Guarding, Searching and Pursuing Evaders using Multiagent Systems

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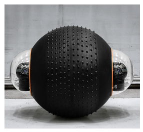
Computer Vision and Active Perception (CVAP)

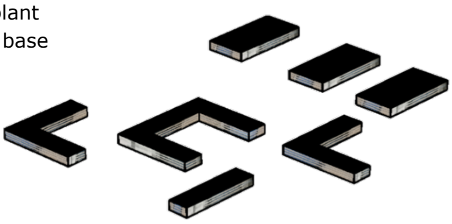
Todays topics

- **Cooperative guarding**
  - Static guards
- **Cooperative search**
  - Static targets
- **Cooperative pursuit evasion**
  - Moving targets and guards



Example Scenario

- Airport
- Power plant
- Military base
- Port
- Factory
- …



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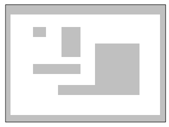
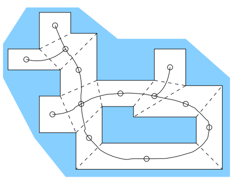


- This field is very broad
  - Overview of problems, results and tools

Common theme: Discretizing the search space

- Partitioning search space into convex sets is often useful
- Create a graph
  - Set <-> vertex
  - Neighbor <-> edge

Some naïve solutions:
- Guarding
  - Each set has guard on border
- Search
  - Travelling salesman



Can we improve on these conservative solutions?

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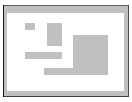
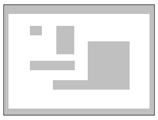
Coordinated Guarding/Coverage

- Applications: Art gallery, Industrial Area, Police positioning
- Possible objectives:
  - Min no of cameras,
  - Max coverage with N cameras,
  - Weighted coverage
- Environment: 2D/3D



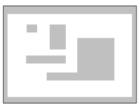
Bounds on number of Guards

- The **General Art Gallery Problem**: What is the smallest number of guards needed to cover **any** polygon with n vertices and h holes.
- For h=0, Chvatal (1975) proved bound: Floor(n/3)
- Hoffmann (1991) proved bound: Floor ((n+h)/3)



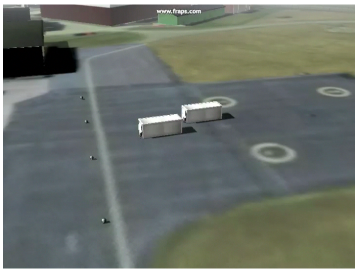
Minimize number of guards (3D etc)

- Problem: (Min number of guards)
- Problem (Minimum set cover) Let E = {e1, . . . , en} be a finite set of elements, and let S = {s1,...,sm} be a collection of subsets of E, i.e. sj ⊆ E. The problem minimum set cover is the problem of finding a minimum subset S′ ⊆ S such that every elements ei ∈ E belongs to at least one subset in S′. We say that E is covered by S′.
  - NP-hard
  - Greedy algorithm performs well , Eidenbenz (2002)



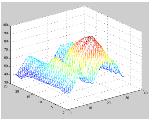
Movie: Guarding with resolution constraints

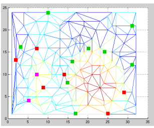
Guarding with resolution constraints



Minimizing number of guards (3D etc)

- Marangoni (2000)
  - Triangulation of 3D environment
  - Vertex coloring to find subset
  - Visibility computation to get candidates
- Efrat (2002) randomized search instead of the greedy





Further reading on Guarding

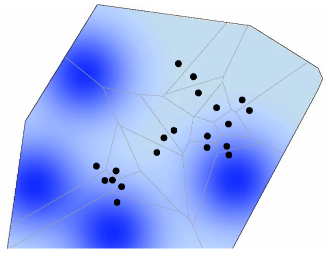
- V. Chvatal. A Combinatorial Theorem in Plane Geometry. Journal of Combinatorial Theory Series B, 18:39–41, 1975.
- F. Hoffmann, M. Kaufmann, and K. Kriegel. The Art Gallery Theorem for Polygons With Holes. Proceedings of the 32nd Annual Symposium on Foundations of Computer Science, pages 39–48, 1991.
- S. Eidenbenz. Approximation Algorithms for Terrain Guarding. Information Processing Letters, 82(2):99–105, 2002.
- M. Marengoni and B. Draper. System to Place Observers on a Polyhedral Terrain in Polynomial Time. Image and Vision Computing, 18(10):773– 780, 2000.
- A. Efrat and S. Har-Peled. Guarding Galleries and Terrains. Proceedings of the IFIP 17th World Computer Congress-TC1 Stream, 2002.
- U. Nilsson, P. Ögren, and J. Thunberg, "Optimal positioning of surveillance UGVs," presented at the 2008 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2008), pp. 2539–2544.
- W.R. Franklin. Siting Observers on Terrain. Symposium on Spatial Data Handling, Ottawa, pages 109–120, 2002.

Bullo Coverage

- Distribute agents p_i to
- Minimize Expected squared distance
  - From random event
  - To nearest agent

$$\mathcal{H}(P,\mathcal{W}) = \sum_{i=1}^{n} \int_{W_i} \|q - p_i\|^2 \phi(q) dq$$

Bullo Coverage $\mathcal{H}(P,\mathcal{W}) = \sum_{i=1}^{n} \int_{W_i} \|q - p_i\|^2 \phi(q) dq$



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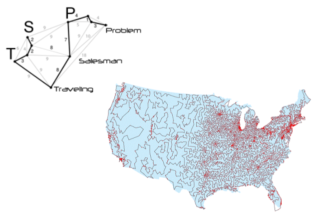
Cooperative Search (Background: TSP)

- Travelling Salesperson Problem (TSP)
- Variations
  - Multi-TSP
  - Metric TSP
  - Vehicle routing problem
  - Max capacity
  - Time windows
  - …

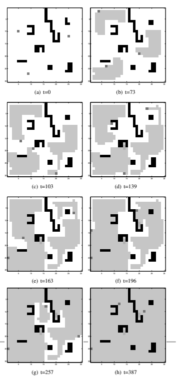


13000 cities, Applegate, Bixby, Cook and Chvatal

"It involves ideas from polyhedral combinatorics and combinatorial optimization, integer and linear programming, computer science data structures and algorithms, parallel computing, software engineering, numerical analysis, graph theory, and more."

Cooperative Search

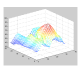
- Sensor range gives two cases
- Range similar to environment size
  - (next slide)
- Range << environment size
  - Shuzhi (2005) proposes solution →

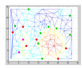


Cooperative Search (long sensor range)

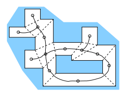
Possible approaches:

- Use Guard positions and solve m-TSP



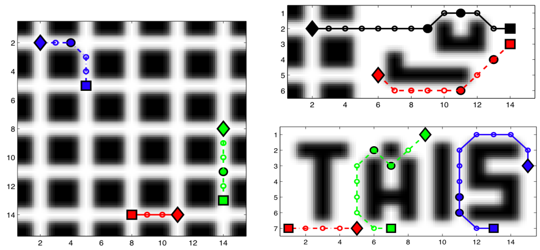


- Discretize to a graph and solve m-TSP



- Use convex cover and solve m-TSP …

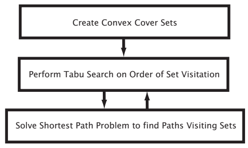
Example of Cooperative search (Anisi 2010)

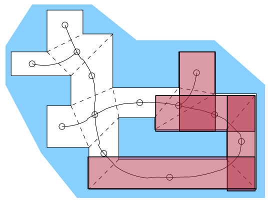


How?

Cooperative Search

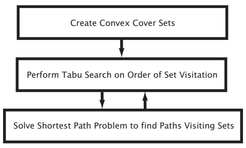
- How can we make search less conservative?
- Replace partition with overlapping convex cover

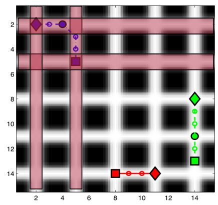




Cooperative Search

- How can we make search less conservative?
- Replace partition with overlapping convex cover





Further reading on cooperative search

- E. Frazzoli and F. Bullo. Decentralized algorithms for vehicle routing in a stochastic time-varying environment. In Proc. of the 43rd IEEE Conference on Decision and Control, CDC, 2004.
- Maria John, David Panton, and Kevin White. Mission planning for regional surveillance. Annals of Operations Research, 108:157–173, Nov. 2001.
- Shuzhi Sam Ge and Cheng-heng Fua. Complete Multi-Robot Coverage of Unknown Environments with Minimum Repeated Coverage. In IEEE International Conference on Robotics and Automation, Barcelona, Spain, pages 727–732, April 2005.
- I. I. Hussein and Stipanovic, "Effective Coverage Control using Dynamic Sensor Networks," presented at the Decision and Control, 2006 45th IEEE Conference on, 2006.
- D. A. anisi, P. Ögren, and X. Hu, "Cooperative Minimum Time Surveillance With Multiple Ground Vehicles," *Automatic Control, IEEE Transactions on*, vol. 55, no. 12, pp. 2679–2691, 2010.

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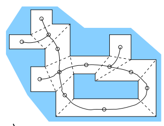
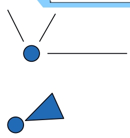
Cooperative **Pursuit Evasion**

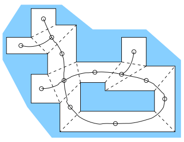


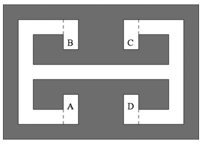
- First introduced by Parsons (1976)
  - Problem on a graph
  - Multiple searchers
- A continuous version: Suzuki et al. (1992).
  - simple polygon
  - single searcher (k-searcher)
- Limited field of view: Gerkey et al. (2006)
  - capability of a robot with a camera
  - (phi-searcher)



Randomized **Pursuit Evasion**

- Randomized strategy: Isler et al. (2005).

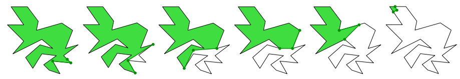




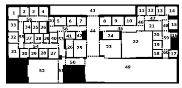
- By repeating a randomized strategy, capture probability can be made arbitrarily high (if simply connected)

Cooperative **Pursuit Evasion**

- Efrat et al.(2000) consider chains of searchers in simple polygons



- Hollinger et al. (2007) a probabilistic similar to *Markov Decision processess* (MDP) and *partially observable MDP* (POMDP)



- Thunberg (2011) MILP/MPC formulation …

MILP/MPC approach to Pursuit Evasion

Problem 2 (MILP) *Given a $T \in \mathbb{Z}^+$ solve the following integer linear program.*

$$\max\ Z = \alpha \sum_{i \in J} \theta_{iT} + (1-\alpha) \sum_{i \in J} \sigma_{iT} \quad (1)$$

subject to

Constraints addressing: λ_{it} *(pursuer locations),*

$$\sum_{i \in J} \lambda_{it} - N = 0, \quad (2)$$

$$N - (N-1)\lambda_{it} - \sum_{j \in M_i} \lambda_{jt} \geq 0, \quad (3)$$

$$\sum_{j \in M_i} \lambda_{jt} - \lambda_{i(t-1)} \geq 0, \quad (4)$$

$$2 - \sum_{j \in M_i} \lambda_{j(t-1)} - \lambda_{it} \geq 0. \quad (5)$$

Constraints addressing: σ_{it} *(seen regions)*

$$\sum_{j \in N_i} \lambda_{jt} - \sigma_{it} \geq 0, \quad (6)$$

$$\sigma_{it} - \lambda_{jt} \geq 0, \quad \forall j \in N_i \quad (7)$$

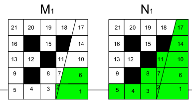
Constraints addressing: θ_{it} *(unseen cleared regions)*

$$\sigma_{jt} + \theta_{jt} - \theta_{it} \geq 0, \quad \forall j \in M_i - \{i\}, \quad (8)$$

$$\sigma_{i(t-1)} + \theta_{i(t-1)} - \theta_{it} \geq 0, \quad (9)$$

$$1 - \sigma_{it} - \theta_{it} \geq 0, \quad (10)$$

$$\theta_{i1} = 0, \quad (11)$$

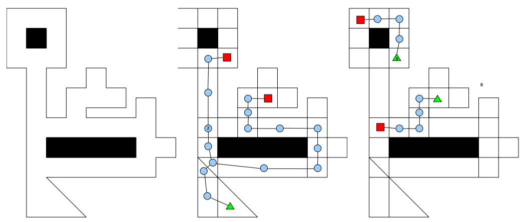


Movement neighborhood

Visible neighborhood

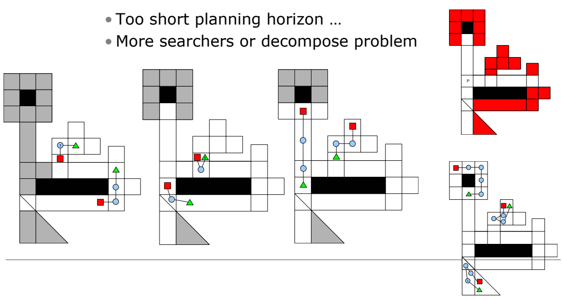
MILP/MPC approach to Pursuit Evasion

- Long planning horizon …



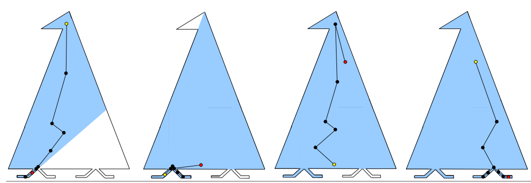
MILP approach to Pursuit Evasion

- Too short planning horizon …
- More searchers or decompose problem



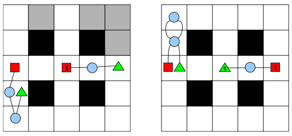
MILP: Recontamination

- Sometimes recontamination is required
- Then Success depends on planning horizon



MILP: Additional constraint

- Additional constraint can be added:
  - Line of sight every 4th timestep



MILP/MPC Pros and Cons

- Handles recontamination (planning horizon)
- Handles connectivity constraints
- Scales poorly with planning horizon length
- Scales poorly with environment size
  - Add static "guard" to partition environment
- Scales well with number of agents

Further reading on pursuit evasion

- T. D. Parsons. Theory and applications of graphs, Lecture Notes in Mathematics, chapter Pursuit-evasion in a graph, pages 426–441. Springer, 1976.
- I. Suzuki and M. Yamashita. Searching for a mobile intruder in a polygonal region. SIAM Journal on Computing, 21(5), 1992.
- V. Isler, S. Kannan, and S. Khanna. Randomized pursuit-evasion in a polygonal environment. IEEE Transactions on Robotics, 21(5), 2005.
- B. P. Gerkey, S. Thrun, and G. Gordon. Visibility-based pursuit-evasion with limited field of view. International Journal of Robotics Research, 25(4), 2006.
- A. Efrat, L. J. Guibas, S. Har-Peled, Lin D. C., J. S. B. Mitchel, and T. M. Murali. Sweeping simple polygons with a chain of guards. In Proceedings of the 11th ACM-SIAM Symposium on Discrete Algorithms, 2000. San Francisco, California, January.
- Geoffrey Hollinger, Athanasios Kehagias, and Sanjiv Singh. Probabilistic Strategies for Pursuit in Cluttered Environments with Multiple Robots. IEEE International Conference on Robotics and Automation, 2007.
- J. Thunberg and P. Ögren, "A Mixed Integer Linear Programming approach to pursuit evasion problems with optional connectivity constraints," *Autonomous Robots*, vol. 31, no. 4, pp. 333–343, Aug. 2011.

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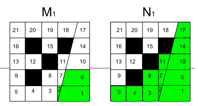

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