FEL3330 Networked and Multi-Agent Control Systems

Lecture 3: Communication constraints

- Maintaining connectivity
- Quantized Consensus
- Event-triggered multi-agent control

Communication and sensing limitations

- Constraints in the neighboring relations: sensing radius, lossy links
- Constraints in the communication exchange between neighbors: quantization, time-delays, sampled data, packet losses

Background: Metzler matrices

- A real matrix with zero row sums and non-positive off-diagonal elements.
- A symmetric Metzler matrix is a weighted Laplacian.
- If the graph corresponding to a symmetric Metzler matrix is connected, then zero is a simple eigenvalue of the matrix with corresponding eigenvector having its elements equal.

Connectivity maintenance

- A general assumption for the validity of most results: the graph stays connected (a path exists between any two nodes)
- How to render connectivity from an assumption to an invariant property?
- Direct strategies: the control law guarantees that if the initial communication graph is connected, then it remains connected for all time
- How to achieve that? First approach: once an edge, always an edge!

Connectivity maintenance

- From $u_i = -\sum_j a_{ij}(x_i - x_j)$ to
  $u_i = -\sum_j a_{ij}(||x_i - x_j||)(x_i - x_j)$
- Apply attraction force that is strong enough whenever an edge between the agents tends to be lost
- Edge definition: $||x_i - x_j|| \leq d \Leftrightarrow (i,j) \in E$

Connectivity maintenance potential

- Define W_{ij} between i and $j \in N_i$
- $W_{ij} = W_{ij}\left(\|x_i - x_j\|^2\right) = W_{ij}(\beta_{ij})$,
- W_{ij} is defined on $\beta_{ij} \in [0, d^2)$,
- $W_{ij} \to \infty$ whenever $\beta_{ij} \to d^2$,
- it is C^1 for $\beta_{ij} \in [0, d^2)$ and
- the term $p_{ij} \triangleq \frac{\partial W_{ij}}{\partial \beta_{ij}}$ satisfies $p_{ij} > 0$ for $0 \leq \beta_{ij} < d^2$.

Connectivity maintenance control law

- $\dot{x}_i = u_i = -\sum_{j \in N_i} \frac{\partial W_{ij}}{\partial x_i} = 2 \sum_{j \in N_i} p_{ij} (x_i - x_j)$
- $\dot{x} = -2Px$, P Metzler matrix
- Use $V = \sum_i \sum_{j \in N_i} W_{ij}$ as a candidate Lyapunov function
- It can be shown that $\nabla V = 4Px$
- What are the dynamics in the $\bar{x}$ space?

Results

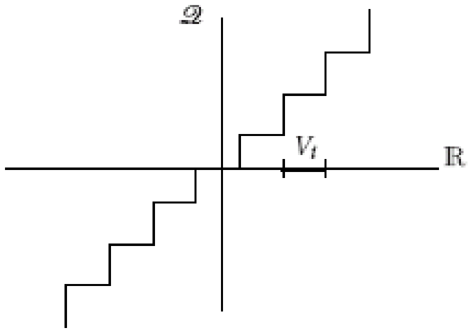
- All agents converge to the initial (invariant) average
- All initial edges are invariant
- Extension: can be extended to consider dynamic addition of edges

Quantized consensus

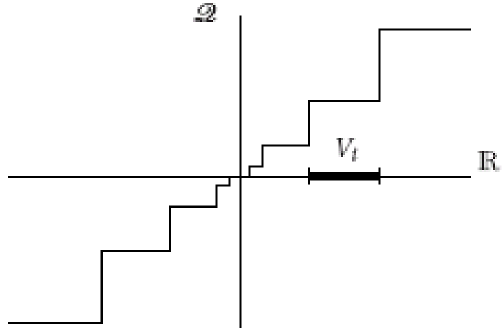
- Each agent has quantized measurements of the form $q(x_i - x_j)$, $q(.)$ quantization function
- Can be extended to multiple dimensions as in the previous cases
- Uniform and logarithmic quantizers

Quantizer types

- Uniform quantizer: $|q_u(a) - a| \leq \delta_u, \forall a \in \mathbb{R}$
- Logarithmic quantizer: $|q_l(a) - a| \leq \delta_l |a|, \forall a \in \mathbb{R}$



(a) Uniform quantizer. The scalar case.



(b) Logarithmic quantizer. The scalar case.

Closed-loop system

- $\dot{x}_i = u_i = -\sum_{j \in \mathcal{N}_i} q\left(x_i - x_j\right)$
- Stack vector form: $\dot{\bar{x}} = -B^T B q\left(\bar{x}\right)$
- We use the positive definiteness of $B^T B$ when G is a tree graph

Results

Theorem: Assume G is a tree.

- In the case of a uniform quantizer, the system converges to a ball of radius $\dfrac{\left\|B^T B\right\| \delta_u \sqrt{m}}{\lambda_{\min}\left(B^T B\right)}$ around $\bar{x} = 0$ in finite time.

- In the case of a logarithmic quantizer, the system is exponentially stabilized to $\bar{x} = 0$, provided that satisfies $\delta_l < \frac{\lambda_{\min}\left(B^T B\right)}{\|B^T B\|}$ Use $V = \frac{1}{2}\bar{x}^T \bar{x}$ as a candidate Lyapunov function

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Event-triggered sampling

- Time-triggered sampling at pre-specified instants: does not take into account optimal resource usage
- A strategy considering better resource usage: event-triggered control
- Actuation updates in asynchronous manner
- Application to multi-agent systems

Event-triggered sampling

- Each agent i broadcast their state at discrete time instants $t_0^i, t_1^i, \ldots$
- $\hat{x}_i(t) = x_i(t_k^i)$: latest update for agent i
- Event-triggered control law:
  $u_i(t) = -\sum_{j \in N_i} (\hat{x}_i(t) - \hat{x}_j(t)))$
- Measurement error:
  $e_i(t) = \hat{x}_i(t) - x_i(t) = x_i(t_k^i) - x_i(t), t \in [t_k^i, t_{k+1}^i)$

Centralized case

- $t_k^i = t_k$ for all agents i
- Event-based design: how to choose t_k?
- Control: $u(t) = -Lx(t_k), t \in [t_k, t_{k+1})$
- Closed loop system: $\dot{x}(t) = -Lx(t_k) = -L(x(t) + e(t))$
- The initial average remains invariant!

Centralized case

- Use $V = \frac{1}{2}x^T Lx$
- $\dot{V}$ is negative definite with respect to the desired equilibria provided that $\|e\| \leq \sigma \frac{\|Lx\|}{\|L\|}$, $0 < \sigma < 1$.
- Triggering rule: $\|e\| \leq \sigma \frac{\|Lx\|}{\|L\|}$, $0 < \sigma < 1$
- Everyone measures and broadcasts its state whenever this condition is violated.

Decentralized case

- In this case $t_k^i \neq t_k^j$ for $i \neq j$ in general
- $u_i(t) = -\sum\limits_{j \in N_i} (\hat{x}_i(t) - \hat{x}_j(t))) =$
  $-\sum\limits_{j \in N_i} (x_i(t) - x_j(t)) - \sum\limits_{j \in N_i} (e_i(t) - e_j(t))$
- Closed loop system: $\dot{x}(t) = -L(x(t) + e(t))$
- The initial average remains invariant in this case as well!
- Event-based design: how to choose t_k^i?

Decentralized case

- Use $V = \frac{1}{2}x^T Lx$
- $\dot{V}$ is negative definite with respect to the desired equilibria provided that $e_i^2 \leq \frac{\sigma_i a(1-a|N_i|)}{|N_i|}(Lx)_i^2$, with $0 < a < \frac{1}{|N_i|}$ and $0 < \sigma_i < 1$
- Triggering rule: $e_i^2 \leq \frac{\sigma_i a(1-a|N_i|)}{|N_i|}(Lx)_i^2$
- Agent i broadcasts its state whenever this condition is violated.

Brooadcast periods

- Strictly positive lower bounds in the centralized case
- Strictly positive lower bounds for at least one agent in the decentralized case

Next Lecture

Formation control 1

- Position-Based formations
- Formation infeasibility
- Connectivity maintenance in formation control
- Flocking
- Distance-Based formation elements