FEL3330 Networked and Multi-agent Control Systems
Lecture 1: Introduction

August 31, 2016

Fall 2016

Automatic Control
School of Electrical Engineering
KTH Royal Institute of Technology

FEL3330 Networked and Multi-Agent Control Systems

- **Disposition**

  7.5 credits, 24h lectures
  ACCESS PhD Course

- **Instructors**

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Homework

Three homework assignments

- First homework given after Lecture 3
- Specific deadline per HW
- Up to teams of two, but no more

Course goal

After the course, you should be able to

- Know the essential theoretical tools to cope with Networked and Multi-Agent Systems
- Know the established problems and results in the area
- Apply the theoretical tools to problems in the area
- Contribute to the research frontier in the area

FEL3330/EL2910 Networked and Multi-Agent Control Systems

Lecture 1: Introduction

- Practical information
- Motivating applications
- What is "Multi-agent" and "Networked"?
- Course outline
- Some Lyapunov theory background tools
- Poll: Take home exam vs. Research Projects?

Today's lecture

- What is the motivation and main theme of the course?
- Lyapunov theory background essentials

Course Information

- All info available at

  http://people.kth.se/~dimos/Networked_and_Multi_HT16.htm

Material

- **Textbook:** Mesbahi+Egerstedt, as well as papers/notes related to each lecture
- **Lecture frames:** Online after each lecture
- **Blackboard:** During the lecture
- **Poll:** Take home exam vs. Research Projects?

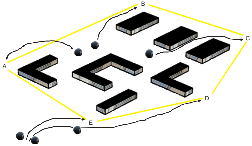
Motivation

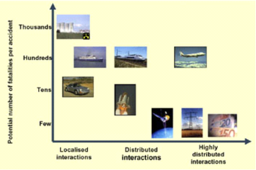
- How to understand and achieve global behaviors from local behaviors
- Multi-robot/vehicle coordination
- Sensor networks
- Social networks
- Power networks
- Bio-inspired coordination

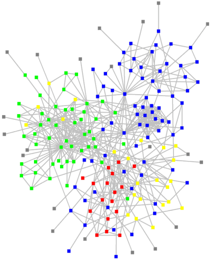
Design issues

- Scalability
- Limited information consideration
- Control objectives

Some nice figures







What is in the course-Why I should take it?

- Decentralized controllers at a high level of abstraction (Simple dynamics/sensing, complicated networks)
- Graph based models of networks with which the first step in control design can be held
- Disclaimer: the selection of topics may be **biased** towards the instructors' interests
- Why I should take the course?
  1. New tools for a large class of control problems
  2. HOT area for the last 15 years and still growing strong!
  3. Lots of really interesting unsolved problems

What is NOT in the course

- Cool computer graphics
- Behavioral robotics
- How to build devices
- Sensing/perception algorithms
- Communication protocols

Course title

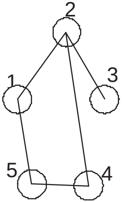
- Q: Why multi-agent?
- A: Agents represent the different entities in each application
- Q: Why networked?
- A: Need to model the limited information on the rest of the group due to sensing and communication limitations
- Agents are the vertices in the graph that represents the network!
- Pairs of agents that can exchange info are the edges!

Limited Sensing and Communication aspects

- Limited Sensing: Vision based sensors, range sensors (sonars, laser scanners,...)
- Limited Communication: communication channel, bandwith, coding,...

Graph theoretic approach

- Limitations in communication/sensing do now allow each agent to communicate with everyone else
- Modelling of limitations through graphs



$$G = (V, E)$$

- Agents are the vertices $V = \{1, \ldots, N\}$
- Edges $E \subset V \times V$ are pairs of agents that can communicate

Some graphs of interest

- Simple graphs
- Undirected graphs
- Weighted graphs

Relating graphs to networks

- Static networks
- Random networks
- State dependent/Dynamic networks

Lyapunov stability

Let $x^* = 0$ be an equilibrium point of $\dot{x} = f(x)$. It is called

- Stable, if for all $\epsilon > 0$, there exists $\delta = \delta(\epsilon) > 0$ such that $\|x(0)\| < \delta$ implies $\|x(t)\| < \epsilon, \forall t \geq 0$.
- Asymptotically stable (A.S.), if stable and there exists $\delta > 0$ such that $\|x(0)\| < \delta$ implies $\lim_{t \to \infty} x(t) = 0$. It is Globally Asymptotically Stable (G.A.S.) if A.S. when the system is arbitrarily initialized.
- Exponentially stable (E.S.) if there exist $\delta, c, \lambda > 0$ such that $\|x(0)\| < \delta$ implies $\|x(t)\| < c\|x(0)\| \exp(-\lambda t), \forall t \geq 0$.

Lyapunov's Second Method

Let $x^* = 0$ be an equilibrium point of $\dot{x} = f(x)$. If there exists a C^1 function $V : \mathbb{R}^n \to \mathbb{R}$ such that

$$
\begin{aligned}
&V(0) = 0 \\
&V(x) > 0, \ \ \forall x \neq 0 \\
&\dot{V}(x) \leq 0, \ \ \forall x \in \mathbb{R}^n,
\end{aligned}
$$

then x^* is stable. In this case, V is called a weak Lyapunov function. If $\dot{V}(x) < 0$, for all $x \neq 0$, then x^* is asymptotically stable, and V is called a Lyapunov function.

Lyapunov's Second Method: GAS and ES

If in addition the C^1 function $V : \mathbb{R}^n \to \mathbb{R}$ satisfies $V(x) \to \infty$ when $\|x\| \to \infty$ then it is called radially unbounded.

- Existence of a radially unbounded Lyapunov function implies that the origin is GAS.
- Existence of a Lyapunov function V fulfilling $\dot{V} \leq -aV$ along the trajectories of $\dot{x} = f(x)$ for some $a > 0$ implies that the origin is ES.

Lyapunov Function for Linear System

$\mathrm{Re}(\lambda_i(A)) < 0$ for all i if and only if for every positive definite $Q = Q^T$ there exists a positive definite $P = P^T$ such that

$$PA + A^T P = -Q$$

A Lyapunov function for a linear system

$$\dot{x} = Ax$$

is given by

$$V(x) = x^T P x$$

In particular,

$$\dot{V}(x) = x^T P \dot{x} + \dot{x}^T P x = x^T (PA + A^T P) x = -x^T Q x < 0$$

LaSalle's Invariance Principle

Let Ω be a compact set that is positively invariant with respect to $\dot{x} = f(x)$. Let V be a C^1 function with $\dot{V} \leq 0$ in Ω. Let E be the set of all points in Ω where $\dot{V} = 0$. Let M be the largest invariant set in E. Then, every solution starting in Ω approaches M as $t \to \infty$.

Previous notions can be extended to discrete-time systems.

Switched Systems

Abstracted model of hybrid systems where continuous part is highlighted more

- $f_p, p \in \mathcal{P}$ family of functions
- $\dot{x} = f_p(x), p \in \mathcal{P}$ family of systems
- Example: $f_p(x) = A_p x$, $\mathcal{P} = \{1, ..., m\}$

Switching signals

- Switched system $\dot{x}(t) = f_{\sigma(t)}(x(t))$ defined based on *switching signal* $\sigma : [0, \infty) \to \mathcal{P}$
- Discontinuities of $\sigma(t)$: switching times
- $\sigma(t) = \lim_{\tau \to t^+}$ for all $t \geq 0$.

Stability of switched systems

- What can we say about behavior of overall system based on stability of individual subsystems?
- Answer: not much...

Some conclusions

- Unconstrained switching may destabilize a switch system even if all subsystems are stable.
- Even if all subsystems are unstable, it may be still possible to stabilize the switched system by constraining the switching sequence appropriately.

- Stability under arbitrary switching.
- Stability under constrained switching. How to define switching sequence appropriately?

Switched system stability notions for the course

The switched system $\dot{x}(t) = f_{\sigma(t)}(x(t))$, $\sigma : [0, \infty) \to \mathcal{P}$, is (globally) uniformly asymptotically stable ((G.)U.A.S.) if it is (globally) asymptotically stable for every switching signal. It is (globally) existentially asymptotically stable if there exists at least one switching signal that renders the system (globally) asymptotically stable.

Stability under arbitrary switching

- GUAS theorem: Suppose that $f_p(0) = 0, \forall p \in \mathcal{P}$, $f_p(.)$ is continuous and $\mathcal{P}$ is finite. Then the switched system $\dot{x}(t) = f_{\sigma(t)}(x(t))$, $\sigma : [0, \infty) \to \mathcal{P}$, is (globally) uniformly asymptotically stable if there exists a common (radially unbounded) Lyapunov function V, ie, a function that is positive definite and fulfills $\frac{\partial V}{\partial x} f_p(x) < 0, \forall x \neq 0, \forall p \in \mathcal{P}$.
- LaSalle-like theorem: Let V be a common weak Lyapunov function for each of the subsystems $\dot{x} = f_p(x), p \in \mathcal{P}$ and M_p the largest invariant set under $\dot{x} = f_p(x)$ contained in

$$\{x \in \mathbb{R}^n : \frac{\partial V(x)}{\partial x} f_p(x) = 0\}$$

  If $M_i = M_j$ for all $i, j \in \mathcal{P}$ then x will converge asymptotically to this set.

Some notes

- Less conservative results exist.
- Stability under constrained switching dealt with multiple Lyapunov functions theorems. (cf. Hybrid MSc/PhD courses for an overview).

Research Projects

- Up to teams of four.
- Pick up one of the course related topics (eg: current or previous versions) and get back to me asap.
- I assign four papers per topic. Purpose of the project: write a report based on these papers, and additionally identify other papers of interest within the topic, and propose new research directions.
- Short presentation in the final lecture.

Next Lecture

Graphs and Matrices

- Graph theory essentials
- Relating graphs to matrices (algebraic graph theory)