FEL3330 Networked and Multi-Agent Control Systems

Spring 2011

Automatic Control
School of Electrical Engineering
Royal Institute of Technology

FEL3330 Networked and Multi-Agent Control Systems

- **Disposition**

  7.5 credits, 24h lectures

  PhD Course. MS students can get a certificate to use later.

- **Instructors**

  Dimos Dimarogonas, lecturer and course responsible, `dimos@kth.se`

  Iman Shames, lecturer, `imansh@kth.se`

  Milos Stankovic , lecturer, `milsta@kth.se`

Course goal

After the course, you should be able to

- Know the essential theoretical tools to cope with Networked and Multi-Agent Systems
- Know the established problems and results in the area
- Develop a research project and give presentations in the area
- Contribute to the research frontier in the area

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Lecture 1: Introduction

- Practical information
- Motivation applications
- What is “Multi-agent” and “Networked”?
- Course outline
- Some algebraic graph theoretic tools
- Poll: Homeworks vs. Research Projects?

Today's lecture

- What is the motivation and main theme of the course?
- Algebraic Graph Theory essentials

Course Information

- All info available at

  `www.s3.kth.se/~dimos/Networked_and_Multi.htm`

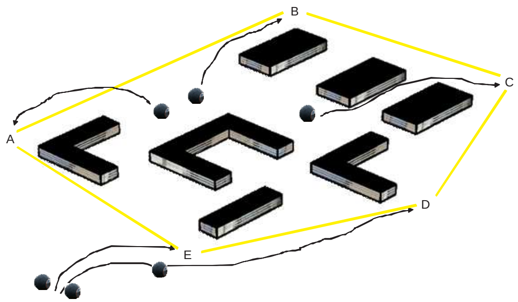
Material

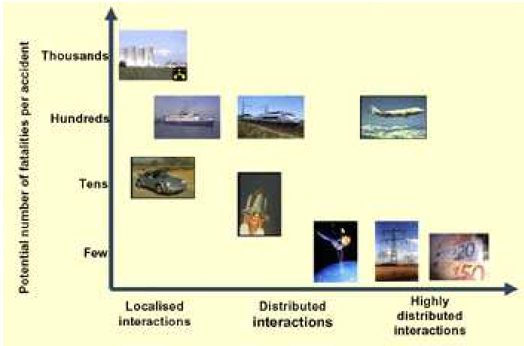
- **Textbook:** No textbook, but papers/notes related to each lecture
- **Lecture slides:** Available online after each lecture)
- **Blackboard:** During the lecture
- **Poll:** Homeworks vs. Research Projects vs. Both?

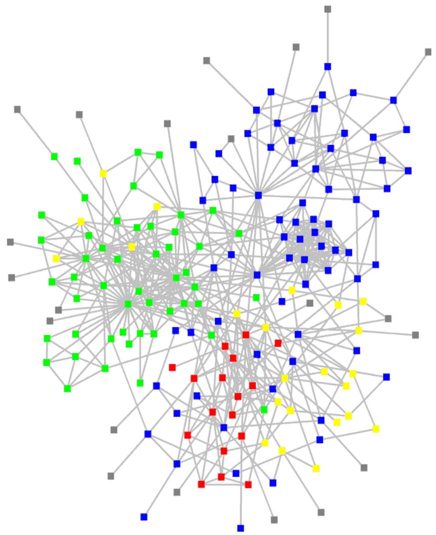
Motivation

- How to understand and achieve global behaviors from local behaviors
- Multi-robot/vehicle coordination
- Sensor networks
- Social networks
- Bio-inspired coordination
- Local behaviors arise due to limitations in sensing and/or communication

Some nice figures







What is in the course-Why I should take it?

- Decentralized controllers at a high level of abstraction (Simple dynamics/sensing, complicated networks)
- Graph based models of networks with which the first step in control design can be held
- Disclaimer: the selection of topics may be **biased** towards the instructors' interests
- Why I should take the course?
  1. New tools for a large class of control problems
  2. HOT area for the last 8 years and still growing strong!
  3. Lots of really interesting unsolved problems

What is NOT in the course

- Cool computer graphics
- Behavioral robotics
- How to build devices
- Sensing/perception algorithms
- Communication protocols

Course title

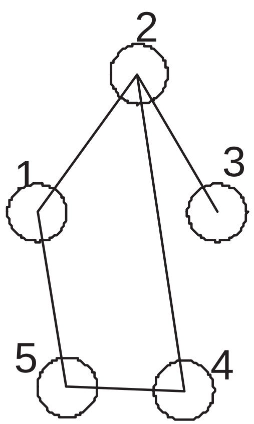
- Q: Why multi-agent?
- A: Agents represent the different entities in each application
- Q: Why networked?
- A: Need to model the limited information on the rest of the group due to sensing and communication limitations
- Agents are the vertices in the graph that represents the network!
- Pairs of agents that can exchange info are the edges!

Limited Sensing and Communication aspects

- Limited Sensing: Vision based sensors, range sensors (sonars, laser scanners,...)
- Limited Communication: communication channel, bandwith, coding,...

Graph theoretic approach

- Limitations in communication/sensing do now allow each agent to communicate with everyone else
- Modelling of limitations through graphs



$$G = (V, E)$$

- Agents are the vertices $V = \{1, \ldots, N\}$
- Edges $E \subset V \times V$ are pairs of agents that can communicate

Some graphs of interest

- Simple graphs
- Undirected graphs
- Weighted graphs

Relating graphs to networks

- Static networks
- Random networks
- State dependent/Dynamic networks

The adjacency matrix and the degree matrix

- We want to associate matrices with graphs.
- Neighboring set: $N_i = \{j \in V | (i,j) \in E\}$
- Adjacency matrix (undirected, simple graph)

$$A = A(G) = [a_{ij}], a_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E, \\ 0 & \text{if otherwise.} \end{cases}$$

- Degree matrix

$$\Delta = \Delta(G) = \mathrm{diag}(d_1, \ldots, d_N), d_i = \sum_j a_{ij} = |N_i|$$

The Laplacian matrix and its eigenvalues

- $L = L(G) = \Delta(G) - A(G)$
- Symmetric and positive semi-definite matrix
- Eigenvalues $0 = \lambda_1(G) \leq \lambda_2(G) \leq \ldots \leq \lambda_N(G)$
- For a connected G, $L(G)$ has a simple zero eigenvalue with the corresponding eigenvector $\mathbf{1} = [1, \ldots, 1]^T$.
- Thus $\lambda_2(G) > 0$ for a connected graph.

Cheeger's inequality

- Let $S \subset V$ and $S^C = V \setminus S$
- $\varepsilon(S) =$ card$\{(i,j) \in E | (i \in S, j \in S^C) \vee (i \in S^C, j \in S)\}$
- $\varepsilon(S)$ is # edges needed to be cut to separate S from S^C
- $\varphi(S) = \frac{\varepsilon(S)}{\min\{|S|,|S^C|\}}$
- Isoperimetric number of G: $\phi(G) = \min_S\{\varphi(S)\}$
- Cheeger's inequality: $\phi(G) \geq \lambda_2(G) \geq \frac{\phi(G)^2}{2\max_{i \in V}\{d_i\}}$
- $\phi(G)$ and thus $\lambda_2(G)$ are a metric of connectivity of the graph

The incidence matrix

- Orientation of G: assignment of direction to each edge.
- Incidence matrix of oriented graph with $M = |E|$ edges:

$$B = [b_{ij}], b_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E \quad \text{and } i \text{ head of } (i,j), \\ -1 & \text{if } (i,j) \in E \quad \text{and } i \text{ tail of } (i,j), \\ 0 & \text{if } (i,j) \notin E. \end{cases}$$

- Laplacian representation: $L = BB^T$

Eigenvalues of the incidence matrix

- If G is a tree, then $B^T B$ is positive definite.
- Thus if G is a tree, then $\lambda_{\min}(B^T B) > 0$

Next Lecture

Consensus

- Definition of consensus/rendezvous
- Consensus for undirected fixed graphs
- Consensus for undirected switching graphs
- Graph theoretic issues for directed graphs
- Consensus for directed fixed graphs