Nonlinear Event-Triggered Platooning Control with Exponential Convergence

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Abstract: In this paper the control of a platoon with a nonlinear controller under event-triggered communication is investigated. The proposed nonlinear controller is a predecessor-following controller for a certain class of nonlinear functions. For the event-triggered communication scheme every agent decides based on its own state when it transmits its state information. Therefore, a trigger rule is designed that guarantees exponential convergence of the state error while it excludes Zeno behavior. The results are extended to allow heterogeneous trigger rules under certain conditions. Furthermore the case of heterogeneous controllers is analyzed and exponential convergence and exclusion of Zeno behavior is still guaranteed. The theoretical results of the paper are supported by numerical simulations.

Keywords: Event-triggered Control, Platooning, Nonlinear Control

1. INTRODUCTION

Increasing road safety and capacity are among the most important goals in traffic systems. Since platooning is a concept that tries to achieve these goals it is an important point in research to improve control and communication for platoons of vehicles. Recent developments in this area investigate vehicles that are equipped with a communication network and therefore one goal is to reduce the network load while keeping the performance of the platooning controller. In this paper we try to reduce the network load by using an event-triggered control scheme. The design of a trigger rule, that guarantees desired properties, in our case exponential convergence and exclusion of Zeno behavior, is the crucial task in this case, since we allow the platooning controller to be nonlinear. The use of a nonlinear controller can be very meaningful for platooning. For example one could think of a saturation like controller that can be approximated with the nonlinearities used in this paper.

The topic of platooning control has a large history in research. Already Hedrick et al. (1991) worked on the design of longitudinal vehicle controllers for platooning. One of the future research directions was the analysis of string stability for platoons with countably infinite number of vehicles as in Swaroop and Hedrick (1996), which is not the focus of the work at hand. A good overview about many investigations and extensions for two widely used control architectures is given in Hao and Barooah (2013). One of these control architectures is the predecessor-following architecture considered in this paper. Hao and Barooah (2013) also gives the motivation why we investigate the nonlinear controller, since it works better than a linear controller with the same architecture.

The use of event-triggered control schemes started initially with the work in Årzén (1999). It experienced increasing relevance through Tabuada (2007). In his work a trigger rule is presented that guarantees asymptotic stability of a nonlinear system and exclusion of Zeno behavior under the condition that a controller with certain properties exists. This concept was enhanced to perform event-triggered control for distributed systems in Wang and Lemmon (2011) and Dimarogonas et al. (2012). A new approach, where each agent transmits its state information only when the difference of its current state and the last transmitted one crosses a time dependent threshold, was given in Seyboth et al. (2013). The concept of time-dependent trigger functions gives the possibility that one vehicle decides solely on its own absolute measurements when to trigger the next event, whereas for state dependent trigger rules the agent has to know at least some continuous relative measure to neighboring agents. Thus this concept was used in Linsenmayer and Dimarogonas (2015). In this paper, firstly a linear event-triggered symmetric bidirectional controller is analyzed. A first analysis of the nonlinear predecessor-following controller, which is used here, is given as well. The result states a trigger rule, that guarantees boundedness of the state error. Furthermore there is a statement, that guarantees convergence of the state error if the trigger error converges. However, in Linsenmayer and Dimarogonas (2015) there is no tool to derive a trigger rule that has this convergence property while Zeno behavior (see Johansson et al. (1999)), i.e., the occurrence of infinitely many events in finite time, can be excluded. This gap is closed through the work at hand. Therefore it is an crucial extension to existing work in this field. The assumptions on the trigger rule derived in this paper provide a nice similarity to the linear case in Linsenmayer and Dimarogonas (2015) since the condition says that the trigger rule must have a slower decrease than the system dynamics.

⋆ The authors Steffen Linsenmayer and Frank Allgöwer would like to thank the German Research Foundation (DFG) for financial support of the project within the Cluster of Excellence in Simulation Technology (EXC 310/2) at the University of Stuttgart. The work of Dimos Dimarogonas has been supported by the Swedish Research Council (VR)

The quantification on how fast the system decreases is done using the concept of input-to-state exponential stability, being defined in the next section, where one can also find a precise problem definition. In Section 3 the crucial proposition is stated that gives the possibility to derive the trigger rules for our nonlinear problem in Section 4. The theoretical analysis is completed by numerical simulations in Section 5 and a concluding Section 6.

2. PROBLEM STATEMENT AND DEFINITIONS

The following section gives a precise statement of the observed problem. At the end of the section, the necessary definitions used in the main part are collected.

2.1 Problem statement

This paper treats the problem of event-triggered platooning with a nonlinear predecessor-following controller from Hao and Barooah (2013) which has partly been investigated in Linsenmayer and Dimarogonas (2015). Every vehicle is modeled as a double integrator

$$\ddot{p}_i(t) = u_i(t), \quad i \in \mathcal{N} := \{1, \ldots, N\}, \tag{1}$$

where $p_i(t) \in \mathbb{R}$ describes the position of vehicle i at time t. The desired position of vehicle i at time t is denoted by $p_i^*(t) \in \mathbb{R}$. The goal is to follow a fictitious reference vehicle, starting in $p_{0,0}$ and moving with constant velocity v_0, i.e.,

$$p_0(t) \equiv p_0^*(t) = v_0 t + p_{0,0} \tag{2}$$

with desired constant gaps $\Delta_{(i-1,i)}$ between vehicle i and $i-1$. To achieve this goal we use the nonlinear predecessor-following controller for all $i \in \mathcal{N}$, i.e.,

$$u_i = -f(p_i - p_{i-1} + \Delta_{(i-1,i)}) - g(\dot{p}_i - \dot{p}_{i-1}) \tag{3}$$

where f, g are odd, nonlinear functions that are globally Lipschitz and fulfill certain sector nonlinearities as in Hao and Barooah (2013).

The event-triggered approach relies on that each vehicle i transmits its state information, i.e., p_i and $\dot{p}_i$, to its follower only at discrete event times t_k^i, where $k \in \mathbb{N}$. To state the decision rule that determines the event times we define the transmitted errors due to outdated information

$$\begin{aligned} e_i(t) &= \hat{p}_i + (t - t_k^i)\hat{\dot{p}}_i - p_i(t) =: p_{i,foh}(t, t_k^i) - p_i(t) \\ e_{d_i}(t) &= \hat{\dot{p}}_i - \dot{p}_i(t) =: \dot{p}_{i,zoh}(t_k^i) - \dot{p}_i(t), \end{aligned} \tag{4}$$

for all $t_k^i \leq t < t_{k+1}^i$ with $\hat{p}_i = p_i(t_k^i)$ and $\hat{\dot{p}}_i = \dot{p}_i(t_k^i)$ being the last transmitted state information, $p_{i,foh}(t, t_k^i)$ being the first-order hold position estimation at time t with information from t_k^i and $\dot{p}_{i,zoh}(t_k^i)$ the corresponding zero-order hold velocity information.

The discrete event times are determined through the trigger rule

$$\sigma_i(t, h_i(t)) > 0$$

where $h_i(t) = [e_i(t) \;\; e_{d_i}(t)]^T$, i.e., the state information of vehicle i is transmitted to its follower as soon as $\sigma_i(\cdot) > 0$. As our event-triggered predecessor-following controller we apply for all $t : \max\{t_k^i, t_l^{i-1}\} \leq t < \min\{t_{k+1}^i, t_{l+1}^{i-1}\}$, $l \in \mathbb{N}$

$$\begin{aligned} u_i(t) = &-f\left(p_{i,foh}(t, t_k^i) - p_{i-1,foh}(t, t_l^i) + \Delta_{(i-1,i)}\right) \\ &-g\left(\dot{p}_{i,zoh}(t_k^i) - \dot{p}_{i-1,zoh}(t_l^i)\right) \end{aligned} \tag{5}$$

where we use a zero-order hold velocity estimation and a first-order hold position estimation as introduced in (4). Furthermore we define the state errors for all $t \geq 0$,

$$\tilde{p}_i(t) = p_i(t) - p_i^*(t), \; \dot{\tilde{p}}_i(t) = \dot{p}_i(t) - v_0, \tag{6}$$

where $p_i^*(t) = p_0^*(t) - \sum_{j=1}^{i} \Delta_{(j-1,j)} =: p_0^*(t) - \Delta_{(0,i)}$ and $x_i(t) := \left[\tilde{p}_i(t) \;\; \dot{\tilde{p}}_i(t)\right]^T$. With the definitions in (6) and (4), using (1) and applying (5) for all $t \geq 0$, we conclude

$$\begin{aligned} \ddot{\tilde{p}}_i(t) = &-f\left(p_i(t) - p_{i-1}(t) + e_i(t) - e_{i-1}(t) + \Delta_{(i-1,i)}\right) \\ &-g\left(\dot{p}_i(t) - \dot{p}_{i-1}(t) + e_{d_i}(t) - e_{d_{i-1}}(t)\right), \; t \geq 0. \end{aligned} \tag{7}$$

Thus, with (6) and the fact that $\Delta_{(0,i)} - \Delta_{(0,i-1)} = \Delta_{(i-1,i)}$ the closed-loop of the event-triggered predecessor-following platooning controller can be computed as

$$\begin{aligned} \ddot{\tilde{p}}_i(t) = &-f\left(\tilde{p}_i(t) - \tilde{p}_{i-1}(t) + e_i(t) - e_{i-1}(t)\right) \\ &-g\left(\dot{\tilde{p}}_i(t) - \dot{\tilde{p}}_{i-1}(t) + e_{d_i}(t) - e_{d_{i-1}}(t)\right), \; t \geq 0. \end{aligned} \tag{8}$$

The goal of this work is now to derive a trigger function $\sigma_i(t, h_i(t))$ that guarantees exponential convergence of $\|x(t)\|$ to zero, i.e.,

$$\|x(t)\| \leq re^{-\alpha t} \tag{9}$$

where $x(t) \in \mathbb{R}^{2N}$ is the stack vector of state errors for all vehicles. Furthermore Zeno behavior needs to be excluded.

2.2 Definitions

In this paper we use comparison functions, i.e. class $\mathcal{K}$ and class $\mathcal{K}_\infty$ as defined in Definition 4.2 in Khalil (2002). According to this definition a continuous function $\gamma : [0, a) \to [0, \infty)$ belongs to class $\mathcal{K}$ if it is strictly increasing and $\gamma(0) = 0$ and it belongs to class $\mathcal{K}_\infty$ if $a = \infty$ and $\gamma(r) \to \infty$ as $r \to \infty$. Furthermore if the inequality $\alpha x^2 \leq x\gamma(x) \leq \beta x^2$ holds with $\alpha, \beta \in \mathbb{R}$ and $\beta \geq \alpha$, the function $\gamma : \mathbb{R} \to \mathbb{R}$ is said to fulfill the sector nonlinearity $\gamma \in [\alpha, \beta]$.

The main part of this work uses a special case of the well-known input-to-state stability (ISS, Sontag (1989)), called input-to-state exponential stability (ISES), introduced in Grüne et al. (1999).

Definition 1. (ISES, Grüne et al. (1999)). A system $\dot{x} = f(t, x(t), u(t))$ is ISES, if there exist $k \geq 1$, $\lambda > 0$ and $\gamma \in K_\infty$, such that

$$\|x(t)\| \leq \max\left\{ke^{-\lambda(t-t_0)}\|x(t_0)\|, \gamma\left(\sup_{t_0 \leq \nu \leq t}\|u(\nu)\|\right)\right\}.$$

3. EXPONENTIAL CONVERGENCE

In this section we deal with the ISES property at two points. Firstly we investigate which statements we can make about how fast the state of an ISES system converges under an exponentially converging input. Then we show, that each of the vehicles in our platooning control scheme can be seen as an ISES system.

The following proposition is important for this paper. It states that for ISES systems and a sufficiently slow exponentially decreasing input the state of the system can be upper bounded by a function that converges as fast as the input.

Proposition 2. Consider a nonlinear system with state x and input u that is ISES with $\gamma(r) = c_\gamma(r)$ being a linear function, i.e.,

$$\|x(t)\| \leq \max\left\{ke^{-\lambda(t-t_0)}\|x(t_0)\|, c_\gamma \sup_{t_0 \leq \nu \leq t}\|u(\nu)\|\right\}. \tag{10}$$

Assume the input satisfies

$$\|u(t)\| \leq ce^{-\alpha t} \tag{11}$$

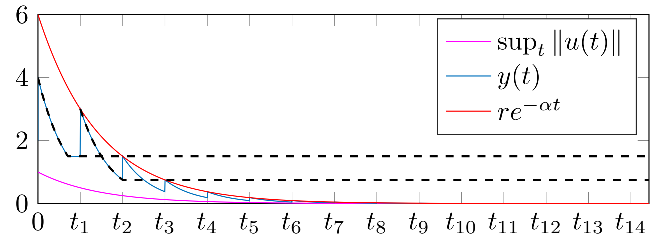

Fig. 1. Figure to express the idea of the proof

with $0 < \alpha < \lambda$, $c > 0$ then

$$\|x(t)\| \le re^{-\alpha t} \tag{12}$$

holds with

$$r = \max\left\{kc_\gamma ce^{\alpha\tau}, k\|x(0)\|\right\} \tag{13}$$

where

$$\tau = \max\left\{\frac{1}{\lambda}\ln\frac{k\|x(0)\|}{c_\gamma c}, \frac{\ln k}{\lambda-\alpha}\right\}. \tag{14}$$

Proof. First of all we define the function $y : \mathbb{R} \to \mathbb{R}$ with $y(0) = \|x(0)\|$ and $y(t) = \max\left\{ky(m\tau)e^{-\lambda(t-m\tau)}, c_\gamma ce^{-\alpha m\tau}\right\}$ for all $t \in (m\tau, (m+1)\tau]$ with $m \in \mathbb{N}_0$. To illustrate why we introduce $y(t)$ Fig. 1 shows the bound on the norm of $u(t)$ in magenta. Furthermore the dotted black line starting at $t = 0$ represents the bound we gain by using (10) with $t_0 = 0$. If we reuse the bound (10) after $\tau > 0$ we gain the bound represented by the second dotted black line. Therefore iterative use of the bound (10) leads to the blue line. Notice, that the blue line is exactly the function $y(t)$. Thus we know $\|x(t)\| \le y(t)$ for all $t \ge 0$. It remains to explain the red line. This line represents an upper bound on $y(t)$, and therefore also on $\|x(t)\|$ which is stated as an exponentially decreasing function $re^{-\alpha t}$. Therefore to prove the proposition we need to show that under the given assumptions on α and r it holds that $y(t) \le re^{-\alpha t}$ for all $t \ge 0$.

For the proof of the existence of such an upper bound the choice of τ is crucial. Therefore we start with computing a lower bound on τ_1 such that $y(\tau_1) = c_\gamma c$, i.e.,

$$k\|x(0)\|e^{-\lambda\tau_1} \le c_\gamma c \quad\Leftrightarrow\quad \tau_1 \ge \frac{1}{\lambda}\ln\frac{k\|x(0)\|}{c_\gamma c} \tag{15}$$

where we assumed $t_0 = 0$ without loss of generality. The next step is to compute a value for $\tau_2 := t_2 - t_1 = t_2 - \tau_1$ that guarantees $y(t_2) = c_\gamma ce^{-\alpha t_1}$ under the assumption $y(t_1) = c_\gamma c$, i.e.,

$$k\underbrace{y(t_1)}_{c_\gamma c}e^{-\lambda\tau_2} = c_\gamma ce^{-\alpha t_1} \quad\Leftrightarrow\quad \tau_2 = \frac{\ln k + \alpha t_1}{\lambda}. \tag{16}$$

If we iterate this procedure we gain the condition

$$\tau_1 \ge \frac{1}{\lambda}\ln\frac{k\|x(0)\|}{c_\gamma c}, \quad \tau_{i+1} = \frac{\ln k + \alpha\tau_i}{\lambda}, \forall i \in \mathbb{N}. \tag{17}$$

In the next step of the proof we compute one global τ that guarantees the demanded bounds from above, i.e., $y(t_i) = c_\gamma ce^{-\alpha t_{i-1}}$ instead of different τ_i. To compute such a τ we analyze the conditions from (17). We will demand the sequence of τ_i to be nonincreasing and show that this can be stated as a condition on τ_1, i.e.,

$$\tau_{i+1} \le \tau_i \quad\Leftrightarrow\quad \frac{\ln k + \alpha\tau_i}{\lambda} \le \frac{\ln k + \alpha\tau_{i-1}}{\lambda}. \tag{18}$$

Thus, the condition

$$\tau_2 = \frac{\ln k + \alpha\tau_1}{\lambda} \le \tau_1 \quad\Leftrightarrow\quad \tau_1 \ge \frac{\ln k}{\lambda-\alpha} \tag{19}$$

guarantees the monotonicity. As a consequence of (15) and (19) we choose

$$\tau = \max\left\{\frac{1}{\lambda}\ln\frac{k\|x(0)\|}{c_\gamma c}, \frac{\ln k}{\lambda-\alpha}\right\} \tag{20}$$

to conclude $y(m\tau) = c_\gamma ce^{-(m-1)\alpha\tau} = c_\gamma ce^{\alpha\tau}e^{-\alpha m\tau}$ and therefore $y(m\tau^+) \le kc_\gamma ce^{\alpha\tau}e^{-\alpha m\tau}$ with $y(m\tau^+) = \lim_{t\to m\tau^+, t>m\tau} y(t)$. Up to now this bound holds only for $m \ge 1$. In the case of $m = 0$ we have the condition $y(0^+) \le k\|x(0)\|$. Thus with $r = \max\left\{kc_\gamma ce^{\alpha\tau}, k\|x(0)\|\right\}$ and $\alpha < \lambda$ we conclude $\|x(t)\| \le re^{-\alpha t}$.

If we see the input as our transmitted error, the proposition can be used in event-triggered control. It serves as a tool to derive a trigger rule for a desired convergence of the state error.

Each vehicle in our platoon with nonlinear predecessor-following controller has dynamics as the system in the following lemma and the special input as in Corollary 4. This Corollary uses Lemma 3 to show that each vehicle with a nonlinear predecessor-following controller that fulfills certain conditions is ISES with respect to the inputs $h_i(t)$, $h_{i-1}(t)$ and $x_{i-1}(t)$. Therefore we can use Proposition 2 to derive our trigger rule in the next section.

Lemma 3. Assume that f, g are globally Lipschitz with Lipschitz constants L_1,L_2 and that they fulfill the following sector nonlinearities; $f \in [\varepsilon_1, K_1]$ and $g \in [\varepsilon_2, K_2]$, where $\varepsilon_1, \varepsilon_2, K_1, K_2 > 0$. Then the system

$$\begin{aligned}\dot{y}_1 &= y_2\\ \dot{y}_2 &= -f(y_1-u_1) - g(y_2-u_2)\end{aligned} \tag{21}$$

is ISES with respect to $u = [u_1 \ u_2]^T$, with $\gamma(r) = c_\gamma r$ being a linear function, i.e.,

$$\|y(t)\| \le \max\left\{ke^{-\lambda(t-t_0)}\|y(t_0)\|, c_\gamma \sup_{t_0\le\nu\le t}\|u(\nu)\|\right\}, \tag{22}$$

where $c_\gamma = \sqrt{\frac{c_2}{c_1}}c_4$, $\lambda = \frac{c_3}{2c_2}$, $k = \sqrt{\frac{c_2}{c_1}}$ with

$$\begin{aligned}c_1 &= \lambda_{\min}(P),\\ c_2 &= \frac{\lambda_{\max}(P) + \eta K_1}{2},\\ c_3 &= \frac{1}{2}(1-\theta)\min\{\varepsilon_1, (\eta\varepsilon_2 - 1)\},\\ c_4 &= \frac{4\eta\max\{L_1, L_2\}}{\theta\min\{\varepsilon_1, (\eta\varepsilon_2 - 1)\}},\end{aligned} \tag{23}$$

where $\eta > \max\left\{1, \frac{1}{\varepsilon_2} + \frac{(1+K_2)^2}{\varepsilon_1\varepsilon_2}\right\}$, $P = \begin{bmatrix}1 & 1\\ 1 & \eta\end{bmatrix}$, $0 < \theta < 1$ and $\lambda_{\min/\max}(P)$ refers to the smallest/largest eigenvalue of the symmetric matrix P.

Proof. From the proof of Proposition 1 in Hao and Barooah (2013) and Lemma 4 in Linsenmayer and Dimarogonas (2015) we resume, that with $V(y) = \frac{1}{2}y^TPy + \eta\int_0^{y_1} f(z)\mathrm{d}z$ we have $c_1\|y\|^2 \le V(y) \le c_2\|y\|^2$ and

$$\dot{V} \le -c_3\|y\|^2 \quad \forall\|y\| \ge c_4\|u\|. \tag{24}$$

Due to the fact that $\|y\| \ge \sqrt{\frac{V}{c_2}}$, (24) implies

$$\begin{aligned}\dot{V} \le -c_3\|y\|^2 \le -\frac{c_3}{c_2}V \qquad & \forall\sqrt{\frac{V}{c_2}} \ge c_4\|u\|\\ \Leftrightarrow \qquad \dot{V} \le -\frac{c_3}{c_2}V \qquad & \forall V \ge c_2(c_4\|u\|)^2\\ \Leftrightarrow V(t) \le V(t_0)e^{-\frac{c_3}{c_2}(t-t_0)} \qquad & \forall V \ge c_2(c_4\|u\|)^2.\end{aligned} \tag{25}$$

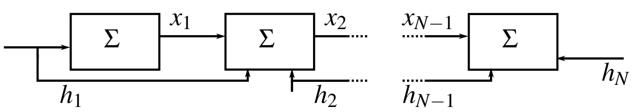

Fig. 2. Interconnection structure

Since $c_1\|y\|^2 \leq V(y) \leq c_2\|y\|^2$ one can conclude

$$c_1\|y\|^2 \leq c_2\|y(t_0)\|^2 e^{-\frac{c_3}{c_2}(t-t_0)} \qquad \forall c_1\|y\|^2 \geq c_2(c_4\|u\|)^2$$
$$\Leftrightarrow \|y(t)\| \leq \sqrt{\frac{c_2}{c_1}}\|y(t_0)\| e^{-\frac{c_3}{2c_2}(t-t_0)} \qquad \forall \|y\| \geq \sqrt{\frac{c_2}{c_1}} c_4\|u\| \tag{26}$$

and hence

$$\|y(t)\| \leq \max\left\{\sqrt{\frac{c_2}{c_1}}\|y(t_0)\| e^{-\frac{c_3}{2c_2}(t-t_0)}, \sqrt{\frac{c_2}{c_1}} c_4 \sup_{t_0 \leq \nu \leq t} \|u(\nu)\|\right\}. \tag{27}$$

holds for all $t \geq 0$. □

The following corollary applies the results of the foregoing lemma to the vehicle dynamics in the platooning case.

Corollary 4. Assume that f,g are globally Lipschitz with Lipschitz constants L_1,L_2 and that they fulfill the following sector nonlinearities, $f \in [\varepsilon_1, K_1]$ and $g \in [\varepsilon_2, K_2]$, where $\varepsilon_1, \varepsilon_2, K_1, K_2 > 0$. Then the state error dynamics of each vehicle

$$\begin{aligned}\ddot{\tilde{p}}_i(t) = &-f\left(\tilde{p}_i(t) - \tilde{p}_{i-1}(t) + e_i(t) - e_{i-1}(t)\right)\\ &-g\left(\dot{\tilde{p}}_i(t) - \dot{\tilde{p}}_{i-1}(t) + e_{d_i}(t) - e_{d_{i-1}}(t)\right)\end{aligned} \tag{28}$$

are ISES with respect to $u(t) = h_i(t) - h_{i-1}(t) - x_{i-1}(t)$, i.e.,

$$\begin{aligned}\|x_i(t)\| \leq \max\Big\{&ke^{-\lambda(t-t_0)}\|x_i(t_0)\|,\\ &c_\gamma \sup_{t_0 \leq \nu \leq t}\left(\|h_i(\nu)\| + \|h_{i-1}(\nu)\| + \|x_{i-1}(\nu)\|\right)\Big\}.\end{aligned} \tag{29}$$

The value for $c_\gamma = \sqrt{\frac{c_2}{c_1}} c_4$, $\lambda = \frac{c_3}{2c_2}$, $k = \sqrt{\frac{c_2}{c_1}}$ can be computed as in Lemma 3.

Proof. Firstly we apply Lemma 3 with $y(t) = \begin{bmatrix}\tilde{p}_i(t)\\ \dot{\tilde{p}}_i(t)\end{bmatrix} = x_i(t)$ and $u(t) = h_i(t) - h_{i-1}(t) - x_{i-1}(t)$ to compute

$$\begin{aligned}\|x_i(t)\| \leq \max\Big\{&ke^{-\lambda(t-t_0)}\|x_i(t_0)\|,\\ &c_\gamma \sup_{t_0 \leq \nu \leq t}\left(\|h_i(\nu) - h_{i-1}(\nu) - x_{i-1}(\nu)\|\right)\Big\}.\end{aligned} \tag{30}$$

With the triangle inequality, i.e.,

$$\begin{aligned}&\|h_i(\nu) - h_{i-1}(\nu) - x_{i-1}(\nu)\|\\ &\leq \|h_i(\nu)\| + \|h_{i-1}(\nu)\| + \|x_{i-1}(\nu)\|,\end{aligned} \tag{31}$$

equation (29) directly follows. □

4. NONLINEAR EVENT-TRIGGERED PLATOONING

In our event-triggered predecessor-following platooning scheme we have an interconnection like in Fig. 2, where all subsystems Σ have the ISES property as shown in Corollary 4. Therefore the following lemma describes the evolution of the norm of the state error for the whole platoon.

Lemma 5. Assume we have an interconnection of N systems as in Fig. 2 where each system is ISES with respect to the inputs h_{i-1}, h_i and x_{i-1} as in (29). Furthermore assume that

$$\|h_j(t)\| \leq ce^{-\alpha t} \tag{32}$$

holds for all $j \in \mathcal{N}$ with $0 < \alpha < \lambda$, $c > 0$. Then there exists $r > 0$, s.t.,

$$\|x(t)\| = \|\left[x_1(t)\ \ldots\ x_N(t)\right]^T\| \leq re^{-\alpha t}. \tag{33}$$

Proof. Since Σ is ISES and (32) holds for $h_1(t)$ we can use Proposition 2 to conclude that there exists a positive $r_1 \in \mathbb{R}$ such that $\|x_1(t)\| \leq r_1 e^{-\alpha t}$. Thus, for the second system we know that $\|x_1(t)\| \leq r_1 e^{-\alpha t}$, $\|h_1(t)\| \leq ce^{-\alpha t}$, $\|h_2(t)\| \leq ce^{-\alpha t}$ and with Proposition 2 the existence of a positive scalar r_2 is guaranteed such that $\|x_2(t)\| \leq r_2 e^{-\alpha t}$ holds. With the same calculation we can iteratively guarantee that a positive $r_i \in \mathbb{R}$ exists such that $\|x_i(t)\| \leq r_i e^{-\alpha t}$ holds for all $i \in \mathcal{N}$. Therefore by choosing $r = \|\left[r_1, \ldots, r_N\right]^T\|$ the statement $\|x(t)\| \leq re^{-\alpha t}$ holds. □

Now we are able to state the main theorem, that states a trigger rule that guarantees asymptotic convergence of the state error and the exclusion of Zeno behavior.

Theorem 6. Suppose a platoon of vehicles modeled as double-integrators (1), controlled with a nonlinear event-triggered predecessor-following controller (5) with f,g being globally Lipschitz with Lipschitz constants L_1,L_2 and fulfilling the following sector nonlinearities; $f \in [\varepsilon_1, K_1]$ and $g \in [\varepsilon_2, K_2]$, where $\varepsilon_1, \varepsilon_2, K_1, K_2 > 0$. Consider the trigger function

$$\sigma_i(t, h_i(t)) = \|h_i(t)\| - ce^{-\alpha t} \tag{34}$$

with $0 < \alpha < \lambda$, $c > 0$, thus guaranteeing the bound $\|h_i(t)\| \leq ce^{-\alpha t}$ for all $i \in \{1, \ldots, N\}$.

Then the norm of the state error exponentially converges to 0, i.e.,

$$\|x(t)\| \leq re^{-\alpha t} \tag{35}$$

and Zeno behavior is excluded.

Proof. The first part of the proof is covered by Corollary 4 and Lemma 5. It remains to show, that Zeno behavior is excluded. As in the proof of Theorem 6 in Linsenmayer and Dimarogonas (2015) we compute $\|\dot{h}_i(t)\| \leq \|h_i(t)\| + |u_i(t)| \leq \|h_i(t)\| + 2(L_1 + L_2)(\|x(t)\| + \|h_i(t)\|)$ where we use the Lipschitz property of f and g. Therefore we can conclude

$$\|\dot{h}_i(t)\| \leq 2(L_1 + L_2)re^{-\alpha t} + (1 + 2(L_1 + L_2))ce^{-\alpha t}$$

and for all $t_k^i \leq t \leq t_{k+1}^i$

$$\|\dot{h}_i(t)\| \leq 2(L_1 + L_2)re^{-\alpha t_k^i} + (1 + 2(L_1 + L_2))ce^{-\alpha t_k^i}$$

and thus

$$\|h_i(t)\| \leq \left(2(L_1 + L_2)re^{-\alpha t_k^i} + (1 + 2(L_1 + L_2))ce^{-\alpha t_k^i}\right)(t - t_k^i).$$

By the definition of the trigger rule the next event is triggered as soon as $\|h_i(t)\| > ce^{-\alpha t}$. Therefore the equation

$$\left(2(L_1 + L_2)re^{-\alpha t_k^i} + (1 + 2(L_1 + L_2))ce^{-\alpha t_k^i}\right)\tau_i = ce^{-\alpha t}$$

multiplied with $e^{\alpha t_k^i}$, i.e.,

$$(2(L_1 + L_2)r + (1 + 2(L_1 + L_2))c)\,\tau_i = ce^{-\alpha \tau_i}$$

gives an implicit lower bound on the inter-execution time $\tau^i = t_{k+1}^i - t_k^i$. This equation shows, that τ^i is the intersection of a linear function with finite gain and a decreasing function with positive initial value. Therefore it is guaranteed, that τ^i is lower bounded and therefore Zeno behavior is excluded. □

With the previous theorem we derived a trigger rule that guarantees exponential convergence of the state error while excluding Zeno behavior. In this case the trigger condition is the same for each vehicle. The following corollary generalizes this statement by allowing different values for α_i and c_i in the triggering rule for each agent. The values for c_i can be arbitrary positive constants while the α_i's need to fulfill certain conditions.

Corollary 7. Consider a platoon of vehicles modeled as double-integrators (1) being controlled with an event-triggered nonlinear predecessor-following controller (5) with f, g being globally Lipschitz with Lipschitz constants L_1,L_2 and fulfilling the following sector nonlinearities, $f \in [\varepsilon_1, K_1]$ and $g \in [\varepsilon_2, K_2]$, where $\varepsilon_1, \varepsilon_2, K_1, K_2 > 0$. Use the trigger functions

$$\sigma_i(t, h_i(t)) = \|h_i(t)\| - c_i e^{-\alpha_i t} \tag{36}$$

with $\lambda > \alpha_1$, $\alpha_i \geq \alpha_{i+1}$, $\alpha_N > 0$ and $c_i > 0$, thus guaranteeing the bound $\|h_i(t)\| \leq c_i e^{-\alpha_i t}$ for all $i \in \mathcal{N}$. Then the norm of the state error exponentially converges to 0 with

$$\|x(t)\| \leq re^{-\alpha_N t} \tag{37}$$

and Zeno behavior is excluded.

Proof. Since Σ is ISES and (36) holds for $h_1(t)$ we conclude from Proposition 2 that a positive $r_1 \in \mathbb{R}$ exists, such that$\|x_1(t)\| \leq r_1 e^{-\alpha_1 t}$ holds. Now we need the condition $\alpha_{i+1} \leq \alpha_i$, i.e., $\|x_1(t)\| \leq r_1 e^{-\alpha_1 t} \leq r_1 e^{-\alpha_2 t}$ and $\|h_1(t)\| \leq c_1 e^{-\alpha_1 t} \leq c_1 e^{-\alpha_2 t}$, to do the same steps as in the proof of Lemma 5 to conclude the existence of positive real values r_i such that $\|x_i(t)\| \leq r_i e^{-\alpha_i t}$ holds for all $i \in \{1, \ldots, N\}$. Therefore by choosing $r = \|[r_1, \ldots, r_N]^T\|$ it holds that $\|x(t)\| \leq re^{-\min_{i \in \mathcal{N}} \alpha_i t} = re^{-\alpha_N t}$.

It remains to show that Zeno behavior is excluded. As before

$$\begin{aligned} \|\dot{h}_i(t)\| &\leq \|h_i(t)\| + |u_i(t)| \\ &\leq c_i e^{-\alpha_i t}(1 + L_1 + L_2) + (L_1 + L_2) \times \\ &\quad (r_i e^{-\alpha_i t} + r_{i-1} e^{-\alpha_{i-1} t} + c_{i-1} e^{-\alpha_{i-1} t}) \end{aligned} \tag{38}$$

and with $\alpha_i \leq \alpha_{i-1}$, for all $t_k^i \leq t \leq t_{k+1}^i$ we get

$$\begin{aligned} \|\dot{h}_i(t)\| \leq{}& c_i e^{-\alpha_i t_k^i}(1 + L_1 + L_2) + (L_1 + L_2) \times \\ & (r_i e^{-\alpha_i t_k^i} + r_{i-1} e^{-\alpha_i t_k^i} + c_{i-1} e^{-\alpha_i t_k^i}). \end{aligned} \tag{39}$$

By the definition of the trigger rule the next event is triggered, as soon as $\|h_i(t)\| > c_i e^{-\alpha_i t}$. Therefore the equation

$$\begin{aligned} \big(c_i(1 + L_1 + L_2) + (L_1 + L_2)(r_i + r_{i-1} + c_{i-1})\big) e^{-\alpha_i t_k^i} \tau_i \\ = c_i e^{-\alpha_i t} \end{aligned} \tag{40}$$

multiplied with $e^{\alpha_i t_k^i}$, i.e.,

$$\begin{aligned} \big(c_i(1 + L_1 + L_2) + (L_1 + L_2)(r_i + r_{i-1} + c_{i-1})\big) \tau_i \\ = c_i e^{-\alpha_i \tau_i} \end{aligned} \tag{41}$$

gives an implicit lower bound on the inter-execution time $\tau^i = t_{k+1}^i - t_k^i$ and we conclude the exclusion of Zeno behavior as in the proof of Theorem 6. □

The foregoing corollary generalizes Theorem 6 towards the use of heterogeneous trigger rules. We can go even one step further by taking into account heterogeneous event-triggered controllers, i.e.,

$$u_{i,et} = -f_i(p_{i,et} - p_{i-1,et} + \Delta_{(i-1,i)}) - g_i(\dot{p}_{i,et} - \dot{p}_{i-1,et}). \tag{42}$$

The nonlinear functions f_i, g_i are chosen in a way that the ISES property for each system can again be concluded from Corollary 4. The main difference is, that (29) holds now with individual positive scalars $c_{\gamma,i}$, k_i and λ_i, i.e.,

$$\begin{aligned} \|x_i(t)\| \leq \max \Big\{ & k_i e^{-\lambda_i(t-t_0)} \|x_i(t_0)\|, \\ & c_{\gamma,i} \sup_{t_0 \leq v \leq t} \big(\|h_i(v)\| + \|h_{i-1}(v)\| + \|x_{i-1}(v)\|\big) \Big\}. \end{aligned} \tag{43}$$

The result is given in the following Corollaray.

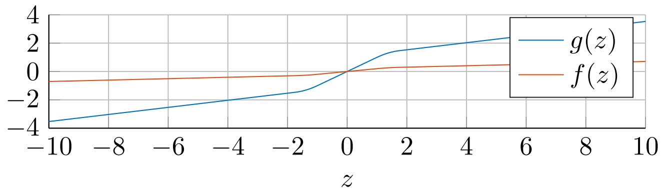

Fig. 3. Nonlinear functions $f(z), g(z)$ in the Simulation example

Corollary 8. Suppose a platoon of vehicles modeled as double-integrators (1), controlled with individual event-triggered nonlinear predecessor-following controllers (42) with f_i, g_i being globally Lipschitz with Lipschitz constants $L_{1,i}$,$L_{2,i}$ and fulfilling the following sector nonlinearities, $f_i \in [\varepsilon_{1,i}, K_{1,i}]$ and $g_i \in [\varepsilon_{2,i}, K_{2,i}]$, where $\varepsilon_{1,i}, \varepsilon_{2,i}, K_{1,i}, K_{2,i} > 0$, guaranteeing that each vehicle is ISES as in (43). Consider the trigger functions

$$\sigma_i(t, h_i(t)) = \|h_i(t)\| - c_i e^{-\alpha_i t} \tag{44}$$

with $0 < \alpha_i < \lambda_i$, $\alpha_i \geq \alpha_{i+1}$ and $c_i > 0$, thus guaranteeing the bound $\|h_i(t)\| \leq c_i e^{-\alpha_i t}$ for all $i \in \mathcal{N}$. Then the norm of the state error exponentially converges to 0 with

$$\|x(t)\| \leq re^{-\alpha_N t} \tag{45}$$

and Zeno behavior is excluded.

Proof. Since Σ_1 is ISES due to Corollary 4 and (44) holds for $h_1(t)$ with $\alpha_1 < \lambda_1$ we conclude the existence of a positive scalar r_1 such that $\|x_1(t)\| \leq r_1 e^{-\alpha_1 t}$ due to Proposition 2. Now we use the conditions $\alpha_{i+1} \leq \alpha_i$, i.e., $\|x_1(t)\| \leq r_1 e^{-\alpha_1 t} \leq r_1 e^{-\alpha_2 t}$ as well as $\|h_1(t)\| \leq c_1 e^{-\alpha_1 t} \leq c_1 e^{-\alpha_2 t}$, and $\alpha_i < \lambda_i$ to do the same steps as in the proof of Lemma 5 to conclude that $\|x_i(t)\| \leq r_i e^{-\alpha_i t}$ for all $i \in \{1, \ldots, N\}$. Therefore by choosing $r = \|[r_1, \ldots, r_N]^T\|$ the statement $\|x(t)\| \leq re^{-\alpha_N t}$ holds. If we compare the framework of Corollary 8 to Corollary 7 we see that the trigger rules are the same and, as shown before the bounds on the state error for each agent decrease with the same rate. Therefore the same computation as in the proof of Corollary 7 excludes Zeno behavior here. □

5. SIMULATION

To illustrate our theoretical results we simulate a platoon consisting of $N = 10$ vehicles, equipped with the nonlinear event-triggered predecessor-following controllers. The chosen nonlinear functions f and g, shown in Fig. 3, are globally Lipschitz and lie in a certain sector. Therefore, Corollary 4 guarantees, that the dynamics of each vehicle is ISES. A lower bound on the parameter λ with these nonlinearities of $\lambda \geq 0.05$, verified through simulations, is used to design the trigger function $\sigma_i = \|h_i(t)\| - e^{-\alpha t}$ with $\alpha = 0.08$. Therefore Theorem 6 guarantees exponential convergence of the state error. In the top subplot of Fig. 4 the evolution of the norm of the state error under event-triggered control $\|x(t)\|_{et}$ confirms this statement. In this simulation the average time between two events of one agent is $\tau_{avg} = 1.6182s$. As a comparison the second subplot in Fig. 4 shows the evolution of the state error under time-triggered information exchange with $\tau_{tt,h} = 0.001s$, while the third subplot is simulated with $\tau_{tt,avg} = 1.6s$. Regarding these plots one can see, that the event-triggered controller attains almost the same performance as the time-triggered controller with very frequent information exchange, and it outperforms the time-triggered controller with constant inter-event times equal to the average time by far.

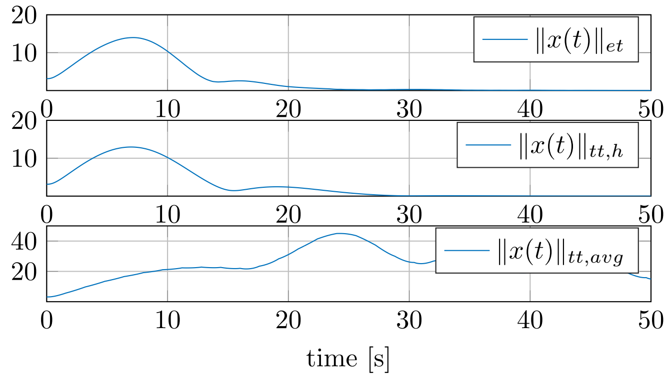

Fig. 4. Evolution of the norm of the state error

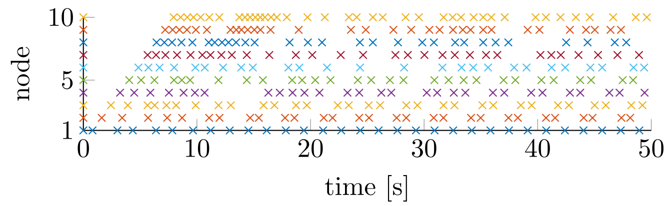

Fig. 5. Inter-event times with common trigger rule

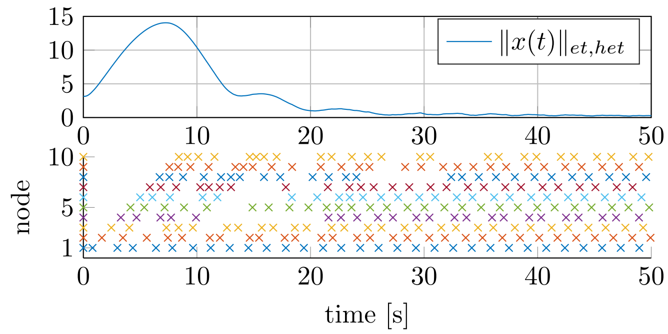

Fig. 6. Evolution of the state error and inter-event times with heterogeneous trigger rule

Taking now a closer look on the inter-event times for this event-triggered controller, one can see in Fig. 5, that for increasing $i \in \mathcal{N}$, the inter-event times of agent i become shorter. This confirms the motivation for allowing individual trigger rules in Corollary 7. As a comparison we simulate the same platoon and controller with the individual trigger functions $\sigma_i = \|h_i(t)\| - e^{-\alpha t}$ with $\alpha_i = 0.08 \cdot 0.85^i$ fulfilling the conditions $\alpha_i \leq \alpha_{i-1}$ and $\alpha_N > 0$. The simulation results in Fig. 6 show that this trigger rule increases the inter-event times for the mentioned agents and therefore also the average to $\tau_{avg} = 1.85s$ while preserving the transient behavior. The asymptotic convergence is slower compared to the previous simulation with a common trigger rule as shown in Corollary 7, since it depends on α_N.

6. CONCLUSION

In this paper we considered the platooning control problem with a nonlinear controller under event-triggered communication. We were able to design a trigger rule that guarantees exponential convergence of the state error for the use of a certain nonlinear event-triggered predecessor-following controller, shown in Theorem 6. Furthermore it is shown, that Zeno behavior is excluded. In a second step we investigated individual trigger rules for individual vehicles, while we can still guarantee exponential convergence and the exclusion of Zeno behavior under some additional conditions on the parameters of the trigger rule in Corollary 7. Further conditions were investigated in Corollary 8 for the case that the nonlinear functions in the controller vary for the different agents. Therefore, in combination with the simulation results, this paper gives a good exploration of the potential of event-triggered communication in combination with nonlinear platooning control. As a future work it will be interesting to explore how mechanisms to guarantee collision avoidance as in Bechlioulis et al. (2014) can be used in an event-triggered platooning framework.

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