

Distributed event-based Model Predictive Control for Multi-Agent systems under disturbances

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Abstract—In this paper, we propose an aperiodic formulation of Distributed Model Predictive Control for the cooperation of multi-agent systems under additive bounded disturbances. In the proposed method, each agent solves an Optimal Control Problem only when certain control performances cannot be guaranteed according to certain triggering rules. This could lead to the reduction of energy consumption and the alleviation of over-usage of communication loads under critical resource constraints in networked control systems, such as limited communication power and the life-time of the battery. The triggering rule is derived for event-based case, where control inputs are executed based on the current state measurement. Our proposed method is also verified through a simple simulation example.

I. INTRODUCTION

The analysis of formation control of cooperating distributed agents such as autonomous vehicles or mobile robots, is an important area of research in the last decades. This has been motivated due to the fact that it increases the efficiency and lowers the overall loads and costs by completing tasks together working as a team of agents. One of the most common problems of formation control involves distributed agents cooperatively achieving their desired formations and destinations. In this case, the control method has to be designed to deal with several problems, such as how to avoid their collisions, how to achieve their desired formation, and how to deal with the actuator limitations. One of the most attractive control schemes is the use of Distributed Model Predictive Control (DMPC), and an extensive research has already been done, e.g., [2], [7], [8], [14], [15], [16], [17]. In this control scheme, each agent tries to solve a Finite Horizon Optimal Control Problem (FHOC) on-line given the current state of the plant and the information of the neighbors to predict their behavior. The most common approach to DMPC is the case of decoupled sub-systems, where each sub-system is not directly influenced by the others. The cost function is instead, coupled and affected by others as a part of the total cost. This implies that choosing the controller is influenced by the cooperation with the neighbors and consequently, may indirectly affect the individual dynamics.

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Another attractive research deals with how to reduce energy consumption and the over-usage of communication resources in large scale interconnected networks. The battery discharge could be one of the serious concerns due to the communication and computational loads when using limited life-time of battery powered devices. The event-triggered control scheme has been increasing its interest in recent years in order to alleviate these network constraints. In the event-triggered control method, control inputs are executed only when certain desired control performances (such as closed loop stability) cannot be guaranteed. This is in contrast to the conventional time-triggered control where the control inputs are executed periodically, which may lead to redundant execution of control inputs and the over-usage of communication resources among sub-systems. Thus the event-triggered control method could alleviate the communication load by executing control input aperiodically while saving desired control performances.

Event-based and self-triggered control are two main different aperiodic control approaches, see, e.g., [9], [13], [19], [20] for the event-based case, and [21], [13], [22] for the self-triggered case. The major difference between event-based and self-triggered control is that, the event-based control scheme requires current measurements of states of the plant, while in the self-triggered case the next control execution can be pre-computed without measuring the states of the system based on the prediction and the dynamics.

Although the DMPC framework requires to solve the FHOC periodically, and thus not only the periodic communication among agents but also computation loads to solve FHOC are required, the combination between the event-triggered control and DMPC has received some attention in recent years; the readers can refer to [1], [4], [5], [6], [10], [12] for some recent results. In [1], the authors consider deriving event-based DMPC for distributed agents having nonlinear dynamics with no additive disturbances, and the FHOC is aperiodically solved according to an event-based triggering rule with respect to the information error from the neighboring agents based on stability analysis. While these methods may lead to the reduction of the energy consumption, the triggering rules could be conservative as the size of the network increases, since the information error for all of the agent's neighbors is taken into account for the stability criterion. In [10], the authors consider a self-triggered MPC for single agent with linear dynamics. A sub-optimal control input is designed, through which the infinite horizon quadratic cost is evaluated. The event-triggered rule described in [10] takes over the fundamental concept of

the event-triggered control; the control input is executed in an aperiodic way, which renders it different from the other papers where only the FHOCP is aperiodically solved and thus the control execution can be periodic.

In this paper, we consider deriving event-based DMPC for cooperation of distributed agents having nonlinear dynamics, where the FHOCP is aperiodically solved according to certain triggering rules. A main novelty with respect to earlier approaches is that we derive a triggering condition which does not involve the neighbors' information by changing the expression of the Lyapunov function candidate, and thus the triggering rules could be less conservative with respect to earlier approaches and the periodic usage of communication is not needed in this case. Moreover, additive bounded disturbances are taken into account, and the corresponding triggering rule is obtained by robust stability of DMPC. This is clearly more practical than the conventional methods, as we cannot avoid any disturbances when applying control schemes to the real world. Therefore, our proposal could lead not only to a reduction of the computational load of DMPC but also to the relaxation of over-usage of communication resources for the cooperation of perturbed multi-agent systems. The feasibility and stability analysis for our proposals are also provided in detail.

The rest of this paper is organized as follows. In Section II, the mathematical modeling and the problem formulation of DMPC for distributed agents are described along with several assumptions. In Section III, a triggering condition for event-based DMPC is described. Feasibility and the stability under our proposed scheme are shown in Section IV and V. Simulation results verify our proposal in Section VI, and finally a summary of the results of this paper is given in Section VII.

II. PROBLEM FORMULATION

In this section the problem formulation is going to be presented. At first, the mathematical modeling is provided and then the design for DMPC for each agent is formulated.

A. Modeling

We consider a distributed system consisting of M agents, where each agent is controlled by a local Model Predictive Controller. The nominal model for each agent is given by a nonlinear difference equation

$$\hat{x}_i(k+1) = f_i(\hat{x}_i(k), u_i(k)) \quad (1)$$

where for each $i = 1, \dots, M$, $\hat{x}_i(k) \in \mathbb{R}^n$ is the state of i -th agent \mathcal{A}_i at time k and $u_i(k) \in \mathbb{R}^m$ is control variable. This general class of nonlinear system is considered in this paper so that various type of systems can be included, such as the kinematics of the autonomous vehicles or flights, see e.g., [16], [17]. We further consider that the agent evolves under the influence of a certain disturbance. That is, the dynamics of the actual state is given by a perturbed model;

$$x_i(k+1) = f_i(x_i(k), u_i(k)) + w_i(k), \quad w_i(k) \in \mathcal{W} \quad (2)$$

where w_i is an additive disturbance which belongs to a compact set $w_i(k) \in \mathcal{W}$ and is assumed to be bounded by $\|w_i(k)\| \leq \bar{w}$, where $\|\cdot\|$ denotes the Euclidean norm. We assume $f_i(0, 0) = 0$ and the constraints for the state and control input on each agent are of the form $x_i(k) \in \mathcal{X}_i, u_i(k) \in \mathcal{U}_i$. Moreover, we assume that the nonlinear function $f_i(x, u)$ is Lipschitz continuous in $x \in \mathcal{X}_i$ and its Lipschitz constant is L_{f_i} , i.e.,

$$\|f_i(x_1, u) - f_i(x_2, u)\| \leq L_{f_i} \|x_1 - x_2\|$$

for $x_1 \in \mathcal{X}_i$ and $x_2 \in \mathcal{X}_i$. This assumption is later used to derive event-based condition for DMPC and the stability property.

B. DMPC Formulation

In the proposed DMPC framework, each agent \mathcal{A}_i solves a FHOCP at current time k , involving the predictive states $\hat{x}_i(k+l|k)$ for future time $k+l$ obtained from the nominal model (1) and predictive control inputs $u_i(k+l|k)$, based on its current state $x_i(k)$ and the information from its neighbors. The current and predictive states and control inputs are denoted in vector format as

$$\mathbf{x}_i(k) = \{\hat{x}_i(k+l|k)\}_{l=0}^N, \quad \mathbf{u}_i(k) = \{u_i(k+l|k)\}_{l=0}^{N-1}$$

with $\hat{x}_i(k|k) = x_i(k)$ and N is the prediction horizon. Furthermore, we consider a partially connected structure; each agent \mathcal{A}_i can exchange the state information with neighboring cooperating agents $\mathcal{G}_i = \{\mathcal{A}_j, j \in G_i\}$, where G_i denotes the set of indices of agents belonging to the set \mathcal{G}_i . We assume that \mathcal{G}_i is non-empty and the information flow is bidirectional; that is, every agent has at least one neighbor and the information can be exchanged between neighboring agents.

The information vector from the neighboring agent j is denoted as $\mathbf{z}_j(k) = \{z_j(k+l)\}_{l=0}^N$, where each component $z_j(k+l)$ is the predictive state of the j -th agent at $k+l$. This information vector depends on whether the FHOCP is solved or not, and thus more specific definitions of the information vector are provided later in this text. The overall information which \mathcal{A}_i obtains from all of its neighboring agents at time k are collected as one stack vector $\mathbf{z}_{G_i}(k)$

$$\mathbf{z}_{G_i}(k) = \text{col}(\mathbf{z}_j(k), j \in G_i)$$

Given the state $x_i(k)$ at time k and its neighbors' information vector $\mathbf{z}_{G_i}(k)$, we define the following type of cost function to be minimized (see e.g., [1], [2], [16], [17]) ;

$$\begin{aligned} & J_i(\mathbf{x}_i(k), \mathbf{u}_i(k), \mathbf{z}_{G_i}(k)) \\ &= J_i^H(\mathbf{x}_i(k), \mathbf{u}_i(k)) + J_i^Q(\mathbf{x}_i(k), \mathbf{z}_{G_i}(k)) \end{aligned}$$

where the easier notation $J_i(k) = J_i^H(k) + J_i^Q(k)$ is used for the rest of this paper. The cost consists of two terms; $J_i^H(k)$ is the cost for the agent itself and is given by

$$\begin{aligned} J_i^H(k) &= \sum_{l=0}^{N-1} \{h_i(\hat{x}_i(k+l|k), u_i(k+l|k))\} \\ &\quad + V_i(\hat{x}_i(k+N|k)) \end{aligned}$$

where $\hat{x}_i(k+l|k)$ is the predictive state obtained from the nominal model (1), $V_i(\hat{x}_i(k+N|k))$ is a terminal cost, and N is the prediction horizon. The second cost $J_i^Q(k)$ involves the information from the neighbors and is given by

$$J_i^Q(k) = \sum_{l=0}^{N-1} \sum_{j \in G^i} q_{ij}(\hat{x}_i(k+l|k), z_j(k+l))$$

where q_{ij} is a coupling cost between neighboring agents \mathcal{A}_i and \mathcal{A}_j . In this paper we assume that the coupling cost q_{ij} is given by quadratic form $q_{ij}(x_i, z_j) = (x_i - z_j + d_{ij})^T Q_{ij}(x_i - z_j + d_{ij})$, where Q_{ij} is positive definite weighted matrix, and d_{ij} is a desired distance vector between agents \mathcal{A}_i and \mathcal{A}_j . Thus, this coupling cost is used in order to achieve the desired formation.

The FHOCP for the perturbed model (2) is now ready to be formulated;

$$\begin{aligned} \min \quad & J_i(\mathbf{x}_i(k), \mathbf{u}_i(k), \mathbf{z}_{G_i}(k)) \\ & \mathbf{u}_i(k) \\ \text{s.t.} \quad & \hat{x}_i(k+l+1|k) = f_i(\hat{x}_i(k+l|k), u_i(k+l|k)) \\ & \hat{x}_i(k+l|k) \in \mathcal{X}_i^l \\ & u_i(k+l|k) \in U_i \\ & \hat{x}_i(k+N|k) \in \mathcal{X}_{f_i} \\ & J_i^H(\mathbf{x}_i(k), \mathbf{u}_i(k)) \leq \gamma_i(k) \end{aligned} \quad (3)$$

where the constraint for $\hat{x}_i(k+l|k)$ is narrowed to $\hat{x}_i(k+l|k) \in \mathcal{X}_i^l \subseteq \mathcal{X}_i$ to make sure that there is a robust positively invariant set for the closed loop system where a solution to the FHOCP exists, see [3] for more detailed explanation. More specifically, the restricted constraint set \mathcal{X}_i^l is given by $\mathcal{X}_i^l = \mathcal{X}_i \sim B_i^l$, where $B_i^l = \{x \in \mathbb{R}^n : \|x\| \leq \frac{L_{f_i}^l - 1}{L_{f_i} - 1}\}$ and \sim denotes Pontryagin difference. The terminal constraint \mathcal{X}_{f_i} is a set given by $\mathcal{X}_{f_i} = \{x \in \mathbb{R}^n : V_i(x) \leq \alpha_{v_i}\}$ and $\mathcal{X}_{f_i} \subseteq \mathcal{X}_i$. The last constraint $J_i^H(\mathbf{x}_i(k), \mathbf{u}_i(k)) \leq \gamma_i(k)$ is imposed for ensuring the stability for each agent. A more specific definition of $\gamma_i(k)$ is formulated in the next section. We further make following assumptions for the stability analysis :

Assumption 1: The running costs $h_i(x_i, u_i)$ is Lipschitz continuous in $x_i \in \mathcal{X}_i$, with Lipschitz constant L_{h_i} . Furthermore, the terminal cost $V_i(x)$ is Lipschitz in $x \in \Phi_i$ with Lipschitz constant L_{V_i} .

Assumption 2: There exists a local stabilizing controller $\kappa_i(x) \in U_i$ in the sense that

$$V_i(f_i(x, \kappa_i(x))) - V_i(x) \leq -h_i(x, \kappa_i(x)) \quad (4)$$

for all $x \in \Phi_i$, where Φ_i is a compact set given by $\Phi_i = \{x \in \mathbb{R}^n : V_i(x) \leq \alpha_i\}$ and $\Phi_i \subseteq \mathcal{X}_i^{N-m}$ for $m = 1, \dots, N-1$.

Assumption 3: The set $\mathcal{X}_{f_i} = \{x \in \mathbb{R}^n : V_i(x) \leq \alpha_{v_i}\}$ is such that for all $x \in \Phi_i$, $f(x, \kappa_i(x)) \in \mathcal{X}_{f_i} \subseteq \Phi_i$.

Remark 1: All of these assumptions are fairly general for guaranteeing stability property of MPC under additive bounded disturbances; see e.g., [3], [18], where the same assumptions are used. In this paper we make use of these assumptions in order to derive our proposed triggering rules as described in the next section. Note that the set \mathcal{X}_i^l could

be very small if the Lipschitz constant L_{f_i} is relatively large, especially when it is larger than 1. This problem is also addressed in [3], where several methods to reduce this conservativeness are given. For example, the control parametrization method can be used by giving a feedback structure to reduce L_{f_i} ; $u_i(k) = Kx_i(k) + v_i(k)$, where $v_i(k)$ is a new control variable.

The solution to the FHOCP gives an optimal control input sequence and the corresponding predictive states denoted by

$$\mathbf{x}_i^*(k) = \{\hat{x}_i^*(k+l|k)\}_{l=0}^N, \quad \mathbf{u}_i^*(k) = \{u_i^*(k+l|k)\}_{l=0}^{N-1}$$

where $\hat{x}_i^*(k|k) = x_i(k)$. Then, in the event-triggered formulation, some part of this optimal input is applied to the system, i.e.,

$$u_i^{ap}(k+l|k) = u_i^*(k+l|k), \quad l = 0, \dots, m-1$$

where $k+m$ denotes the next time step when the FHOCP is going to be solved obtained by the triggering condition. Before deriving the triggering condition, some useful Lemmas are given;

Lemma 1: The difference between the true state $x_i(k+l)$ and the predictive state $\hat{x}_i(k+l|k)$ is bounded by $\|\hat{x}_i(k+l|k) - x_i(k+l)\| \leq \delta_i(l)$ where $\delta_i(l) = \frac{L_{f_i}^l - 1}{L_{f_i} - 1} \bar{w}$.

Lemma 2: Let \mathcal{X}_i^l be given by $\mathcal{X}_i^l = \mathcal{X}_i \sim B_i^l$, where $B_i^l = \{x \in \mathbb{R}^n : \|x\| \leq \frac{L_{f_i}^l - 1}{L_{f_i} - 1}\}$ for $l \geq 1$, and let $x \in \mathcal{X}_i^l$ and $y \in \mathbb{R}^n$ be such that $\|x - y\| \leq L_{f_i}^{l-1} \bar{w}$. Then $y \in \mathcal{X}_i^{l-1}$. For the proofs, the reader can refer to [3].

III. DERIVING EVENT-BASED CONDITION FOR DMPC

In this section the event-based triggering rule is derived. ¹ Assuming that we solved the FHOCP at k , then this provides an optimal control sequence $\mathbf{u}_i^*(k)$ and the corresponding optimal cost denoted by $J_i^*(k) = J_i^{H^*}(k) + J_i^{Q^*}(k)$, where $J_i^{H^*}(k) = J_i^H(\mathbf{x}_i^*(k), \mathbf{u}_i^*(k))$ and $J_i^{Q^*}(k) = J_i^Q(\mathbf{x}_i^*(k), \mathbf{z}_{G_i}(k))$. Then, consider that the following control sequence $\bar{\mathbf{u}}_i(k+m) = \{\bar{u}_i(k+l|k+m)\}_{l=m}^{m+N-1}$ where $1 \leq m < N$ is used to obtain the predictive state sequence $\bar{\mathbf{x}}_i(k+m) = \{\bar{x}_i(k+l|k+m)\}_{l=m}^{m+N}$ (given $\bar{x}_i(k+m|k+m) = x_i(k+m)$) from $k+m$: for $m = 1$,

$$\bar{u}_i(k+l|k+m) = \begin{cases} u_i^*(k+l|k) & (\text{for } l = 1, \dots, N-1) \\ \kappa_i(\bar{x}(k+l|k+m)) & (\text{for } l = N) \end{cases} \quad (5)$$

This means that the optimal control inputs are used until $N-1$, and the local stabilizing controller (4) is used at the last step. For $1 < m < N$, $\bar{\mathbf{u}}_i(k+m)$ is given by

$$\bar{u}_i(k+l|k+m) = \begin{cases} \bar{u}_i(k+l|k+m-1) & (\text{for } l = m, \dots, N+m-2) \\ \kappa_i(\bar{x}(k+l|k+m)) & (\text{for } l = N+m-1) \end{cases} \quad (6)$$

which means that the control inputs at $k+m$ is based on ones at the previous step; $\bar{u}_i(k+l|k+m-1)$ is used for $\bar{u}_i(k+l|k+m)$ until $N+m-2$, and the local stabilizing

¹In the extended version submitted as the journal publication, the self-triggered condition and the feasibility and stability are also derived in detail.

controller is used at the last step. The corresponding cost is simply denoted as $\bar{J}_i(k+m) = \bar{J}_i^H(k+m) + \bar{J}_i^Q(k+m)$, where $\bar{J}_i^H(k+m) = J_i^H(\bar{\mathbf{x}}(k+m), \bar{\mathbf{u}}_i(k+m))$ and $\bar{J}_i^Q(k+m) = J_i^Q(\bar{\mathbf{x}}(k+m), \mathbf{z}_{G_i}(k+m))$.

At $k+m$, we will assume that this (admissible) control input sequence $\bar{\mathbf{u}}_i(k+m)$ is going to be applied instead of solving the FHOCP, and check triggering conditions if the stability is still guaranteed. Specifically, we propose the following triggering rule.

(Triggering rule): The FHOCP is solved only when $J_i^H(k)$ is not guaranteed to decrease.

This means that we take $J_i^H(k) = J_i^H(\mathbf{x}_i(k), \mathbf{u}_i(k))$ as a Lyapunov candidate, instead of using the total cost $J_i(k)$. The reason for this is that if we had the total cost as a Lyapunov candidate, we would need to take into account the information $\mathbf{z}_{G_i}(k)$ to evaluate triggering conditions and thus the periodic usage of communication resources would be required. Furthermore, the uncertainties of this information vector would be considered for all the neighbors, and thus the triggering condition might have been more conservative. Thus by taking only $J_i^H(k)$ as a Lyapunov candidate, which involves only the information of the agent i itself, the reduction of not only the utilization of communication resources but also of the conservativeness of the triggering rule can be achieved.

The problem when the partial cost is used here is, however, that the optimal control inputs are obtained by evaluating the total cost $J_i(k)$ and thus $\bar{J}_i^H(k)$ does not necessarily follow $J_i^{H*}(k) \leq \bar{J}_i^H(k)$, and the stability conditions are hard to verify. Motivated by this, we impose here an additional constraint for $J_i^H(k)$ in $J_i^H(\mathbf{x}_i(k), \mathbf{u}_i(k)) \leq \gamma_i(k)$, which corresponds to $J_i^H(\mathbf{x}_i(k+m), \mathbf{u}_i(k+m)) \leq \gamma_i(k+m)$ when the FHOCP is triggered at $k+m$. The upper bound $\gamma_i(k+m)$ is defined as

$$\begin{aligned} \gamma_i(k+m) &= \bar{J}_i^H(k+m-1) + L_{pi} \cdot \bar{w} \\ &\quad - h_i(x_i(k+m-1), u_i^*(k+m-1|k)) \end{aligned} \quad (7)$$

where $L_{pi} = L_{h_i} \frac{L_i^{N-1} - 1}{L_i - 1} + L_{V_i} L_{f_i}^{N-1}$, and $\bar{J}_i^H(k+m-1)$ is replaced with the optimal cost $J_i^{H*}(k)$ when $m=1$.

Having obtained $J_i^{H*}(k)$ at k , we first check if this cost is guaranteed to decrease from k to $k+1$. Consider that an admissible control input $\bar{\mathbf{u}}_i(k+1)$ is used for $k+1$ and obtain $\bar{J}_i^H(k+1)$. If $\bar{J}_i^H(k+1) < J_i^{H*}(k)$, the stability is already guaranteed without having to solve the FHOCP at $k+1$. Similarly, we check $\bar{J}_i^H(k+m) < \bar{J}_i^H(k+m-1)$ for $2 \leq m < N$ if the cost is guaranteed to decrease from $k+m-1$ to $k+m$. The following theorem provides more detailed condition if the cost is decreasing:

Theorem 1: Let $J_i^{H*}(k)$ be the optimal cost obtained at k , and $\bar{J}_i^H(k+m)$ be the cost obtained by applying (5) or (6). Then, a sufficient condition to satisfy $\bar{J}_i^H(k+1) < J_i^{H*}(k)$ is given by

$$L_{pi} \bar{w} \leq \sigma \cdot h_i(x_i(k), u_i^*(k|k)) \quad (8)$$

where $0 < \sigma < 1$. Furthermore, for $2 \leq m < N$, a sufficient

condition to satisfy $\bar{J}_i^H(k+m) < \bar{J}_i^H(k+m-1)$ is

$$L_{pi} \cdot \bar{w} \leq \sigma \cdot h_i(x_i(k+m-1), u_i^*(k+m-1|k)). \quad (9)$$

Proof: See Appendix. ■

The event-based MPC is therefore, formulated as follows;

(Event-based MPC): Assume that the FHOCP is solved at k . Then the FHOCP is solved at $k+1$ only when (8) is violated. For $k+m$ where $2 \leq m < N$, the FHOCP is solved at $k+m$ only when (9) is violated. When the triggering conditions (8) and (9) are satisfied for all $2 \leq m < N$, then the FHOCP is solved at $k+N$.

Definition 1 (Information vector for event-based case):

Let $\mathbf{z}_i(k+1)$ be the information vector that should be transmitted from agent i at time k . This information is sent to its neighbor, e.g., j , only when j decides to solve the FHOCP at $k+1$. When the agent i solves the FHOCP at k , the following optimal predictive states are transmitted

$$\mathbf{z}_i(k+1) = \{\hat{x}_i^*(k+1|k), \dots, \hat{x}_i^*(k+N|k), f_i(\hat{x}_i^*(k+N|k), \kappa_i(\cdot))\}. \quad (10)$$

When it is not solved, the predictive states obtained from $\bar{\mathbf{u}}_i(k)$ are transmitted, i.e.,

$$\mathbf{z}_i(k+1) = \{\bar{x}_i(k+1|k), \dots, \bar{x}_i(k+N|k), f_i(\bar{x}_i(k+N|k), \kappa_i(\cdot))\}. \quad (11)$$

IV. FEASIBILITY ANALYSIS

In this section the feasibility analysis of the FHOCP for each agent is going to be given. Analyzing the feasibility is important in the MPC framework since the FHOCP is solved in on-line fashion. In the event-triggered formulation, it is going to be shown that if we successfully solve the FHOCP at k , the FHOCP is feasible whenever until prediction horizon it is again going to be solved in the future $k+m$, which means that there exists at least one solution satisfying all the constraints imposed on (3) at $k+m$. The main theorem for the feasibility is provided below.

Theorem 2: Let the system be described by (2), and assume that all Assumptions 1-3 are satisfied. Then, the FHOCP solved by agent i is feasible if the disturbance is bounded by

$$\bar{w} \leq \frac{(\alpha_i - \alpha_{v_i})}{L_{V_i} L_{f_i}^{N-1}} \quad (12)$$

for all $i = 1, \dots, M$.

Proof: Assume that we successfully solve FHOCP at time k to get the optimal input sequence $\mathbf{u}_i^*(k)$ and the corresponding optimal cost $J_i^*(k)$, and then from the triggering rule, the next FHOCP is determined to be solved at $k+m$ where $1 \leq m < N$. The FHOCP is shown to be feasible at $k+m$ that there exists a solution satisfying all the constraints in (3) by considering that $\bar{\mathbf{u}}_i(k+m)$ given by either (5) or (6) is applied.

1) $\bar{\mathbf{u}}_i(k+m) \in \mathcal{U}_i$

This is clearly admissible from the expression of $\bar{\mathbf{u}}_i(k+l|k+m)$ given by (5) and (6).

- 2) $\bar{x}_i(k+l|k+m) \in \mathcal{X}_i^{l-m}$ for $l = m+1, \dots, N+m-1$. First we show this for $m = 1$. Since $\bar{u}_i(k+l|k+1) = u_i^*(k+l|k)$, we first obtain

$$\|\bar{x}_i(k+l|k+1) - \hat{x}_i^*(k+l|k)\| \leq L_{f_i}^{l-1} \bar{w}$$

where $\hat{x}_i^*(k+l|k) \in \mathcal{X}_i^l$. Thus from Lemma 2, $\bar{x}_i(k+l|k+1) \in \mathcal{X}_i^{l-1}$ for $l = 2, \dots, N$. For $m = 2$, since $\bar{u}_i(k+l|k+2) = \bar{u}_i(k+l|k+1)$ for $l = 2, \dots, N$, we get

$$\|\bar{x}_i(k+l|k+2) - \bar{x}_i(k+l|k+1)\| \leq L_{f_i}^{l-2} \bar{w}$$

for $l = 3, \dots, N+1$, where $\bar{x}_i(k+l|k+1) \in \mathcal{X}_i^{l-1}$. Thus from Lemma 2, $\bar{x}_i(k+l|k+2) \in \mathcal{X}_i^{l-2}$ for $l = 3, \dots, N+1$. By recursion, we get $\bar{x}_i(k+l|k+m) \in \mathcal{X}_i^{l-m}$ for $l = m+1, \dots, N+m-1$.

- 3) $\bar{x}_i(k+m+N|k+m) \in \mathcal{X}_{f_i}$. First we show $\bar{x}_i(k+m+N-1|k+m) \in \Phi_i$. For $m = 1$, by using

$$\|\bar{x}_i(k+N|k+1) - \hat{x}_i^*(k+N|k)\| \leq L_{f_i}^{N-1} \bar{w}$$

we get that

$$V_i(\bar{x}_i(k+N|k+1)) \leq \alpha_{v_i} + L_{V_i} L_{f_i}^{N-1} \bar{w} \leq \alpha_i$$

Hence $\bar{x}_i(k+N|k+1) \in \Phi_i$, thus by using Assumption 3, we obtain $\bar{x}_i(k+N+1|k+1) \in \mathcal{X}_{f_i}$. Similarly as above, for $m = 2$,

$$\begin{aligned} & \|\bar{x}_i(k+N+1|k+2) - \bar{x}_i(k+N+1|k+1)\| \\ & \leq L_{f_i}^{N-1} \bar{w} \end{aligned}$$

and we get

$$V_i(\bar{x}_i(k+N+1|k+2)) \leq \alpha_{v_i} + L_{V_i} L_{f_i}^{N-1} \bar{w} \leq \alpha_i$$

Hence $\bar{x}_i(k+N+1|k+2) \in \Phi_i$, thus by using Assumption 3, we get $\bar{x}_i(k+N+2|k+2) \in \mathcal{X}_{f_i}$. Therefore we recursively get that $\bar{x}_i(k+m+N-1|k+m) \in \Phi_i$ and $\bar{x}_i(k+m+N|k+m) \in \mathcal{X}_{f_i}$.

- 4) $\bar{J}_i^H(k+m) \leq \gamma_i(k+m)$

First we check for $m = 1$. The difference between $\bar{J}_i^H(k+1)$ and the optimal cost $J_i^{H*}(k)$ is bounded according to (18) from Appendix, and thus we get

$$\begin{aligned} \bar{J}_i^H(k+1) & \leq J_i^{H*}(k) - h_i(x_i(k), u_i^*(k|k)) + L_{p_i} \cdot \bar{w} \\ & = \gamma_i(k+1) \end{aligned} \quad (13)$$

Thus, $\bar{J}_i^H(k+1)$ satisfies $\bar{J}_i^H(k+1) \leq \gamma_i(k+1)$ and so the feasibility for this constraint is guaranteed when $m = 1$. For $1 < m < N$, we obtain from () that

$$\begin{aligned} \bar{J}_i^H(k+m) & \leq \bar{J}_i^H(k+m-1) + L_{p_i} \cdot \bar{w} \\ & \quad - h_i(x_i(k+m-1), u_i^*(k+m-1|k)) \\ & = \gamma_i(k+m) \end{aligned}$$

Therefore, the cost at step $k+m$ satisfies $\bar{J}_i^H(k+m) \leq \gamma_i(k+m)$, so the feasibility for the last constraint is guaranteed for $m = 1, \dots, N-1$. This completes the proof for the feasibility. ■

V. STABILITY ANALYSIS

In this section the stability analysis for DMPC under the event-based control scheme is going to be given. According to [18], it is proven for a single agent that the closed loop system under the standard MPC scheme is Input-to-State Stable (ISS) with respect to bounded disturbances, and that the trajectory of the state evolves towards inside the terminal region Φ_i .

In this paper, we will show that $J_i^H(k)$ is an ISS Lyapunov function, and the same stability property is guaranteed as in [18] by making use of the additional constraint imposed on the FHOCP. According to [18], $J_i^H(k)$ is said to be an ISS Lyapunov function for agent i , if the following is satisfied;

- 1) $J_i^H(k) \geq \alpha_1(x_i(k))$, $\forall x_i(k) \in \mathcal{X}_i$, and $J_i^H(k) \leq \alpha_2(x_i(k)) \forall x_i(k) \in \Phi_i$, where α_1 and α_2 are \mathcal{K}_∞ functions.
- 2) $J_i^H(k+1) - J_i^H(k) \leq -\alpha_3(x_i(k)) + \sigma(\|w\|)$, $\forall x_i(k) \in \mathcal{X}_i$, where α_3 is a \mathcal{K}_∞ function, and σ is a \mathcal{K} -function.

where 1) is obtained in the same way as in [18]. Before showing 2), we must note that $J_i^H(k)$ is not always the cost from the optimal control inputs, and thus this takes values in two different ways; when the FHOCP is solved, $J_i^{H*}(k) = J_i^H(x_i^*(k), u_i^*(k))$ and when not solved, $\bar{J}_i^H(k) = J_i^H(\bar{x}_i(k), \bar{u}_i(k))$. According to the triggering rules, we have already shown 2) for the differences $J_i^H(k+1) - J_i^{H*}(k)$ and $\bar{J}_i^H(k+1) - \bar{J}_i^H(k)$, i.e., when the triggering conditions are satisfied at $k+1$. Therefore, we need to check the difference when the FHOCP is solved at $k+1$, i.e., $J_i^{H*}(k+1) - J_i^{H*}(k)$ and $J_i^{H*}(k+1) - \bar{J}_i^H(k)$ for guaranteeing the ISS property. This is easily shown, since by using the last constraint $J_i^H(k+1) \leq \gamma_i(k+1)$ when the FHOCP is solved at $k+1$, we get from (7) that

$$\begin{aligned} J_i^{H*}(k+1) - J_i^{H*}(k) & \leq -h_i(x_i(k), u_i^*(k|k)) + L_{p_i} \cdot \bar{w} \\ & \leq -\lambda_{\min}(F) \|x_i(k)\|^2 + \sigma(\|w\|) \end{aligned}$$

where $J_i^{H*}(k)$ can be replaced with $\bar{J}_i^H(k)$. Therefore, from [18], it is proved that $J_i^H(k)$ is an ISS Lyapunov function, and the following stability result is concluded.

Theorem 3: Let the system be given by (2), and assume that all the Assumptions 1-3 are satisfied. Furthermore, assume that the additive disturbance is bounded by (12). The control update is given under the event-based condition(9), which is described above. Then, the system for agent i is ISS and thus the trajectory of the state reaches toward inside the terminal region Φ_i where it is ultimately bounded.

VI. SIMULATION RESULTS

The simulation results of the proposed methods are given in this section. Consider the position and formation control of 3 non-holonomic agents having the same dynamics in two dimensions, where the nominal model of each agent is given by

$$\begin{cases} x_i(k+1) = x_i(k) - v_i(k)T \cos \theta_i(k) \\ y_i(k+1) = y_i(k) - v_i(k)T \sin \theta_i(k) \\ \theta_i(k+1) = \theta_i(k) - \omega_i(k)T \end{cases} \quad (14)$$

which is simply denoted as $\chi_i(k+1) = f_i(\chi_i(k), u_i(k))$, where $\chi_i = [x_i, y_i, \theta_i]^T$ for $i = 1, 2, 3$ denotes the state vector consisting of the position of i -th agent and its direction θ_i . $u = [v_i, \omega_i]^T$ is the control input and the constraints are given by $|v_i| \leq \bar{v} = 2.5$ and $|\omega_i| \leq \bar{\omega} = 0.5$. $T = 0.2$ is the sampling time. The information flows are allowed between \mathcal{A}_1 and \mathcal{A}_2 , and between \mathcal{A}_1 and \mathcal{A}_3 , that is, $G_1 = \{2, 3\}$, $G_2 = \{1\}$, $G_3 = \{1\}$.

The matrices for the transition cost are given by $F = 0.5I_3$, $R = 0.1I_2$, and the coupling costs are $Q_{12} = Q_{13} = 0.1I_2$, $Q_{21} = 1.0I_2$, $Q_{31} = 1.0I_2$. The prediction horizon is set to $N = 35$ steps. The terminal cost is given by $V_i = \chi_i^T \chi_i$, and the parameters α_i and α_{v_i} for defining this terminal region are given by $\alpha_i = 2.25$ and $\alpha_{v_i} = 0.68$ according to the procedure in [17]. With computed Lipschitz constants $L_{f_i} = 1.01$, $L_{V_i} = 4.50$, the allowable disturbance for guaranteeing the feasibility is $\bar{w} = 0.25$. The initial points of the agents are $x_1(0) = [15, 10, \pi]^T$, $x_2(0) = [4, 20 - \pi/2]^T$, $x_3(0) = [15, 5, \pi]^T$ and the desired distance vector is $d_{12} = -d_{21} = [0, 2]$, $d_{13} = -d_{31} = [0, -2]$. We assume that their desired goals are $x_{\text{goal}1} = [0, 0, \pi]^T$, $x_{\text{goal}2} = [0, 2, \pi]^T$, $x_{\text{goal}3} = [0, -2, \pi]^T$.

Fig. 1 shows the trajectory of three agents with $\bar{w} = 0.20$ under event-based and standard DMPC schemes. The heading of triangles represent the direction of the agents, and filled triangles show the instants when the FHOCP was solved. Cross marks represent their goals. Fig. 1 shows that the agents $\mathcal{A}_1, \mathcal{A}_2$ and \mathcal{A}_3 solve the FHOCP for 21 steps, 20 steps, and 20 steps out of all 41 time steps respectively. Therefore, we can conclude that the agents could achieve their desired formation and their goals by aperiodically solving the FHOCP, which leads to the reduction of the energy consumption.

VII. CONCLUSIONS

We proposed event-based DMPC for distributed agents having nonlinear dynamics with additive bounded disturbances. In these control methods, each agent aperiodically solves an OCP in order not only to reduce the energy consumption but also to reduce the communication loads, while we can still guarantee the stability and feasibility. Finally our proposal was verified by a simple simulation result.

APPENDIX

(Proof of Theorem 1): The difference between $\bar{J}_i^H(k+1)$ and the optimal cost $J_i^{H*}(k)$ is given by

$$\begin{aligned} \Delta J_{i1}^H &= \bar{J}_i^H(k+1) - J_i^{H*}(k) \\ &= -h_i(x_i(k), u_i^*(k|k)) \\ &\quad + \sum_{l=1}^{N-1} \{h_i(\bar{x}_i(k+l|k+1), \bar{u}_i(k+l|k+1)) \\ &\quad \quad - h_i(\hat{x}_i^*(k+l|k), u_i^*(k+l|k))\} \\ &\quad - V_i(\hat{x}_i^*(k+N|k)) + V_i(\bar{x}_i(k+N+1|k+1)) \\ &\quad + h_i(\bar{x}_i(k+N|k+1), \kappa_i(\cdot)) \\ &\quad - V_i(\bar{x}_i(k+N|k+1)) + V_i(\bar{x}_i(k+N|k+1)) \end{aligned} \quad (15)$$

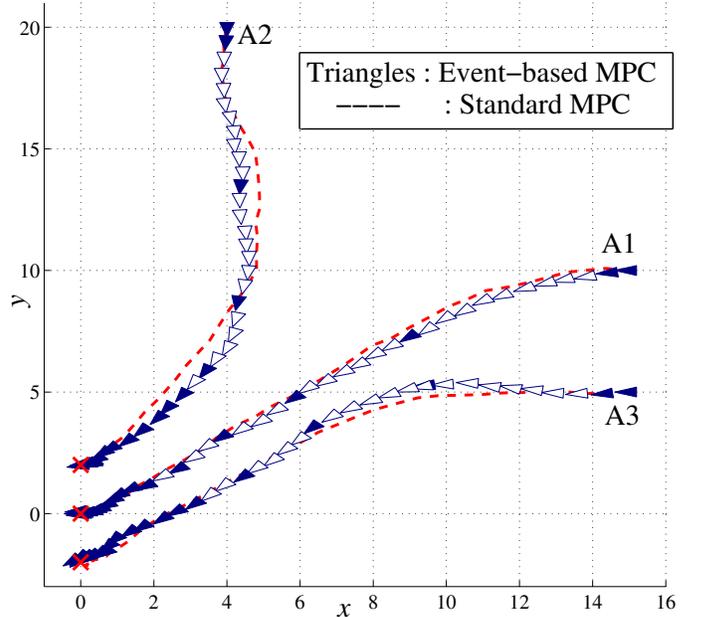


Fig. 1. Trajectories of the agents under the several schemes; triangles represent the trajectory of the agents under event-based MPC, and the dashed line represents the trajectory under standard MPC. The filled triangles show instants when the FHOCP was solved. Cross marks represent their goals.

By using the Assumption 2 for terminal cost and state, we get

$$\begin{aligned} &V_i(\bar{x}_i(k+N+1|k+1)) - V_i(\bar{x}_i(k+N|k+1)) \\ &\leq -h_i(\bar{x}_i(k+N|k+1), \kappa_i(\bar{x}_i(k+N|k+1))). \end{aligned}$$

Since $\bar{u}_i(k+l|k+1) = u_i^*(k+l|k)$ for $l = 1, \dots, N-1$, the difference for the transition cost h_i in the summation can be bounded by the Lipschitz continuity of h_i in Assumption 1:

$$\begin{aligned} &|h_i(\bar{x}_i(k+l|k+1), \bar{u}_i(k+l|k+1)) \\ &\quad - h_i(\hat{x}_i^*(k+l|k), u_i^*(k+l|k))| \\ &\leq L_{h_i} L_{f_i}^{l-1} \bar{w} \end{aligned} \quad (16)$$

Furthermore, the difference for the terminal cost V_i is bounded by the Lipschitz continuity of V_i in Assumption 1:

$$\begin{aligned} &|V_i(\hat{x}_i^*(k+N|k)) - V_i(\bar{x}_i(k+N|k+1))| \\ &\leq L_{V_i} L_{f_i}^{N-1} \bar{w} \end{aligned} \quad (17)$$

Therefore, the bound is given by

$$\begin{aligned} \Delta J_{i1}^H &\leq -h_i(x_i(k), u_i^*(k|k)) \\ &\quad + \sum_{l=1}^{N-1} (L_{h_i} L_{f_i}^{l-1} \bar{w}) + L_{V_i} L_{f_i}^{N-1} \bar{w} \\ &\leq -h_i(x_i(k), u_i^*(k|k)) + L_{p_i} \cdot \bar{w} \end{aligned} \quad (18)$$

Letting

$$L_{p_i} \bar{w} \leq \sigma \cdot h_i(x_i(k), u_i^*(k|k)) \quad (19)$$

where $0 < \sigma < 1$, we obtain

$$\Delta J_{i1}^H \leq (\sigma - 1) \cdot h_i(x_i(k), u_i^*(k|k)) < 0$$

and so the stability is guaranteed. Therefore, a sufficient condition to satisfy $\bar{J}_i^H(k+1) < J_i^{H*}(k)$ is (19). Now

assume that the triggering condition (19) is satisfied and we obtain $\bar{J}_i^H(k+1)$ at $k+1$. Then, we further consider that the admissible control input $\bar{u}(k+2)$ is going to be used for $k+2$ to get the next cost $\bar{J}_i^H(k+2)$, and take a difference from $\bar{J}_i^H(k+1)$ similarly to (15), i.e.,

$$\begin{aligned} \Delta J_{i2}^H &= \bar{J}_i^H(k+2) - \bar{J}_i^H(k+1) \\ &= -h_i(x_i(k+1), u_i^*(k+1|k)) \\ &\quad + \sum_{l=2}^N \{h_i(\bar{x}_i(k+l|k+2), \bar{u}_i(k+l|k+2)) \\ &\quad - h_i(\bar{x}_i(k+l|k+1), \bar{u}_i(k+l|k+1))\} \\ &\quad - V_i(\bar{x}_i(k+N+1|k+1)) \\ &\quad + V_i(\bar{x}_i(k+N+2|k+2)) \\ &\quad + h_i(\bar{x}_i(k+N+1|k+2), \kappa_i(\cdot)) \\ &\quad - V_i(\bar{x}_i(k+N+1|k+2)) \\ &\quad + V_i(\bar{x}_i(k+N+1|k+2)) \end{aligned}$$

From (5), $\bar{u}_i(k+l|k+1) = \bar{u}(k+l|k+2)$ for $l = 2, \dots, N$. By using the Assumption 4 for the terminal cost, we get

$$\begin{aligned} &V_i(\bar{x}_i(k+N+2|k+2)) - V_i(\bar{x}_i(k+N+1|k+2)) \\ &\leq -h_i(\bar{x}_i(k+N+1|k+2), \kappa_i(\bar{x}_i(k+N+1|k+2))) \end{aligned}$$

From Lemma 1, the difference between the transition costs h_i in the summation above is bounded by

$$\begin{aligned} &|h_i(\bar{x}_i(k+l|k+2), \bar{u}_i(k+l|k+2)) \\ &\quad - h_i(\bar{x}_i(k+l|k+1), \bar{u}_i(k+l|k+1))| \\ &\leq L_{h_i} \|\bar{x}_i(k+l|k+2) - \bar{x}_i(k+l|k+1)\| \\ &\leq L_{h_i} L_{f_i}^{l-2} \bar{w} \end{aligned} \quad (20)$$

Furthermore, the the difference for the terminal cost V_i is bounded by

$$\begin{aligned} &|V_i(\bar{x}_i(k+N+1|k+2)) - V_i(\bar{x}_i(k+N+1|k+1))| \\ &\leq L_{V_i} \|\bar{x}_i(k+N+1|k+2) - \bar{x}_i(k+N+1|k+1)\| \\ &\leq L_{V_i} L_{f_i}^{N-1} \bar{w} \end{aligned}$$

which is also the same bound as in (17). Therefore, we get

$$\Delta J_{i2}^H \leq -h_i(x_i(k+1), u_i^*(k+1|k)) + L_{p_i} \cdot \bar{w}$$

Thus, by letting $L_{p_i} \bar{w} \leq \sigma \cdot h_i(x_i(k+1), u_i^*(k+1|k))$, we get $\Delta J_{i2}^H < 0$, so the stability property is guaranteed. By using the same procedure, we get the bound of $\Delta J_{im}^H = \bar{J}_i^H(k+m) - \bar{J}_i^H(k+m-1)$ for $1 \leq m < N$ as

$$\begin{aligned} \Delta J_{im}^H &= \bar{J}_i^H(k+m) - \bar{J}_i^H(k+m-1) \\ &\leq -h_i(x_i(k+m-1), u_i^*(k+m-1|k)) \\ &\quad + L_{p_i} \cdot \bar{w} \end{aligned}$$

and so the triggering rule is given by

$$L_{p_i} \cdot \bar{w} \leq \sigma \cdot h_i(x_i(k+m-1), u_i^*(k+m-1|k)).$$

This completes the proof of Theorem 1.

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