyapunov function
- Results extended to undirected graphs of general topology (Guo and DVD, Automatica 2013).

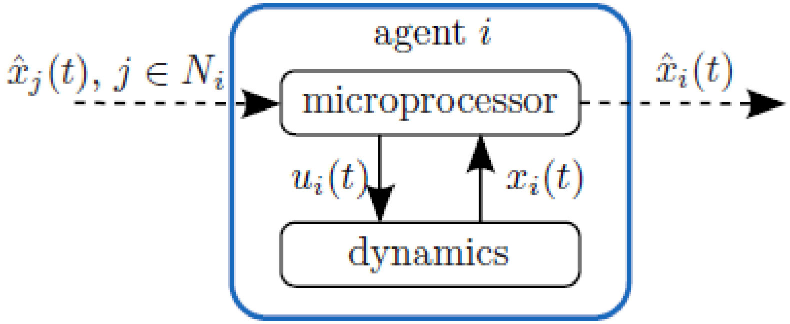
Event-triggered sampling

- Time-triggered sampling at pre-specified instants: does not take into account optimal resource usage
- A strategy considering better resource usage: event-triggered control
- Actuation updates in asynchronous manner
- Application to multi-agent systems

Event-triggered sampling

- Each agent i broadcast their state at discrete time instants $t_0^i, t_1^i, \ldots$
- $\hat{x}_i(t) = x_i(t_k^i)$: latest update for agent i
- Event-triggered control law:
  $u_i(t) = -\sum_{j \in N_i} (\hat{x}_i(t) - \hat{x}_j(t)))$
- Measurement error:
  $e_i(t) = \hat{x}_i(t) - x_i(t) = x_i(t_k^i) - x_i(t), t \in [t_k^i, t_{k+1}^i)$
- Next slides ack to Georg Seyboth

Event-based scheduling of measurement broadcasts:



Event-based broadcasting

$$\hat{x}_i(t) = x_i(t_k^i),\ t \in [t_k^i, t_{k+1}^i[$$

$$0 \leq t_0^i \leq t_1^i \leq t_2^i \leq \cdots$$

- Consensus protocol

$$u_i(t) = -\sum_{j \in N_i} (\hat{x}_i(t) - \hat{x}_j(t))$$

- Measurement errors

$$e_i(t) = \hat{x}_i(t) - x_i(t)$$

- Closed-loop

$$\dot{x}(t) = -L\hat{x}(t) = -L(x(t) + e(t))$$

- Disagreement

$$\delta(t) = x(t) - a\mathbf{1}, \qquad \mathbf{1}^T\delta(t) \equiv 0$$

Trigger mechanism: Define *trigger functions* $f_i(\cdot)$ and trigger when

$$f_i\left(t, x_i(t), \hat{x}_i(t), \bigcup_{j \in N_i} \hat{x}_j(t)\right) > 0$$

Defines sequence of events: $t^i_{k+1} = \inf\{t : t > t^i_k, f_i(t) > 0\}$

Problem statement: Find suitable $f_i(\cdot)$ such that

- no Zeno behavior
- desired convergence properties
- as few inter-agent communications as possible

Intuition:

$$f_i(e_i(t)) = |e_i(t)| - c_0 \quad \Rightarrow \quad \forall t \geq 0 : \quad |e_i(t)| \leq c_0$$

$$\dot{x}(t) = u(t), \qquad u(t) = -L\hat{x}(t) \tag{1}$$

Theorem (constant thresholds)

Consider system (1) with undirected connected graph G. Suppose that

$$f_i(e_i(t)) = |e_i(t)| - c_0,$$

with $c_0 > 0$. Then, for all $x_0 \in \mathbb{R}^N$, the system does not exhibit Zeno behavior and for $t \to \infty$,

$$\|\delta(t)\| \leq \frac{\lambda_N(L)}{\lambda_2(L)} \sqrt{N} c_0.$$

Proof ideas:

- Analytical solution of disagreement dynamics yields

$$\|\delta(t)\| \leq e^{-\lambda_2(L)t}\|\delta(0)\| + \lambda_N(L)\int_0^t e^{-\lambda_2(L)(t-s)}\|e(s)\|ds$$

- Compute lower bound τ on the inter-event intervals

Can we achieve asymptotic convergence?

Theorem (exponentially decreasing thresholds)

Consider system (1) with undirected connected graph G. Suppose that

$$f_i(t, e_i(t)) = |e_i(t)| - c_1 e^{-\alpha t},$$

with $c_1 > 0$ and $0 < \alpha < \lambda_2(L)$. Then, for all $x_0 \in \mathbb{R}^N$, the system does not exhibit Zeno behavior and as $t \to \infty$,

$$\|\delta(t)\| \to 0.$$

Intuition:

- $\lambda_2(L)$ is the rate of convergence for $\delta(t)$ in cont. time
- $\alpha < \lambda_2(L)$ means that the threshold $c_1 e^{-\alpha t}$ decreases slower!

Remark:

– Vanishing thresholds cause problems (measurement noise, numerics)

Combine the advantages:

Theorem (exponentially decreasing thresholds with offset)

Consider system (1) with undirected connected graph G. Suppose that

$$f_i(t, e_i(t)) = |e_i(t)| - (c_0 + c_1 e^{-\alpha t}),$$

with $c_0, c_1 \geq 0$, at least one positive, and $0 < \alpha < \lambda_2(L)$. Then, for all $x_0 \in \mathbb{R}^N$, the system does not exhibit Zeno behavior and for $t \to \infty$,

$$\|\delta(t)\| \leq \frac{\lambda_N(L)}{\lambda_2(L)} \sqrt{N} c_0.$$

Remarks:

+ Size of the final region is tunable
+ Larger thresholds for small t increase inter-event time
+ Problems due to noise or numerics are avoided

Lower bound τ on the inter-event intervals:

For $c_0 > 0$:

$$\tau = \frac{c_0}{k_1 + k_2 + k_3}$$

For $c_0 = 0$:

$$(k_1 + k_2)\tau = c_1 e^{-\alpha\tau}$$

with

$$k_1 = \lambda_N(L)\|\delta(0)\|$$
$$k_2 = \lambda_N(L)\sqrt{N}c_1\left(1 + \frac{\lambda_N(L)}{\lambda_2 - \alpha}\right)$$
$$k_3 = \lambda_N(L)\sqrt{N}c_0\left(1 + \frac{\lambda_N(L)}{\lambda_2}\right)$$

Next Lecture

Formation control 1

- Position-Based formations
- Formation infeasibility
- Connectivity maintenance in formation control
- Flocking
- Distance-Based formation elements